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二维各向异性介质中地震波前面参数方程的建立*

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关键词 各向异性介质、地震波、参数方程、法向方程、三叉区现象

本文从二维各向异性介质中地震波动偏微分方程组入手, 建立了各向异性介质中波前面的法向方程, 并利用特征带法求解常规的一阶偏微分方程获得了二维各向异性介质中地震波前面的参数方程.

1 波前面法向方程的建立

假定二维各向异性介质中, 扰动置于坐标系原点($x=z=0$), $t=0$ 处形成的波前随时间增加由该点向外传播. 设 $\varphi(x, z, t)=0$ 为波前面方程, 同时考虑到在波前面上存在 $\frac{\partial^2 u_x}{\partial \varphi^2}$, $\frac{\partial^2 u_y}{\partial \varphi^2}$ 与 $\frac{\partial^2 u_z}{\partial \varphi^2}$ 二阶位移导数跃变, 而其它导数保持连续.

$$Lu=0 \tag{1}$$

变成如下问题的求解:

由一、二阶导数锁链法则, 可将二维各向异性介质中地震波动问题[1, 2]:

$$Mu_{\varphi\varphi}=0, \tag{2}$$

其中 $[u=u_x\ u_y\ u_z]^T$, $u_{\varphi\varphi}=\left[\frac{\partial^2 u_x}{\partial \varphi^2}\quad \frac{\partial^2 u_y}{\partial \varphi^2}\quad \frac{\partial^2 u_z}{\partial \varphi^2}\right]^T$,

u_x, u_y 与 u_z 分别为 x, y 与 z 向位移分量, 偏微分算子矩阵 L 的组成元素详见参考文献 [2]. 偏微分算子矩阵 M 的组成元素分别为:

$$M_{11}=A_{11}\left(\frac{\partial \varphi}{\partial x}\right)^2+A_{55}\left(\frac{\partial \varphi}{\partial z}\right)^2+2A_{15}\frac{\partial \varphi}{\partial x}\cdot\frac{\partial \varphi}{\partial z}-\left(\frac{\partial \varphi}{\partial t}\right)^2,$$

$$M_{12}=A_{16}\left(\frac{\partial \varphi}{\partial x}\right)^2+A_{45}\left(\frac{\partial \varphi}{\partial z}\right)^2+(A_{14}+A_{56})\frac{\partial \varphi}{\partial x}\cdot\frac{\partial \varphi}{\partial z},$$

$$M_{13}=A_{15}\left(\frac{\partial \varphi}{\partial x}\right)^2+A_{35}\left(\frac{\partial \varphi}{\partial z}\right)^2+(A_{13}+A_{55})\frac{\partial \varphi}{\partial x}\cdot\frac{\partial \varphi}{\partial z},$$

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$$M_{22}=A_{66}\left(\frac{\partial\varphi}{\partial x}\right)^2+A_{44}\left(\frac{\partial\varphi}{\partial z}\right)^2+2A_{46}\frac{\partial\varphi}{\partial x}\cdot\frac{\partial\varphi}{\partial z}\left(\frac{\partial\varphi}{\partial t}\right)^2,$$

$$M_{23}=A_{56}\left(\frac{\partial\varphi}{\partial x}\right)^2+A_{34}\left(\frac{\partial\varphi}{\partial z}\right)^2+(A_{36}+A_{45})\frac{\partial\varphi}{\partial x}\cdot\frac{\partial\varphi}{\partial z},$$

$$M_{33}=A_{55}\left(\frac{\partial\varphi}{\partial x}\right)^2+A_{33}\left(\frac{\partial\varphi}{\partial z}\right)^2+2A_{35}\frac{\partial\varphi}{\partial x}\cdot\frac{\partial\varphi}{\partial z}-\left(\frac{\partial\varphi}{\partial t}\right)^2,$$

$$M_{21}=M_{12},$$

$$M_{31}=M_{13},$$

$$M_{32}=M_{23},$$

这里，$A_{ij}=C_{ij}/\rho$，$C_{ij}(i,\ j=1,\ 2,\cdots 6)$ 为各向异性介质弹性参数，ρ 为介质密度.

由于二阶导数跃变 $u_{\varphi\varphi}$ 各元素不同时为零，因而可以得到波前面 $\varphi(x,\ z,\ t)=0$ 上的波前面特征偏微分方程:

$$Q\left(\frac{\partial\varphi}{\partial x},\ \frac{\partial\varphi}{\partial z},\ \frac{\partial\varphi}{\partial t}\right)=0, \tag{3}$$

其中

$$Q\left(\frac{\partial\varphi}{\partial x},\ \frac{\partial\varphi}{\partial z},\ \frac{\partial\varphi}{\partial t}\right)=\left[A_{11}\left(\frac{\partial\varphi}{\partial x}\right)^2+A_{55}\left(\frac{\partial\varphi}{\partial z}\right)^2+2A_{15}\frac{\partial\varphi}{\partial x}\cdot\frac{\partial\varphi}{\partial x}\left(\frac{\partial\varphi}{\partial t}\right)^2\right]$$

$$\cdot\left[A_{66}\left(\frac{\partial\varphi}{\partial x}\right)^2+A_{44}\left(\frac{\partial\varphi}{\partial z}\right)^2+2A_{46}\frac{\partial\varphi}{\partial x}\cdot\frac{\partial\varphi}{\partial z}-\left(\frac{\partial\varphi}{\partial t}\right)^2\right]\cdot\left[A_{55}\left(\frac{\partial\varphi}{\partial x}\right)^2+A_{33}\left(\frac{\partial\varphi}{\partial z}\right)^2\right.$$

$$\left.+2A_{35}\frac{\partial\varphi}{\partial x}\cdot\frac{\partial\varphi}{\partial z}-\left(\frac{\partial\varphi}{\partial t}\right)^2\right]+2\left[A_{16}\left(\frac{\partial\varphi}{\partial x}\right)^2+A_{45}\left(\frac{\partial\varphi}{\partial z}\right)^2+(A_{14}+A_{56})\frac{\partial\varphi}{\partial x}\cdot\frac{\partial\varphi}{\partial z}\right]$$

$$\cdot\left[A_{56}\left(\frac{\partial\varphi}{\partial x}\right)^2+A_{34}\left(\frac{\partial\varphi}{z}\right)^2+(A_{36}+A_{45})\frac{\partial\varphi}{x}\cdot\frac{\partial\varphi}{z}\right]\cdot\left[A_{15}\left(\frac{\partial\varphi}{\partial x}\right)^2+A_{35}\left(\frac{\partial\varphi}{\partial z}\right)^2+(A_{13}+A_{55})\right.$$

$$\left.\frac{\partial\varphi}{\partial x}\cdot\frac{\partial\varphi}{\partial z}\right]-\left[A_{66}\left(\frac{\partial\varphi}{\partial x}\right)^2+A_{44}\left(\frac{\partial\varphi}{\partial z}\right)^2+2A_{46}\frac{\partial\varphi}{\partial x}\cdot\frac{\partial\varphi}{\partial z}-\left(\frac{\partial\varphi}{\partial t}\right)^2\right]\cdot\left[A_{15}\left(\frac{\partial\varphi}{\partial x}\right)^2\right.$$

$$\left.+A_{35}\left(\frac{\partial\varphi}{\partial z}\right)^2+(A_{13}+A_{55})\frac{\partial\varphi}{\partial x}\cdot\frac{\partial\varphi}{\partial z}\right]^2-\left[A_{11}\left(\frac{\partial\varphi}{\partial x}\right)^2+A_{55}\left(\frac{\partial\varphi}{\partial z}\right)^2+2A_{15}\frac{\partial\varphi}{\partial x}\cdot\frac{\partial\varphi}{\partial z}\right.$$

$$\left.-\left(\frac{\partial\varphi}{\partial t}\right)^2\right]\cdot\left[A_{56}\left(\frac{\partial\varphi}{\partial x}\right)^2+A_{34}\left(\frac{\partial\varphi}{\partial z}\right)^2+\left(A_{36}+A_{45}\right)\frac{\partial\varphi}{\partial x}\cdot\frac{\partial\varphi}{\partial z}\right]^2-\left[A_{55}\left(\frac{\partial\varphi}{\partial x}\right)^2+A_{33}\left(\frac{\partial\varphi}{\partial z}\right)^2\right.$$

$$\left.+2A_{35}\frac{\partial\varphi}{\partial x}\cdot\frac{\partial\varphi}{\partial z}-\left(\frac{\partial\varphi}{\partial t}\right)^2\right]\cdot\left[A_{16}\left(\frac{\partial\varphi}{\partial x}\right)^2+A_{45}\left(\frac{\partial\varphi}{\partial z}\right)^2+(A_{14}+A_{56})\frac{\partial\varphi}{\partial x}\cdot\frac{\partial\varphi}{\partial z}\right]^2.$$

由偏微分方程理论，一 二阶及其更高阶间断将会沿特征面传播. 因此，各向异性介质中地震波前面为如下特征面:

$$\varphi(x,\ z,\ t)=\psi(x,\ z)-t=0. \tag{4}$$

由以上结果可以得到以下一阶偏微分方程:

$$Q(\psi_x,\ \psi_z,\ -1)=0. \tag{5}$$

假定波前法向曲线 $\psi_x=p$, $\psi_z=q$, 则有法向曲线满足如下六阶方程:

$$Q(p,\ q,\ -1)=F(p,\ q)=0, \tag{6}$$

通过引入极坐标

$$\begin{aligned} p&=R\cos\theta,\\ q&=R\sin\theta \qquad (0\leqslant\theta<2\pi), \end{aligned} \tag{7}$$

可知各向异性介质中波前面法向方程为

$$F(R,\ \theta)=AR^6+BR^4+CR^2-1=0, \tag{8}$$

其中

$$\begin{aligned} &A(\theta)=A_1(\theta)A_2(\theta)A_3(\theta)+2A_4(\theta)A_5(\theta)A_6(\theta)-[A_2(\theta)A_6^2(\theta)+A_1(\theta)A_5^2(\theta)+A_3(\theta)A_4^2(\theta)],\\ &B(\theta)=A_4^2(\theta)+A_5^2(\theta)+A_6^2(\theta)-\{[A_1(\theta)+A_2(\theta)]A_3(\theta)+A_1(\theta)A_2(\theta)\},\\ &C(\theta)=A_1(\theta)+A_2(\theta)+A_3(\theta),\\ &A_1(\theta)=A_{11}\cos^2\theta+A_{55}\sin^2\theta+A_{15}\sin^2 2\theta,\\ &A_2(\theta)=A_{66}\cos^2\theta+A_{44}\sin^2\theta+A_{46}\sin^2 2\theta,\\ &A_3(\theta)=A_{55}\cos^2\theta+A_{33}\sin^2\theta+A_{35}\sin^2 2\theta,\\ &A_4(\theta)=A_{16}\cos^2\theta+A_{45}\sin^2\theta+\frac{1}{2}(A_{14}+A_{56})\sin^2 2\theta,\\ &A_5(\theta)=A_{56}\cos^2\theta+A_{34}\sin^2\theta+\frac{1}{2}(A_{36}+A_{45})\sin^2 2\theta,\\ &A_6(\theta)=A_{15}\cos^2\theta+A_{35}\sin^2\theta+\frac{1}{2}(A_{13}+A_{55})\sin^2 2\theta, \end{aligned}$$

显然(8)式6个根对应于传播方向相反的三类波.

2　波前面参数方程的建立

我们知道,波前面满足如下方程:

$$\psi(x,\ z)-t=0, \tag{9}$$

通过假定地震波在原点(0, 0)处起始激发,因而有 $\psi(0,\ 0)=0$. 利用特征带法解一阶偏微分方程(法向方程):

$$F(p,\ q)=0, \tag{10}$$

可以得到

$$\begin{aligned} &\frac{\mathrm{d}x}{\mathrm{d}s}=F_p, \quad \frac{\mathrm{d}z}{\mathrm{d}s}=F_q,\\ &\frac{\mathrm{d}p}{\mathrm{d}s}=-(F_\psi\cdot q+F_x)=0,\\ &\frac{\mathrm{d}q}{\mathrm{d}s}=-(F_\psi\cdot p+F_z)=0, \end{aligned} \tag{11}$$

而且

$$\frac{\mathrm{d}\psi}{\mathrm{d}s}=pF_p+qF_q. \tag{12}$$

由初始条件:

$$x(0)=z(0)=0, \quad p(0)=p_0,\ q(0)=q_0,\ \psi(0)=0, \tag{13}$$

可知沿特征曲线 S, p 与 q 均为常数.

经过系列推导可以得到

$$S=\frac{t}{pF_p+qF_q},$$
$$X=\frac{tF_p}{pF_p+qF_q}, \tag{14}$$
$$Z=\frac{tF_p}{pF_p+qF_q}.$$

由(7)式微分有

$$\mathrm{d}p=(R_\theta\cos\theta-R\sin\theta)\,\mathrm{d}\theta,$$
$$\mathrm{d}q=(R_\theta\sin\theta+R\cos\theta)\,\mathrm{d}\theta,$$
$$\frac{\mathrm{d}p}{\mathrm{d}q}=\frac{R_\theta\cos\theta-R\sin\theta}{R_\theta\sin\theta+R\cos\theta}=\frac{-v_\theta\cos\theta-v\sin\theta}{-v_\theta\sin\theta+v\cos\theta},$$

这里

$$v(\theta)=\frac{1}{R(\theta)}, \tag{15}$$
$$v_\theta=\frac{\partial v(\theta)}{\partial\theta}=-\frac{R_\theta}{R^2}.$$

考虑到 $F_p\,dp+F_g\,dq=0$ 并结合以上各式,经过系列推导可以得到二维各向异性介质波前面的参数方程:

$$X=t[v(\theta)\sin\theta+v_\theta\cos\theta],$$
$$Z=t[v(\theta)\cos\theta-v_\theta\sin\theta].$$

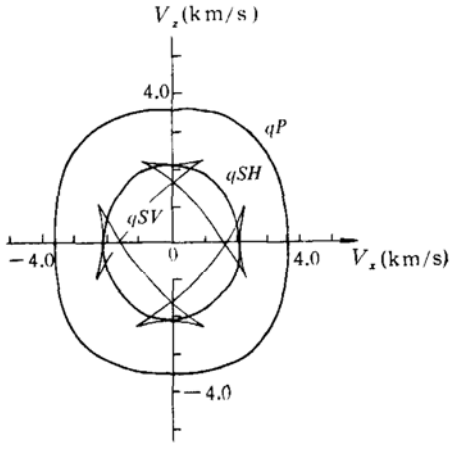

图1 $qP_{h,l}SV$ 和 qSH 波前面

3 计算与分析

我们利用以上研究结果计算了视弹性参数 (C_{ij}/ρ) 分别为 $A_{11}=7.84$, $A_{12}=1.568$, $A_{13}=2.613$, $A_{23}=1.568$, $A_{22}=2.852$, $A_{33}=7.84$, $A_{44}=1.568$, $A_{55}=2.613$, $A_{66}=1.568$ 的正交各向异性介质中三类地震波 qP, qSV 和 qSH 的波前面. 由图1可见, qSH 波前面为椭圆, qP, qSH 波前面很复杂,不仅表现出 qSV 与 qSH 速度差异与方向性,还显示出各向异性诱导的特异现象即波前面上的三叉区现象,并值得进一步研究与利用.

参 考 文 献

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