



$A \approx 40$ 质量区轴对称和三轴形变晕核的预言

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摘要 近40年来, 科学家一直致力于探索可能存在晕现象的原子核. 迄今, 实验上已确认或建议了约20例晕核或候选晕核, 其中³⁷Mg是目前已知的最重晕核. 随着实验能力的提升, $A \approx 40$ 质量区新晕核的探索已成为可能; 同时, 可靠的理论预言将提供有意义的参考. 相对论密度泛函理论能够统一自洽描述核素图中几乎所有原子核, 是研究晕核的重要工具. 本文简要回顾形变相对论连续谱Hartree-Bogoliubov (deformed relativistic Hartree-Bogoliubov theory in continuum, DRHBc)理论在描述已知晕核和构建原子核质量表等方面取得的成功, 主要介绍DRHBc理论预言形变晕核³⁹Na和⁴²Mg以及新发展的三轴相对论连续谱Hartree-Bogoliubov理论预言三轴形变晕核⁴²Al.

关键词 晕核, 相对论密度泛函理论, 轴对称形变, 三轴形变

在晕核研究中取得成功的形变相对论连续谱Hartree-Bogoliubov (deformed relativistic Hartree-Bogoliubov theory in continuum, DRHBc)理论微观自洽考虑轴对称形变自由度、对关联和连续谱效应, 适用于描述从稳定谷到滴线附近的原子核^[1-4]. 2018年, 为了基于DRHBc理论构建同时考虑形变和连续谱效应的原子核质量表, 统一预言实验未知原子核的基态性质, 为探索元素起源等基本问题提供可靠的微观输入, 北京大学发起并组织了DRHBc质量表合作组^[5]. 在过去几年, 合作组不断壮大, 成员单位数已从初始的10家左右发展为现在的超过30家. 具体的成员单位信息见合作组网站: <https://drhbctable.jcnp.org/>.

2020年, DRHBc质量表合作组发展了基于点耦合

密度泛函的DRHBc理论, 从轻核到重核开展了详尽的数值检验, 确定了质量表构建的数值细节^[6]. 2022年, 合作组成功构建了DRHBc偶偶核质量表, 在质子数 $8 \leq Z \leq 120$ 范围内预言了2583个束缚的偶偶核, 对637个已知实验数据的描述精度为1.52 MeV^[7]. 为了高效计算核子数为奇数的原子核, 合作组发展了基于等填充近似的点耦合DRHBc理论, 实现了自动堵塞方案^[8]. 2024年, 合作组成功构建了DRHBc偶Z核质量表, 在质子数 $8 \leq Z \leq 120$ 范围内预言了4829个束缚的偶Z核, 对1244个已知实验数据的描述精度为1.43 MeV^[9]. 已建立的DRHBc偶偶核与偶Z核质量表给出了原子核质量微观描述的最高精度之一, 吸引了国内外同行的广泛关注. 包含奇Z核的完整DRHBc质量表数据正在整理

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之中。

DRHBc质量表合作组的目标不仅局限于提供大规模计算得到的基态性质数据,而且在于紧密结合实验,例如通过Kr、Sr、Pb和Au同位素的电荷半径建议它们的基态形状演化^[10~12]、通过 $20 \leq Z \leq 30$ 原子核的电荷半径与中子分离能等揭示传统中子闭壳的演化^[13]、解释Hg同位素电荷半径奇偶振荡的起源^[14]、探索原子核形状长椭优势的微观起源^[15]、研究重核的 α 衰变^[16,17]和U同位素的裂变位垒^[18]、计算质子发射核¹⁴⁹Lu的寿命^[19]等。同时,DRHBc理论的预言能为将来的实验提供参考,例如 $10 < Z \leq 20$ 同位素链的中子滴线以及形变效应的影响^[20]、滴线外可能存在的束缚核组成的丰中子半岛^[21~24]、 $72 \leq Z \leq 80$ 核区同时存在气泡结构和形状共存的候选核^[25]、对称能和双核子分离能的关系^[26]等。另一方面,在质量表构建的过程中,合作组在理论方法的优化和发展方面取得了许多成果,包括:原子核密度多极展开随质量数和形变的收敛性检验^[27]、Dirac Woods-Saxon基^[28]的优化^[29]、基于DRHBc理论的角动量投影^[30,31]和两维集体哈密顿量^[32]方法、奇时间DRHBc理论^[33]和三轴相对论连续谱Hartree-Bogoliubov (triaxial relativistic Hartree-Bogoliubov theory in continuum, TRHBc)理论^[34,35]等。

DRHBc理论对晕核的研究是其紧密结合实验的最好范例^[36~38]。DRHBc理论已应用于描述诸多实验建议的晕核,包括^{17,19}B^[39,40]、^{15,19,22}C^[41,42]、³¹Ne^[33]和³⁷Mg^[43]。尤其是,对于目前实验确定的质量数 A 大于30的晕核³¹Ne和³⁷Mg,基于DRHBc理论提供的微观核结构输入,Glauber模型成功再现了它们的核反应实验可观测量,实现了从结构性质到反应动力学的统一自洽描述^[44,45]。除了中子晕核,DRHBc和TRHBc理论近期对滴线核²²Al中是否存在质子晕现象进行了探讨^[46]。这些成功应用无疑证明了DRHBc理论的可靠性。值得一提的是,同样在相对论密度泛函理论的框架下成功研究晕核的还包括相对论连续谱Hartree-Bogoliubov (relativistic continuum Hartree-Bogoliubov, RCHB)理论^[47,48]和相对论Hartree-Fock-Bogoliubov理论^[49,50]等。此外,其他研究晕核的理论模型有非相对论密度泛函理论^[51]、少体模型^[52]、壳模型^[53]、反对称分子动力学模型^[54]和有效场理论^[55]等。

随着世界范围内新一代稀有同位素大科学装置的建造和升级,探索 $A \approx 40$ 质量区的新晕核已成为可能。可靠的微观理论预言能为相关实验研究提供重要参考。

DRHBc理论预言了形变晕核^{42,44}Mg及其核芯和中子晕之间的形状退耦^[1,2],被列为滴线附近有趣的新现象之一^[56]。DRHBc理论还预言了2022年发现的新核素³⁹Na中传统中子幻数 $N=28$ 的消失和形变晕的出现^[57]。新发展的TRHBc理论继承了DRHBc理论的诸多优势并进一步考虑了三轴形变自由度,首次微观预言了三轴形变晕现象。TRHBc理论预言⁴²Al是三轴形变晕核,并且其核芯和晕具有不同的三轴形变,即三轴形状退耦^[34]。在一些更重的质量区亦有理论预言存在晕核,例如价空间介质相似重整化群方法建议 $A \approx 50$ 的丰中子Si、P、S同位素可能是晕核^[58];基于相对论密度泛函理论的复动量表象方法预言⁵³Ar和 $A \approx 80$ 的丰中子Ti、Cr、Fe同位素可能是晕核^[59,60];RCHB理论在 $A > 60$ 的丰中子Ca同位素和 $A > 122$ 的丰中子Zr同位素中预言了多于两个中子形成的巨晕现象^[61~63]。然而, $A \approx 50$ 及更重质量区新晕核的实验探索在短期内还无法实现。

本文将根据元素(质子数 Z)顺序,依次介绍DRHBc理论预言³⁹Na和⁴²Mg的形变晕以及TRHBc理论预言⁴²Al的三轴形变晕。

1 ³⁹Na

探索核素图的边界,即滴线位置,一直是核物理研究的重要前沿^[64]。虽然质子滴线已确定至93号Np元素^[65],但是中子滴线仅确定至10号Ne元素^[66]。2022年,实验发现了目前11号Na元素最为丰中子的同位素³⁹Na,但是未能确定中子滴线^[67]。³⁹Na的中子数为 $N=28$,是一个传统幻数。传统幻数的消失和新幻数的出现是滴线附近奇特原子核中的新奇现象。实验已经揭示了³⁹Na同中子素⁴⁰Mg和⁴²Si中 $N=28$ 幻数的消失^[68,69]。另一方面,除Na和Si同位素之外,从He到P的每一条同位素链的滴线均发现了晕核的迹象。因此,有必要理论研究³⁹Na的壳结构,探索丰中子Na同位素中可能的晕现象,为将来的实验研究提供参考。

图1展示了DRHBc理论计算³⁹Na中子费米面附近的单粒子能级随着四极形变的演化^[57]。可以发现,在球形极限下, $2p_{3/2}$ 和 $1f_{7/2}$ 能级近简并,且 $2p_{1/2}$ 能级和它们十分接近。这一特征与传统的单粒子壳模型给出的结果不符。根据传统单粒子壳模型,稳定原子核中的 $2p_{3/2}$ 和 $1f_{7/2}$ 能级之间存在可观的能隙,形成了 $N=28$ 的闭壳,使得体系处于球形形状时能量最低。在³⁹Na中,由于 $2p$ 能级的降低, $N=28$ 闭壳塌缩,系统在四极形变大于0.4时具有更低的基态能量。与此同时,对关联效应使

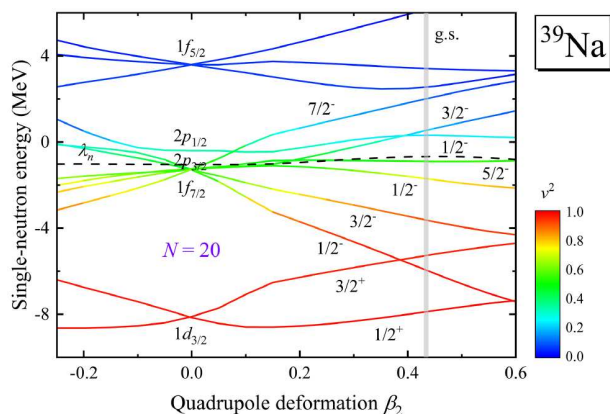


图1 (网络版彩色)DRHBc理论计算 ^{39}Na 中子费米面(虚线)附近的正则基单粒子能级。标记了球形极限下的量子数 nlj 和长椭一侧的量子数 Ω^π 。占据几率 v^2 由颜色标度。灰色竖线对应 ^{39}Na 基态。图片取自文献[57]

Figure 1 (Color online) Single-neutron levels around the Fermi surface λ_n (dashed line) of ^{39}Na in the canonical basis calculated from the DRHBc theory. Their quantum numbers nlj in the spherical limit and Ω^π on the prolate side are labeled. The occupation probability v^2 is scaled by colors. The grey vertical line corresponds to the ground state (g.s.) of ^{39}Na . Taken from Ref. [57]

得 ^{39}Na 的价中子部分地占据弱束缚的和连续谱中的 $\Omega^\pi = 1/2^-$ 和 $3/2^-$ 能级(Ω 是总角动量沿对称轴方向的投影, π 是宇称)。由于形变效应, 这些能级的波函数中存在 p 成分的混合, 而价中子的 p 波成分有利于中子晕的形成。因此, ^{39}Na 中可能会出现晕现象。

图2展示了 $N \geq 20$ 的奇质量Na同位素的中子密度沿对称轴与垂直于对称轴方向的分布[57]。由于 $N=20$ 闭壳, ^{31}Na 的基态是球形的, 而其他更重的Na同位素具有大于0.35的四极形变。因此, 在图2(a)中, 沿着对称轴方向, 从 ^{31}Na 到 ^{33}Na 中子密度分布的显著增加是形变效应导致的。此后, 从 ^{33}Na 到 ^{41}Na , 沿对称轴方向的中子密度分布随着中子数的增加而缓慢增加。在垂直于对称轴方向, 如图2(b)所示, 从 ^{31}Na 到 ^{37}Na 中子密度分布表现出类似的缓慢增加。然而, 在远离中心处, 即使具有长椭圆形变的基态, ^{39}Na 和 ^{41}Na 的密度分布仍十分弥散。这意味着 ^{39}Na 和 ^{41}Na 中存在扁椭的中子晕, 和长椭的核芯发生了形状退耦。关于Na同位素中子滴线位置的讨论以及晕现象形成微观机制的进一步探索, 可参考文献[57]。

2 ^{42}Mg

在2000年左右, 由于缺乏足够的实验证据, 形变原子核中晕现象的存在性还是一个有争议的课题。虽然

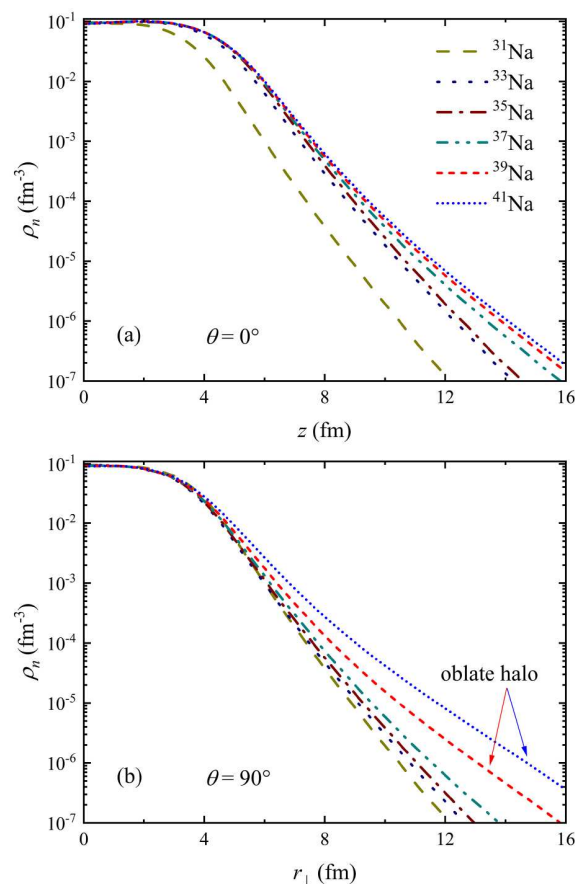


图2 (网络版彩色)DRHBc理论计算的奇质量Na同位素 $^{31-41}\text{Na}$ 的中子密度沿对称轴方向($\theta=0^\circ$) (a)和垂直于对称轴方向($\theta=90^\circ$) (b)的分布。图片取自文献[57]

Figure 2 (Color online) The DRHBc calculated neutron density distributions along ($\theta=0^\circ$) (a) and perpendicular to ($\theta=90^\circ$) (b) the symmetry axis for odd-mass sodium isotopes $^{31-41}\text{Na}$. Taken from Ref. [57]

基于唯象模型开展了一些关于形变晕核的讨论[70,71], 但仍存在着质疑形变晕核或形变晕的研究。例如, 基于球形Woods-Saxon势场的计算, 指出球形满壳可以解释轻核区的滴线位置, 并建议已知的滴线核都是球形的[72]; 基于轴对称形变Woods-Saxon势场的计算, 发现当 $\Omega^\pi = 1/2^+$ 轨道的能量接近连续谱阈值时, 不论形变大小, $s_{1/2}$ 成分均会在其波函数中占绝对主导[73]; 三体模型计算指出, 核子间的关联与核芯形变或激发引起的关联会阻碍晕的形成, 不支持滴线附近的形变晕核[74]。为了解决争议, 需要更多的实验证据和更加微观自洽的理论研究。

鉴于RCHB理论微观研究球形晕核取得的成功[47,61], 在其基础上进一步考虑形变自由度, 发展相应

的理论模型,是微观自洽探索形变晕核的自然选择.虽然RCHB理论在坐标空间求解球形相对论Hartree-Bogoliubov (RHB)方程,但是形变RHB方程的坐标空间求解非常困难^[75].为此,提出了波函数具有正确渐近行为的Woods-Saxon基,并证明了Woods-Saxon基空间求解基本等价于坐标空间求解^[28],为DRHBc理论的发展奠定了基础.

2010年,在Woods-Saxon基空间求解形变RHB方程,利用Legendre多项式展开原子核的密度和势场,发展了DRHBc理论^[1,2].DRHBc理论系统研究了镁同位素链中偶偶核的基态性质,合理地再现了它们的中子分离能、均方根半径和形变的实验值^[2].通过分析角度平均密度分布、沿对称轴以及垂直于对称轴的密度分布,揭示了形变原子核 ^{42}Mg 中的晕现象.图3展示了 ^{42}Mg 晕和核芯的中子密度在 xz 平面的密度分布.可以看到,晕密度显著弥散,尤其是垂直于对称轴方向,呈现出扁椭形状;核芯密度主要沿对称轴方向分布,呈现出长椭形状.因此,DRHBc理论预言 ^{42}Mg 是存在形状退耦的形变晕核.虽然目前实验室合成的最重镁同位素是 ^{40}Mg ^[76],但是蒙特卡罗壳模型^[77]和价空间介质相似重整化群方法^[78]的计算结果均支持 ^{42}Mg 大概率作为束缚原子核存在.DRHBc理论对 ^{37}Mg ^[43]在内的已知晕核的成功描述将促进实验物理学家对 ^{42}Mg 晕现象进行探索和验证.

3 ^{42}Al

非轴对称形变,即三轴形变,是原子核的基本形变自由度之一.它与丰富的核物理现象密切相关,在原子核的手征转动^[79]、摇摆运动^[80]和裂变^[81]等方面的研究中至关重要.最近,物理学家发现相对论重离子碰撞实验可以提取原子核的形变^[82],并在欧洲大型强子对撞机的实验数据中找到了三轴形变原子核的迹象^[83],极大地激发了对三轴形变原子核的研究兴趣.目前,三轴原子核中的晕现象是一个鲜有探索的课题.

2021年,基于三轴形变Woods-Saxon势的研究指出,考虑三轴形变可能会增加晕现象出现的几率^[84].这是因为,引入三轴形变会使轴对称形变下具有不同磁量子数的单粒子态发生混合,从而允许部分弱束缚单粒子态的波函数中出现 s 或 p 波成分.然而,该研究中势场的三轴形变、深度和宽度等均为唯象可调参数,并且没有考虑重要的对关联和连续谱效应.

最近发展的TRHBc理论微观自洽考虑了三轴形变

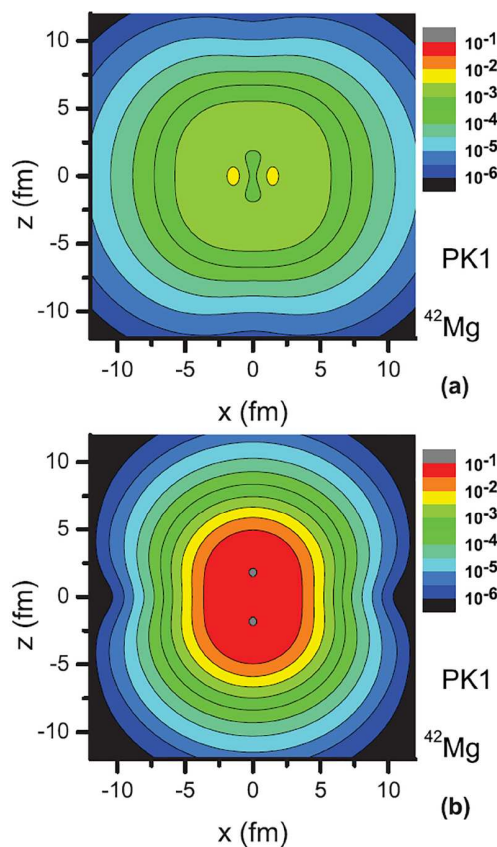


图3 (网络版彩色)基态 ^{42}Mg 的密度分布. (a) 中子晕; (b) 中子核芯. 其中 z 轴是原子核的对称轴. 图片取自文献^[2]

Figure 3 (Color online) Density distributions of the ground state of ^{42}Mg with the z axis as the symmetry axis. (a) Neutron halo; (b) neutron core. Taken from Ref. ^[2]

自由度、对关联和连续谱效应,对Al同位素开展了研究^[34].TRHBc理论正确给出了铝同位素链质子滴线的位置,并高精度再现了铝同位素的中子分离能和电荷半径已有实验数据. TRHBc理论预言最后一个束缚的奇中子铝同位素是 ^{42}Al ,其基态是三轴形变的. TRHBc理论计算 ^{42}Al 的中子分离能约为0.6 MeV,与最新原子核质量评估建议的0.67(64) MeV^[85]相符.弱束缚的价中子有助于三轴形变晕的形成.为了定量估计中等质量原子核内可能的晕现象,提出了晕标度(halo scale)的概念,定义为晕核子带来的均方根半径增量与经验公式描述该增量的比值.计算结果表明,在Al同位素中,仅有弱束缚的 ^{40}Al 和 ^{42}Al 的晕标度大于2. RCHB理论预言的Zr同位素中的巨晕原子核^[61]对应的晕标度也均大于2.因此, ^{40}Al 和 ^{42}Al 中可能存在晕现象.

为了研究弱束缚和连续谱能级,图4展示了 ^{42}Al 中

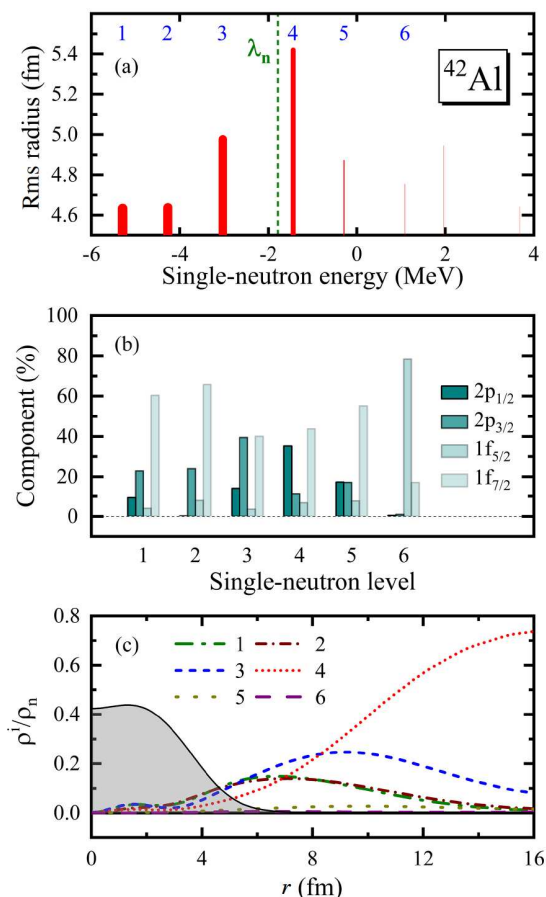


图4 (网络版彩色)(a) ^{42}Al 中子费米面附近正则基单粒子能级的均方根半径对能量. 每条能级的宽度正比于其占据几率. (b) 能级1~6的主要球形成分. (c) 能级对总中子密度的贡献. 总中子密度(任意单位制)由灰色区域表示. 密度的角度依赖已进行平均. 图片取自文献[34]

Figure 4 (Color online) (a) The rms radius versus the energy for the single-particle levels around the neutron Fermi surface in the canonical basis for ^{42}Al . The thickness of each level is proportional to its occupation probability. (b) The main spherical components for the levels 1–6. (c) Contributions of these levels to the total neutron density. The shaded region represents the total neutron density in arbitrary units. Here the angular dependence is averaged. Taken from Ref. [34]

子费米面附近的单粒子能级和它们的半径、成分、对总中子密度的贡献. 在图4(a)中, ^{42}Al 价中子占据的弱束缚能级4的均方根半径显著大于其他附近能级的均方根半径. 这是因为, 能级4的 $2p$ 成分大于45%, 如图4(b)所示. 虽然能级3也具有类似的 $2p$ 成分, 但是它相对较为深束缚, 抑制了其波函数的弥散. 从图4(c)可以看到, 能级4的贡献从 $r > 9$ fm后变为主导, 甚至在 $r > 14$ fm后超过了70%. 因此, 弱束缚的价中子对于 ^{42}Al 总密度的弥散起决定性作用, 符合单中子晕的物理图像.

根据图4, 中子费米面在能量上和空间分布上自然区分了贡献 ^{42}Al 核芯和晕的单粒子能级. 由此, 分别计算了核芯和晕在不同平面的中子密度分布, 如图5所示. 晕的密度显著比核芯密度更加弥散, 尤其是在 yz 平面. 根据密度定量计算得出核芯和晕的均方根半径分别为3.85和5.26 fm, 三轴形变参数(β, γ)分别为(0.38, 50°)和(0.79, -23°). 晕的负 γ 值意味着其密度分布的中间轴是 y 而短轴是 x , 与核芯的情形恰好相反. 利用半径和形变信息, 图5(g)和(h)分别展示了核芯和晕的示意图, 可以清楚地区分它们的短轴、中间轴和长轴. 这种核芯和晕的三轴形状退耦是在三轴形变层次上预言的新奇现象. 关于三轴形变晕更详细的结果和讨论, 可参考文献[34].

TRHBc理论是微观研究三轴形变奇特原子核的有力工具, 其研究范围和DRHBc理论同样广泛. 最近, TRHBc理论揭示质子发射核 ^{149}Lu 的三轴形变, 探索了三轴形变效应对其寿命的影响[86]. 未来更多基于TRHBc理论的研究值得期待.

4 总结与展望

晕现象一经发现就成为了实验和理论核物理研究的前沿热点. 历经近10年的探索, 已经在质子数不超过12、质量数不超过40的约20例原子核中发现了晕的迹象, 其中 ^{37}Mg 是目前已知的最重晕核. 世界范围内稀有同位素大科学装置的升级和建造持续促进着晕核的实验研究, $A \approx 40$ 质量区新晕核的探索已成为可能. 同时, 可靠的理论预言将提供有意义的参考.

相对论密度泛函理论能够统一自洽描述核素图中几乎所有原子核, 是研究晕核最重要的理论工具之一. DRHBc理论不仅成功描述了诸多已知晕核, 而且用于构建首个考虑形变和连续谱效应的原子核质量表. 对于偶偶核与偶 Z 核, DRHBc质量表已成功建立, 给出了原子核质量微观描述的最高精度之一. 在DRHBc质量表的构建过程中, 开展了一系列紧密结合实验的理论研究. 本文简要回顾了这些进展, 重点介绍了DRHBc理论预言形变晕核 ^{39}Na 和 ^{42}Mg 以及新发展的TRHBc理论预言三轴形变晕核 ^{42}Al .

形变晕核的理论研究预言了核芯与晕可能存在各种形状退耦, 包括球形-长椭退耦[33]、长椭-扁椭退耦[1]、高阶形变退耦[43]以及三轴形变退耦[34]. 这意味着形状退耦可能在晕核中普遍存在, 因为晕的形状不取决于核芯, 而是依赖于晕核子占据的能级及其成分.

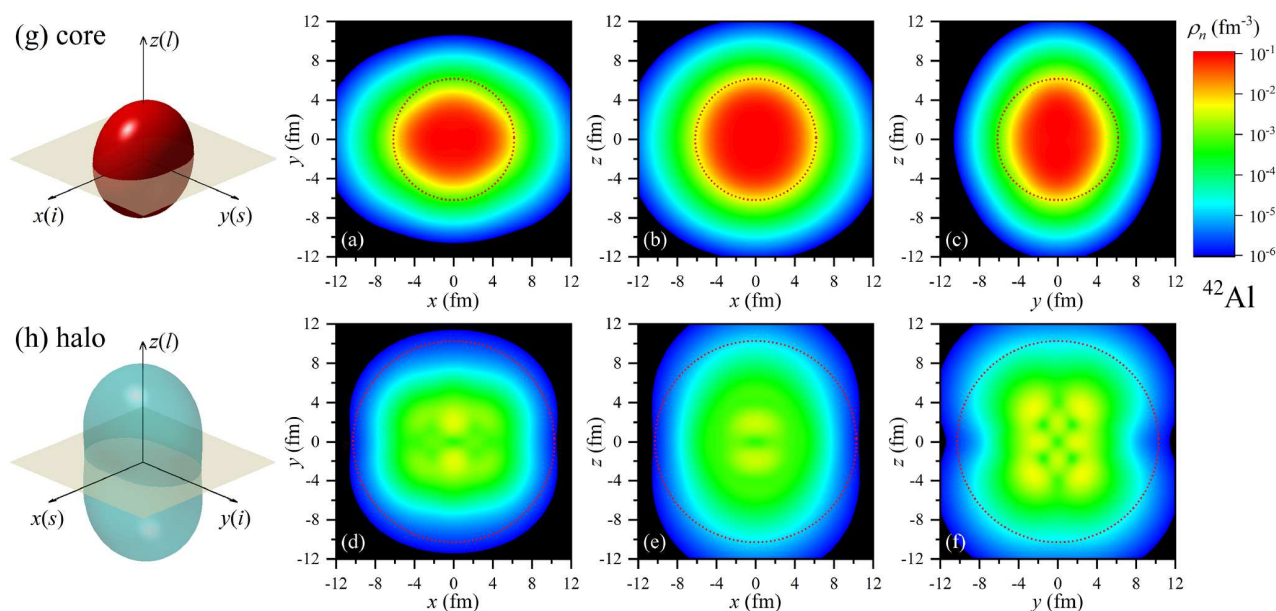


图5 (网络版彩色)TRHBc理论预言的三轴晕核 ^{42}Al 的核芯((a)~(c))和晕((d)~(f))分别在 xy 、 xz 和 yz 平面的中子密度分布。根据密度计算的均方根半径和形变, (g)和(h)分别给出了核芯和晕的示意图, 其中 s 、 i 和 l 分别表示密度分布的短轴、中间轴和长轴。图片取自文献[34], 有轻微修改

Figure 5 (Color online) Neutron density distributions in xy , xz , and yz planes for the core ((a)–(c)) and the halo ((d)–(f)) in the TRHBc predicted triaxial halo nucleus ^{42}Al . With the rms radius and deformation parameters calculated from densities, the corresponding schematic shapes for the core and the halo are given respectively in (g) and (h), where s , i , and l respectively represent the short, intermediate, and long axes. Taken from Ref. [34] and slightly modified

然而, 哪些实验可观测量能够帮助揭示晕核中的形状退耦仍是一个有待回答的问题。结合核反应模型与DRHBc

理论或TRHBc理论, 从核结构到核反应自洽统一描述已知晕核并预言新晕核, 是值得长期研究的课题。

致谢 感谢孟杰和张双全的长期指导和帮助。感谢孟杰、张双全和周善贵对本文提出的宝贵修改意见。感谢与耿立升、Peter Ring、孙保华、Dario Vretenar、赵鹏巍、周善贵等人关于三轴形变晕现象的讨论。感谢与DRHBc质量表合作组全体成员的合作。

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Summary for “ $A \approx 40$ 质量区轴对称和三轴形变晕核的预言”

Prediction of axially and triaxially deformed halo nuclei in the $A \approx 40$ mass region

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The halo phenomenon has long been a frontier in experimental and theoretical nuclear physics research. Over nearly forty years of exploration, signals of halos have been identified in approximately 20 nuclei, with ^{37}Mg being the heaviest among them. The worldwide development of rare isotope beam facilities continues to advance the experimental exploration of new halo nuclei in the $A \approx 40$ mass region. Meanwhile, relevant theoretical predictions are increasingly desirable.

The relativistic density functional theory is a crucial tool for studying halo nuclei. The deformed relativistic Hartree-Bogoliubov theory in continuum (DRHBc) is capable of describing deformed exotic nuclei by incorporating self-consistently axial deformation degrees of freedom, pairing correlations, and continuum effects. It has provided microscopic descriptions for several known halo nuclei, such as ^{31}Ne and ^{37}Mg . The success of the DRHBc theory is further evidenced by the high-precision DRHBc mass table and related applications to various nuclear phenomena. The DRHBc theory has predicted the newly discovered nuclide ^{39}Na and the yet-to-be-synthesized ^{42}Mg as deformed halo nuclei.

It has been found that the lowering of the $2p$ orbitals in the spherical limit causes the collapse of the $N=28$ shell closure in ^{39}Na , leading to a well-deformed ground state. Pairing correlations, together with the mixing of pf components driven by deformation, result in the occupation of weakly bound or continuum p -wave neutron orbitals. Consequently, an oblate halo forms around the prolate core in ^{39}Na .

The experimental neutron separation energies and empirical matter radii of neutron-rich magnesium isotopes, along with the deformed p -wave halo characteristics of ^{37}Mg , are well reproduced by the DRHBc theory. A deformed neutron halo in ^{42}Mg is revealed through the analysis of angle-averaged densities as well as densities along and perpendicular to the symmetry axis for magnesium isotopes. It is shown that the deformation of a potential neutron halo depends essentially on the quantum numbers of the main components of the halo orbital(s).

The triaxial relativistic Hartree-Bogoliubov theory in continuum (TRHBc), which considers triaxiality, pairing correlations, and continuum effects, has been recently developed to explore triaxial halo nuclei. The TRHBc calculated neutron separation energies and charge radii for aluminum isotopes show good agreement with available experimental data. The heaviest observed odd-odd aluminum isotope, ^{42}Al , is predicted to be a triaxial halo nucleus with a novel shape decoupling, characterized by an exchange of intermediate and short axes from the well-deformed core to the super-deformed halo.

Combining nuclear reaction models with the DRHBc or TRHBc theory to provide a unified description of known halo nuclei and predict new ones, from structural properties to reaction dynamics, is a topic worthy of long-term exploration. Further applications of the DRHBc and TRHBc theories to other exotic phenomena near the dripline are also anticipated in the future.

halo nuclei, relativistic density functional theory, axial deformation, triaxial deformation

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