

Joint ternary coprime array and sparse Bayesian learning for underwater acoustic target bearing estimation

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Abstract To enhance underwater acoustic target bearing estimation, a novel scheme integrating a ternary coprime array (TCA) with sparse Bayesian learning (SBL) is proposed. The TCA architecture, comprising three uniform subarrays, is leveraged for its high degrees of freedom (DOFs) and reduced mutual coupling. Notably, the TCA maintains a closed-form DOFs expression regardless of element variations, facilitating modularity and scalability well-suited for complex and dynamic sea environments. For multi-snapshot scenarios, a vectorized SBL algorithm tailored for the TCA is developed. Comprehensive simulations and validation using sea-trial data confirm its superior estimation accuracy over classical sparse arrays across varying signal-to-noise ratios (SNRs) and snapshot numbers in both narrowband and broadband conditions. To address the challenging few-snapshot case, an innovative compressed-grid SBL strategy confined to specific angular subintervals is derived. An estimate matching framework is devised to merge the high-precision yet aliased estimates from sparse subarrays with the aliasing-free but low-precision estimates from the dense subarray. Finally, simulation results demonstrate that this compressed-grid approach offers significant performance advantages over existing methods at identical grid densities.

Keywords Sonar signal processing, Ternary coprime array, Sparse Bayesian learning, Bearing estimation, Grid strategy

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1 Introduction

Passive sonar is a key technology for underwater acoustic target detection, recognition, and localization, serving as the primary mode for submarine systems, hull-mounted sonars on deep-sea unmanned underwater vehicles (UUVs), and underwater beacons, by receiving and addressing acoustic signals from targets and the environment. Passive sonar offers low power consumption and high stealth, with array signal processing being one of its core operating approaches^[1]. By leveraging energy accumulated across array elements and extracting the phase information from the received signals, array signal processing can achieve spatial beamforming gain, thereby extending the sonar system's effective range and enabling robust target detection and tracking as well as precise bearing estimation^[2]. Indeed, direction of arrival (DOA) estimation is foundational to signal-source localization and beamforming, with important applications in radar, sonar, and wireless communications^[3,4]. Two principal factors affect the bearing estimation performance of linear sonar arrays: the

geometry configuration of the array and the DOA estimation algorithm. Under the paradigm of sparsity, the sparse linear array (SLA) architecture design^[5,6] and the sparse Bayesian learning (SBL)^[7,8] based on compressed sensing and sparse signal recovery theory^[9,10] have concurrently become prominent research hotspots.

In the far field, the received signal is often approximated as a plane wave, and the bearing estimation accuracy of a linear array improves with increasing effective aperture. However, the traditional minimum-spacing uniform linear array (ULA), typically using N physical elements with half-wavelength inter-element spacing, can estimate at most $N-1$ sources, which makes it relatively low in cost-effectiveness^[11,12]. Non-uniformly arranging sonar array elements can yield richer pairwise element spacings and a larger aperture, thereby achieving higher resolution, more degrees of freedom (DOFs), and reduced mutual coupling. Nested array (NA)^[13] and prototype coprime array (PCA)^[14] are two classic SLAs that exhibit a clear geometric structure and have closed-form expressions for their element positions. The NA

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consists of two sub-ULAs, a dense sub-array and a sparse sub-array, providing high DOFs, modularity, and scalability. A key drawback is the strong coupling in the dense sub-array, which degrades practical performance. Several upgraded SLAs derived from the NA have been proposed^[15-18]. However, NA and its variants still exhibit notable coupling effects. Similarly, PCA is also composed of two uniform sub-arrays. Although its DOFs are not as prominent as those of some recently developed SLAs, the potential of fully exploiting the coprime property, especially multi-coprime configurations, for high-performance SLA design has attracted sustained attention and investigation^[19-21]. To combine the high DOFs and scalability of NA with the low coupling of PCA, our recent work^[22] introduced a ternary coprime array (TCA), which incorporates an additional sub-ULA with large spacing upon the PCA structure. Compared with PCA, the TCA maintains low mutual coupling due to its coprime-based structure. Furthermore, its three sub-arrays create a richer diversity of inter-element spacings, markedly increasing the DOFs. Additionally, the TCA forms an architecture similar to NA, consisting of both dense and sparse sub-arrays. Thus, it features modularity and scalability. This means that as the number of elements changes, the closed-form expression for the DOFs remains valid, making it well-suited to complex and dynamic sea environments. It allows rapid replacement of faulty elements and flexible adding or removing elements to support different estimation methods as tasks and conditions change. To illustrate this, an example of scaling the TCA is presented in the second set of simulations in Sec. 5.

The traditional DOA estimation methods can be broadly categorized into spatial filtering, subspace, and maximum-likelihood classes. For spatial filtering, the conventional beamforming (CBF) algorithm is a typical representative^[23], which offers robustness and low computational load but is limited by its spatial resolution and sidelobe level. To overcome these limitations, adaptive beamforming has been introduced. The minimum variance distortionless response (MVDR) or Capon beamformer, a robust adaptive technique, was first proposed by Capon in 1969 and applied to geophysical array signal processing^[24]. The good performance of the conventional MVDR algorithm relies on accurate estimation of the true spatial covariance of the sound field, which assumes stationarity over long observation times and thus may not adapt well to complex and dynamic ocean environments. Traditional subspace methods (e.g., multiple signal classification (MUSIC), estimation of signal parameters via rotational invariance techniques (ESPRIT)) achieve excellent performance at high signal-to-noise ratio (SNR) and with ample snapshots but degrade sharply in low SNR, with few snapshots, or in coherent-source scenarios^[25-27]. In the past few years, SBL, as a machine learning algorithm, was initially proposed by Tipping^[28] and subsequently introduced into the sparse signal recovery field by Wipf^[29]. The core idea of SBL

is to discretize the continuous bearing space into an over-complete dictionary, turning DOA estimation into a sparse signal reconstruction problem, and to use SBL to automatically adjust model hyperparameters, achieving high resolution and accuracy in complex scenarios. Compared with traditional DOA methods, SBL offers adaptive regularization, robustness at low SNR and with coherent sources, and does not require prior knowledge of the number of sources. For practical DOA applications of SBL-type algorithms, researchers have developed methods such as relevance vector machine (RVM) DOA^[30], fast block-sparse Bayesian SBL algorithms^[31-33], multi-snapshot broadband SBL algorithms^[34-40], broadband DOA estimation based on variational Bayesian inference (VBI)^[41-43], and higher-order cumulant SBL algorithms^[44], extending SBL from narrowband to broadband scenarios and improving convergence speed.

Grid strategy is a critical research hotspot in SBL, as the inevitable grid mismatch introduces significant estimation errors. While increasing grid density can mitigate quantization errors, it severely hikes computational complexity and violates the restricted isometry property (RIP), causing numerical instability^[45,46]. To address this, the adaptive grid refinement (AGR) strategy^[47] uses interpolation to concentrate grids efficiently but struggles under low-SNR conditions. For off-grid issues, several frameworks have been proposed but show distinct limitations: the sparse total least squares (STLS)^[48] relies on an idealized Gaussian perturbation assumption; Basic SBL (B-SBL)^[7] suffers from truncation errors due to first-order Taylor approximation; and the root-finding framework^[49], despite outperforming B-SBL, is restricted to ULAs and computationally expensive for large arrays. Regarding SBL for sparse arrays, extensive research has also been conducted. Vectorized approaches based on nested or coprime arrays^[50,51] effectively suppress noise but at the cost of reduced array aperture. Improved methods utilizing all virtual elements^[52] or real-valued transformations^[53] have enhanced estimation accuracy and stability. However, they are burdened by high computational loads from large-scale matrix inversions or singular value decomposition (SVD) operations, especially with fine grids. Although grid-free methods^[54] offer a convex optimization alternative to avoid grid issues, their complexity remains a hurdle. Consequently, a comprehensive SBL solution specifically developed for particular SLA configurations deployed in complex dynamic sea environments is still lacking.

In multi-snapshot scenarios, given the inherent advantages of the TCA in DOFs, mutual coupling, and modularity, combining it with vectorized SBL theoretically yields superior performance. This paper validates this premise through simulations and sea-trial data. However, sea environments pose a severe limitation: rapid target motion and channel non-stationarity often restrict observations to limited snapshots. Under such constraints, covariance-based

vectorized methods inevitably fail due to rank deficiency. Consequently, a critical challenge emerges: how to achieve robust high-precision estimation solely by innovating signal processing algorithms, without the luxury of altering the physical array deployment. To tackle this challenge, this work partitions the TCA observations into a “dense sub-array” set designed to resolve ambiguity and a “sparse sub-array” set responsible for high precision. By integrating this configuration information, a novel method named compressed-grid SBL (CG-SBL) is proposed. Specifically, the bearing aliasing mechanism is analyzed from a sampling theorem perspective to derive angular subintervals of a specific width that contain all true or aliased bearings. Grids are then established exclusively within these subintervals. By compressing the same number of grid points into these narrower intervals, higher resolution is achieved; conversely, compared to a full bearing interval dense grid, computational complexity is significantly reduced. Finally, simulations confirm the effectiveness of the proposed CG-SBL.

2 Preliminary analysis for SLA signal processing

This section presents the basic signal processing model of SLA under a narrowband approximation to facilitate the initial analysis. Considering an SLA comprising V physical elements, one element is chosen as the origin, and the positive direction along the array defines a one-dimensional Cartesian coordinate system. The vector $\tilde{\mathbf{P}} = [\tilde{p}_1, \dots, \tilde{p}_V]$ provides the positions of the array elements along the axis. Let f denote the narrowband center frequency and c_w denote the speed of sound in water, the wavelength is $\lambda = c_w / f$. Normalizing $\tilde{\mathbf{P}}$ by $\lambda / 2$ yields an equivalent array-configuration vector that describes the SLA geometry as

$$\mathbf{P} = \frac{\tilde{\mathbf{P}}}{\lambda/2} = [p_1, \dots, p_V]. \quad (1)$$

Suppose K far-field, narrowband, uncorrelated signals impinge on the array, with source directions $\theta_1, \theta_2, \dots, \theta_K$. At time t , the array's received signal at the t th snapshot can be modeled as

$$\mathbf{x}(t) = \sum_{k=1}^K \mathbf{a}(\theta_k) s_k(t) + \mathbf{n}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \quad (2)$$

where $\mathbf{A} = [\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_K)] \in \mathbb{C}^{V \times K}$ is the steering matrix; $\mathbf{a}(\theta_k) = [e^{-j\pi p_1 \sin \theta_k}, \dots, e^{-j\pi p_V \sin \theta_k}]^T \in \mathbb{C}^{V \times 1}$ ($k = 1, 2, \dots, K$) is the steering vector; $\mathbf{s}(t) = [s_1(t), s_2(t), \dots, s_K(t)]^T \in \mathbb{C}^{K \times 1}$ denotes the source signals vector, with $s_k(t)$ being the signal transmitted by the k th source from direction θ_k ; $\mathbf{n}(t) \in \mathbb{C}^{V \times 1}$ is the array noise. The covariance matrix $\mathbf{x}(t)$ can be calculated by

$$\mathbf{R}_{\mathbf{XX}} = \mathbb{E}[\mathbf{x}(t)\mathbf{x}^H(t)] = \mathbf{A}\mathbf{R}_{\mathbf{SS}}\mathbf{A}^H + \sigma_n^2 \mathbf{I}_V, \quad (3)$$

where $\mathbf{R}_{\mathbf{SS}} = \mathbb{E}[\mathbf{s}(t)\mathbf{s}^H(t)] = \text{diag}([\sigma_1^2, \dots, \sigma_K^2])$ is a K -order diagonal matrix, representing the covariance of the source, where σ_k^2 is the signal power of the k th source; σ_n^2 is the noise power of the array element and $\mathbf{I}_V \in \mathbb{R}^{V \times V}$ denotes the identity matrix of order V . The vectorization of the covariance matrix $\mathbf{R}_{\mathbf{XX}}$ yields:

$$\mathbf{z} = \text{vec}(\mathbf{R}_{\mathbf{XX}}) = (\mathbf{A}^* \odot \mathbf{A})\mathbf{J} + \sigma_n^2 \text{vec}(\mathbf{I}_V) = \mathbf{B}\mathbf{J} + \sigma_n^2 \mathbf{H}, \quad (4)$$

where $\mathbf{B} = [\mathbf{a}^*(\theta_1) \otimes \mathbf{a}(\theta_1), \dots, \mathbf{a}^*(\theta_K) \otimes \mathbf{a}(\theta_K)] \in \mathbb{C}^{V^2 \times K}$, $\mathbf{J} = [\sigma_1^2, \sigma_2^2, \dots, \sigma_K^2]^T \in \mathbb{C}^{K \times 1}$, the symbol $\otimes$ denotes the Kronecker product and $\mathbf{H} = \text{vec}(\mathbf{I}_V)$. If the received signal is ergodic, its covariance can be estimated by the time average of $\mathbf{x}(t)\mathbf{x}^H(t)$:

$$\hat{\mathbf{R}}_{\mathbf{XX}} = \frac{1}{T} \sum_{t=1}^T \mathbf{x}(t)\mathbf{x}^H(t), \quad (5)$$

where T is the number of snapshots.

From this perspective, Eqs. (3) and (4) are two different mathematical representations of the same physical quantity. When Eq. (3) is treated as the observation model, the vectorized data $\mathbf{z}$ can be regarded as the signal received by the virtual array; $\mathbf{B}$, the steering matrix for the virtual array, serves as the linear observation matrix; and $\mathbf{J}$ can be seen as the incident signal vector. Equation (4) shows that $\mathbf{B}$ has a higher row rank than $\mathbf{A}$ in Eq. (3). This rank advantage is the fundamental reason that SLA can estimate more source directions than the minimum-spacing ULA with V physical elements when the received signals span multiple snapshots. Specifically, the difference co-array of the SLA corresponding to $\mathbf{P}$ is

$$\mathbf{D} = \{p_i - p_j, \forall p_i, p_j \in \mathbf{P}, 1 \leq i, j \leq V\}. \quad (6)$$

Clearly, the number of elements in set $\mathbf{D}$ is equal to the row rank of matrix $\mathbf{B}$.

There is acoustic coupling between the elements of a hydrophone array, which is typically described by a mutual impedance matrix $\mathbf{C}$, and the coupling leakage γ quantify the degree of mutual coupling of the array:

$$\gamma = \frac{\|\mathbf{C} - \text{diag}(\mathbf{C})\|_F}{\|\mathbf{C}\|_F}, \quad (7)$$

where $\|\cdot\|_F$ represents the Frobenius norm; $\text{diag}(\cdot)$ maps a square matrix to the diagonal matrix consisting of its diagonal elements; $\mathbf{C}$ is approximated by a symmetric Toeplitz matrix with sideband G :

$$[\mathbf{C}]_{i,j} = \begin{cases} c|p_i - p_j|, & |p_i - p_j| \leq G, \\ 0, & |p_i - p_j| > G. \end{cases} \quad (8)$$

The maximum spacing of the array elements that produce the coupling is usually $G = 100$. The coupling coefficients are $c_1 = ce^{j\pi/3}$, $c_g = c_1 e^{-j(g-1)\pi/8} / g$ ($2 \leq g \leq G$). Given Ref. [19], let $c = 0.3$.

When the broadband signal is generated by the source, the resulting K -dimensional signal vector is denoted by $\mathbf{s}^w(t) = [s_1^w(t), s_2^w(t), \dots, s_K^w(t)]^T$, and the array noise vector is denoted by $\mathbf{n}^w(t)$. The broadband frequency range $[f_L, f_H]$ can be divided into D equal subbands in ascending order. The center frequency of the d th subband is f_d , and the corresponding array-configuration vector can be regarded as a function of such frequency:

$$\mathbf{P}(f_d) = \frac{2\tilde{\mathbf{P}}}{c_w/f_d} = [p_1(f_d), \dots, p_V(f_d)]. \quad (9)$$

The t th broadband source signal $\mathbf{s}^w(t)$ and the array receive signal $\mathbf{x}^w(t)$ can be modeled as the superposition of each subband component:

$$\mathbf{s}^w(t) = \sum_{d=1}^D \mathbf{s}(t, f_d), \quad (10)$$

$$\mathbf{x}^w(t) = \sum_{d=1}^D \mathbf{x}(t, f_d) = \sum_{d=1}^D \mathbf{A}(f_d) \mathbf{s}(t, f_d) + \mathbf{n}^w(t), \quad (11)$$

where $\mathbf{A}(f_d) = [\mathbf{a}(\theta_1, f_d), \mathbf{a}(\theta_2, f_d), \dots, \mathbf{a}(\theta_K, f_d)]$ and $\mathbf{a}(\theta_k, f_d) = [e^{-j\pi p_1(f_d) \sin \theta_k}, \dots, e^{-j\pi p_V(f_d) \sin \theta_k}]^T$ denote the d th subband's steering matrix and the steering vector for the θ_k direction, respectively.

Additionally, the broadband receive signal needs to calculate the covariance separately on each subband:

$$\begin{aligned} \mathbf{R}_{\mathbf{xx}}(f_d) &= \mathbb{E}[\mathbf{x}(t, f_d) \mathbf{x}^H(t, f_d)] \\ &= \mathbf{A}(f_d) \mathbf{R}_{\mathbf{ss}} \mathbf{A}^H(f_d) + \sigma_n^2(f_d) \mathbf{I}_V. \end{aligned} \quad (12)$$

Thereafter, the vectorized form is

$$\begin{aligned} \mathbf{z}(f_d) &= (\mathbf{A}(f_d)^* \odot \mathbf{A}(f_d)) \mathbf{J}(f_d) + \sigma_n^2(f_d) \text{vec}(\mathbf{I}_V) \\ &= \mathbf{B}(f_d) \mathbf{J}(f_d) + \sigma_n^2(f_d) \mathbf{H}, \end{aligned} \quad (13)$$

where $\mathbf{J}(f_d)$, $\mathbf{B}(f_d)$, and $\sigma_n^2(f_d)$ represent the signal power, virtual observation matrix, and noise power on the d th subband, respectively.

For underwater acoustic arrays, time-domain samples captured by the hydrophone array are typically real-valued. To reveal the phase information present in the array-received signal, the samples should be partitioned into frames, and a fast Fourier transform (FFT) should be applied to each frame, so that the output frequency-domain coefficient is used as the snapshot in Eqs. (2)–(5). Moreover, the narrowband snapshot $\mathbf{x}(t)$ of Eq. (2) is given by the Fourier coefficient of the central frequency, and the broadband signal

$\mathbf{x}^w(t)$ of Eq. (11) is the sum of the Fourier coefficient $\mathbf{x}(t, f_d)$ of each subband frequency.

3 Ternary coprime array configurations

The PCA is composed of two sub-ULAs overlaid together, with their first elements coinciding and both arrays oriented along the array axis in the forward direction, as shown in Fig. 1. Sub-ULA 1 contains N array elements, and sub-ULA 2 contains M array elements, N and M are coprime. The total number of array elements is $V = M + N - 1$. In addition, sub-ULA 1 has an inter-element spacing of $M(\lambda/2)$, while sub-ULA 2 has an inter-element spacing of $N(\lambda/2)$. The equivalent array-configuration vector of the PCA is

$$\begin{aligned} \mathbf{P}_{\text{PCA}} &= \text{Sort}(\{Mn | 0 \leq n \leq N-1\} \\ &\quad \cup \{Nm | 0 \leq m \leq M-1\}), \end{aligned} \quad (14)$$

where $\text{Sort}(\cdot)$ represents the vector of sorting the finite set elements from small to large. The difference co-array of the PCA can be determined by

$$\text{Card}(\mathbf{D}_{\text{PCA}}) = MN + M + N - 2, \quad (15)$$

where $\text{Card}(\cdot)$ represents the number of different elements in the set and this value is also known as the DOFs of an SLA. When an SLA is used to receive multiple snapshot signals, its DOFs determine the row rank of the observation matrix $\mathbf{B}$ in Eq. (4).

The I-type TCA (TCA-I) is one of the two TCA configurations proposed in Ref. [22]. It is constructed by augmenting the PCA with a sub-ULA 3 containing $Q+1$ elements, whose inter-element spacing is set to $(M+N)(\lambda/2)$. For TCA-I, the total number of elements is $M+N+Q-1$, and the array structure is shown in Fig. 2.

The equivalent array-configuration vector of the TCA-I is given by



Fig. 1 Structure of PCA.

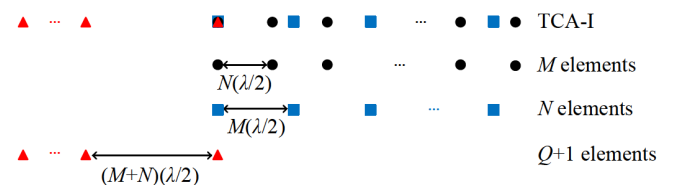


Fig. 2 Structure of TCA-I.

$$\begin{aligned} \mathbf{P}_{\text{TCA-I}} = & \text{Sort}(\{Mn | 0 \leq n \leq N-1\} \\ & \cup \{Nm | 0 \leq m \leq M-1\} \\ & \cup \{-(M+N)q | 1 \leq q \leq Q\}). \end{aligned} \quad (16)$$

The difference co-array of the TCA-I is

$$\begin{aligned} \mathbf{D}_{\text{TCA-I}} = & \{\pm(mN-nM) | 0 \leq m \leq M-1, 0 \leq n \leq N-1\} \\ & \cup \{\pm((M+N)q+Mn) | 1 \leq q \leq Q, 0 \leq n \leq N-1\} \\ & \cup \{\pm((M+N)q+Nm) | 1 \leq q \leq Q, 0 \leq m \leq M-1\}. \end{aligned} \quad (17)$$

The number of elements contained in the $\mathbf{D}_{\text{TCA-I}}$ can be derived as

$$\text{Card}(\mathbf{D}_{\text{TCA-I}}) = MN + (2Q+1)(M+N) - 2(Q+1). \quad (18)$$

The TCA-I consists of three ULAs, supporting modular prefabrication; more importantly, Eq. (18) holds true as Q varies, ensuring straightforward DOFs acquisition during array scaling. With the same number of physical array elements, the DOFs of the TCA-I far exceed those of PCA. Notably, the TCA-I is a modular, scalable array, and even if the number of elements Q in the sub-ULA 3 changes, the close expression for the array's DOFs remains governed by Eq. (18). Equations (14) and (16) provide the SLA equivalent array-configuration vectors under the narrow-band, single-frequency incident signals assumption, and can also be interpreted as a purely geometrical proportional structure not tied to any physical background.

When the SLA is applied to the actual broadband hydroacoustic signal, the equivalent array configuration vector $\mathbf{P}(f_d)$ at a specific frequency can be obtained by combining purely geometrical proportional structure and f_d . At this time, although the elements in $\mathbf{P}(f_d)$ are no longer presented as integers like Eqs. (14) and (16), the number of difference co-array set elements, i.e., DOFs for an SLA, is still held by Eqs. (15) and (17).

4 Sparse Bayesian learning for TCA-I

4.1 Basic SBL for DOA estimation

Underwater acoustic signal sources are sparse in the spatial domain, as shown in Fig. 3. In view of this, within the SBL

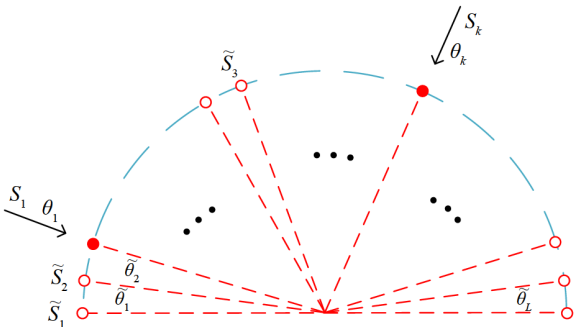


Fig. 3 Spatial grid division diagram.

framework, the spatial range of -90° – 90° is uniformly divided into L grids and the grid point is denoted as $\{\tilde{\theta}_1, \tilde{\theta}_2, \dots, \tilde{\theta}_L\}$. Each grid corresponds to a potential incident signal direction, and the number K is much smaller than L , to ensure the sparsity of the azimuth angle space. In Fig. 3, “•” indicates the actual location of the incident signal source, while “○” indicates that there is no real source. The steering matrix $\tilde{\mathbf{A}}$ is formed by expanding the original steering matrix $\mathbf{A}$ from K columns to L columns; it can be treated as an over complete dictionary, with $\tilde{\mathbf{A}} = [\mathbf{a}(\tilde{\theta}_1), \mathbf{a}(\tilde{\theta}_2), \dots, \mathbf{a}(\tilde{\theta}_L)] \in \mathbb{C}^{I \times L}$. The grid resolution increases with L ; a larger L yields a finer discretization of azimuth angles, causing the incident signal to be closer to the grid point. For sufficiently high grid resolution, $\{\theta_1, \theta_2, \dots, \theta_K\} \subset \{\tilde{\theta}_1, \tilde{\theta}_2, \dots, \tilde{\theta}_L\}$. Correspondingly, the t th snapshot of the incident signal is also extended to an L -dimensional sparse vector, denoted as $\tilde{\mathbf{s}}(t) = [\tilde{s}_1(t), \tilde{s}_2(t), \dots, \tilde{s}_L(t)]^T \in \mathbb{C}^{L \times 1}$, with K nonzero entries at the actual source directions, and the other $L-K$ elements set to zero, indicating no sources at those directions.

When a minimum-spacing ULA is employed, or an SLA with limited snapshots, and given $\tilde{\mathbf{A}}$, $\tilde{\mathbf{s}}(t)$, and the direct observations $\mathbf{x}(t)$, the resulting model is an underdetermined linear system; the SBL seeks a sparse solution to this model:

$$\mathbf{x}(t) = \tilde{\mathbf{A}}\tilde{\mathbf{s}}(t) + \mathbf{n}(t), \quad (19)$$

where the noise vector $\mathbf{n}(t) \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I}_I)$. If an array receives T snapshots, the reception model can be written as

$$\mathbf{X} = \tilde{\mathbf{A}}\tilde{\mathbf{S}} + \mathbf{N}, \quad (20)$$

where $\mathbf{X} = [\mathbf{x}(1), \mathbf{x}(2), \dots, \mathbf{x}(T)] \in \mathbb{C}^{I \times T}$ is the received signal, $\tilde{\mathbf{S}} = [\tilde{\mathbf{s}}(1), \tilde{\mathbf{s}}(2), \dots, \tilde{\mathbf{s}}(T)] \in \mathbb{C}^{L \times T}$ is the transmitted signal, and $\mathbf{N} = [\mathbf{n}(1), \mathbf{n}(2), \dots, \mathbf{n}(T)] \in \mathbb{C}^{I \times T}$ is the array noise. Obviously, DOA estimation using the model in Eq. (20) essentially involves maximizing the likelihood $p(\mathbf{X}; \tilde{\mathbf{S}}, \sigma_n^2)$. However, such high-dimensional nonlinear optimization generally does not admit a closed-form solution, and the resulting sparsity is not guaranteed. Therefore, in SBL, the components of the vector $\tilde{\mathbf{s}}(t)$ are assumed to be independent of one another, with each component $\tilde{s}_l(t)$ following a complex Gaussian distribution with zero mean and variance δ_l . In other words, for all t , $\tilde{\mathbf{s}}(t) \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Omega})$, where $\mathbf{\Omega} = \text{diag}[\delta_1, \delta_2, \dots, \delta_L]$.

In this hierarchical model, σ_n^2 and $\mathbf{\Omega}$ are treated as hyperparameters. Given priors on the hyperparameters, the posterior distribution $p(\tilde{\mathbf{S}} / \mathbf{X}, \sigma_n^2, \mathbf{\Omega})$ can be computed. The posterior is then used to form the expectation $E_{p(\tilde{\mathbf{S}} / \mathbf{X}, \sigma_n^2, \mathbf{\Omega})} [\ln p(\mathbf{X}, \tilde{\mathbf{S}}; \sigma_n^2, \mathbf{\Omega})]$, which can be maximized with respect to

σ_n^2 and Ω in closed form via the EM algorithm. The concrete steps are as follows.

In the E-step (compute the posterior distribution):

$$\boldsymbol{\mu}_{\tilde{\mathbf{s}}(t)} = \Omega \tilde{\mathbf{A}}^H \left[\sigma_n^2 \mathbf{I}_V + \tilde{\mathbf{A}} \Omega \tilde{\mathbf{A}}^H \right]^{-1} \mathbf{x}(t), \quad t=1, \dots, T, \quad (21)$$

$$\Sigma_{\tilde{\mathbf{s}}} = \Omega - \Omega \tilde{\mathbf{A}}^H \left[\sigma_n^2 \mathbf{I}_V + \tilde{\mathbf{A}} \Omega \tilde{\mathbf{A}}^H \right]^{-1} \tilde{\mathbf{A}} \Omega. \quad (22)$$

In the M-step (update the hyperparameters):

$$\Omega = \frac{\sum_{t=1}^T \boldsymbol{\mu}_{\tilde{\mathbf{s}}(t)} \boldsymbol{\mu}_{\tilde{\mathbf{s}}(t)}^H}{T} + \Sigma_{\tilde{\mathbf{s}}}, \quad t=1, \dots, T, \quad (23)$$

$$\sigma_n^2 = \frac{\sum_{t=1}^T \left\| \mathbf{x}(t) - \tilde{\mathbf{A}} \boldsymbol{\mu}_{\tilde{\mathbf{s}}(t)} \right\|_F^2}{V \cdot T} + \frac{\text{Tr}(\tilde{\mathbf{A}} \Sigma_{\tilde{\mathbf{s}}} \tilde{\mathbf{A}}^T)}{V}, \quad (24)$$

where $\boldsymbol{\mu}_{\tilde{\mathbf{s}}(t)}$ and $\Sigma_{\tilde{\mathbf{s}}}$ are the posterior mean and the posterior covariance, respectively, and $\text{Tr}(\cdot)$ denotes the matrix trace. Equations (23) and (24) are updated iteratively until the maximum number of iterations is reached or the convergence rate falls below a preset threshold. This method is referred to as B-SBL.

4.2 Vectorized SBL for DOA estimation

After sparsely extending the steering matrix $\mathbf{A}$ into an overcomplete dictionary $\tilde{\mathbf{A}}$, the virtual observation equation of Eq. (4) can be expressed as

$$\mathbf{z} = (\tilde{\mathbf{A}}^* \odot \tilde{\mathbf{A}}) \tilde{\mathbf{J}} + \sigma_n^2 \text{vec}(\mathbf{I}_V) = \tilde{\mathbf{B}} \tilde{\mathbf{J}} + \sigma_n^2 \mathbf{H}, \quad (25)$$

where $\tilde{\mathbf{J}} = [\sigma_1^2, \sigma_2^2, \dots, \sigma_L^2]^T$ is the extended signal power vector, only the position corresponding to the actual arrival angle has a non-zero value. In the virtual observation model given in Eq. (25), the non-zero elements of vector $\mathbf{H}$ are sparsified, and the original observation noise σ_n^2 only affects a few elements in the virtual observation $\mathbf{z}$. Under these circumstances, assuming that the observation vector $\mathbf{z}$ is uniformly corrupted by noise across all elements deviates from the true model. This leads to inaccurate noise variance estimation and severely hampers the performance of B-SBL^[52]. On the other hand, given the high DOFs of the TCA-I, removing the zero-valued rows from matrix $\tilde{\mathbf{B}}$ in Eq. (25) has a negligible impact. By simultaneously removing the rows in matrix $\mathbf{H}$ with element values of 1 and merging identical row vectors, Eq. (25) can be reformulated as

$$\mathbf{z}' = \tilde{\mathbf{B}}' \tilde{\mathbf{J}} + \mathbf{n}', \quad (26)$$

where $\mathbf{n}'$ represents the physical array noise. Rather, it is interpreted as a measure of the mismatch between the observations $\mathbf{z}'$ and the model prediction $\tilde{\mathbf{B}}' \tilde{\mathbf{J}}$ ^[28]. Following a

similar setup to $\tilde{\mathbf{s}}(t)$ and $\mathbf{n}(t)$, $\tilde{\mathbf{J}}$ and $\mathbf{n}'$ are modeled as $\tilde{\mathbf{J}} \sim \mathcal{CN}(\mathbf{0}, \Omega_{\tilde{\mathbf{J}}})$, $\mathbf{n}' \sim \mathcal{CN}(\mathbf{0}, (\sigma')^2 \mathbf{I}')$, respectively, where $\Omega_{\tilde{\mathbf{J}}} = \text{diag}\{\delta_{\tilde{\mathbf{J}}}^1, \delta_{\tilde{\mathbf{J}}}^2, \dots, \delta_{\tilde{\mathbf{J}}}^L\}$, and $\mathbf{I}'$ denotes an identity matrix with a dimension equal to the DOFs of the SLA minus 1. When applying TCA-I, according to Eq. (18), the order of $\mathbf{I}'$ becomes $MN + (2Q + 1)(M + N) - 2(Q + 1) - 1$, which matches the dimension of $\mathbf{n}'$. This value is denoted as V' . The iterative update rules for the hyperparameters $\Omega_{\tilde{\mathbf{J}}}$ and $(\sigma')^2$ in the EM algorithm are given as follows.

In the E-step (compute the posterior distribution):

$$\boldsymbol{\mu}_{\tilde{\mathbf{J}}} = \Omega_{\tilde{\mathbf{J}}} (\tilde{\mathbf{B}}')^H \left[(\sigma')^2 \mathbf{I}' + \tilde{\mathbf{B}}' \Omega_{\tilde{\mathbf{J}}} (\tilde{\mathbf{B}}')^H \right]^{-1} \mathbf{z}', \quad (27)$$

$$\Sigma_{\tilde{\mathbf{J}}} = \Omega_{\tilde{\mathbf{J}}} - \Omega_{\tilde{\mathbf{J}}} (\tilde{\mathbf{B}}')^H \left[(\sigma')^2 \mathbf{I}' + \tilde{\mathbf{B}}' \Omega_{\tilde{\mathbf{J}}} (\tilde{\mathbf{B}}')^H \right]^{-1} \tilde{\mathbf{B}}' \Omega_{\tilde{\mathbf{J}}}. \quad (28)$$

In the M-step (update the hyperparameters):

$$\Omega_{\tilde{\mathbf{J}}} = \boldsymbol{\mu}_{\tilde{\mathbf{J}}} \boldsymbol{\mu}_{\tilde{\mathbf{J}}}^H + \Sigma_{\tilde{\mathbf{J}}}, \quad (29)$$

$$(\sigma')^2 = \frac{\left\| \mathbf{z}' - \tilde{\mathbf{B}}' \boldsymbol{\mu}_{\tilde{\mathbf{J}}} \right\|_F^2}{V'} + \frac{\text{Tr}(\tilde{\mathbf{B}}' \Sigma_{\tilde{\mathbf{J}}} (\tilde{\mathbf{B}}')^T)}{V'}. \quad (30)$$

This method is essentially consistent with that in Ref. [50] and is referred to herein as vectorized SBL (V-SBL).

4.3 The CG-SBL based on TCA-I

In scenarios with limited snapshots, the V-SBL method described earlier becomes inapplicable, and using the basic-SBL directly on TCA-I fails to fully leverage its structural advantages. Based on the special structure of the TCA-I, this section designs a targeted SBL method, named CG-SBL, which provides improved bearing estimation performance in the small-snapshot signal regime.

As is well known, the signal incident on the array can be regarded as spatial sampling. Defined as the projection of cyclic variation of a signal with wavelength λ onto the array axis, the normalized spatial frequency k_λ is given by $\sin\theta$ for an incident signal from direction θ , as illustrated in Fig. 4(a). Thus, the bearing range of -90° – 90° is mapped to the normalized spatial-frequency interval $[-1, 1]$. For simplicity, the bearing spatial spectrum is set to a unit-height rectangle: $sf_{\text{bearing}}(k_\lambda) = U(k_\lambda + 1) - U(k_\lambda - 1)$, where $U(\cdot)$ denotes the unit step function. Correspondingly, the normalized spatial sampling frequency of a ULA is denoted by $k_s = 2/d_s$, with d_s representing the inter-element spacing normalized by a half-wavelength. According to the Nyquist sampling theorem, $d_s = 1$ is the threshold for aliasing. When $d_s > 1$, the incident bearing at the array becomes aliased, as shown in Fig. 4(b). When $d_s = 3$, the entire bearing spatial spectrum $sf_{\text{bearing}}(k_\lambda)$

overlaps, causing aliasing for all incident bearings, as illustrated in Fig. 4(c). The bearing spatial spectrum after sampling at the normalized spatial sampling frequency k_s is

$$sf_{\text{bearing}}^{\text{sampled}}(k_\lambda) = \sum_{m \in \mathbb{Z}} sf_{\text{bearing}}(k_\lambda - mk_s). \quad (31)$$

As a result, for the bearing θ , the alias bearing θ_{alias} can be searched from

$$\sin \theta_{\text{alias}} = \sin \theta + m(2/d_s), \quad (32)$$

where $m \in \mathbb{Z}$. Aliasing is typically something to overcome, but interestingly, when combined with grid-based modeling of SBL, it may become a favorable factor. According to Eq. (31), array sampling means repeatedly shifting $sf_{\text{bearing}}(k_\lambda)$ by $2/d_s$ and summing the shifted spectra. As illustrated in Fig. 4(c), taking $d_s = 3$ as an example to demonstrate the principle of grid compression, the bearing spatial spectrum $sf_{\text{bearing}}(k_\lambda)$ spanning the range of $[-1, 1]$ is partitioned into 3 distinct segments: the light blue region corresponds to subinterval $[-1, -1/3]$, the blue region to subinterval $[-1/3, 1/3]$, and the dark blue region to subinterval $[1/3, 1]$. According to Eq. (31), the sampling process causes $sf_{\text{bearing}}(k_\lambda)$ to be shifted and accumulated, resulting in the overlapping of these distinct color blocks in the spatial-frequency domain.

Therefore, it is evident that for any integer d_s , owing to sampling-induced spectral shifts, every sub-interval of width $2/d_s$ within the range $[-1, 1]$ contains exactly one complete copy of the spatial spectrum $sf_{\text{bearing}}(k_\lambda)$. Consequently, by establishing an overcomplete dictionary within such a sub-interval, the B-SBL algorithm is guaranteed to capture the bearing θ , identifying it either directly or via its aliased value. Figure 4(d) illustrates the grid construction within the sub-range $-\arcsin(1/d_s) - \arcsin(1/d_s)$ with $d_s = 3$.

For on-grid SBL with a fixed grid, the true bearing θ_{true} is assumed to be uniformly distributed within the grid interval $[-\Delta/2, \Delta/2]$ around the nearest grid point, where Δ denotes the grid spacing. Under this assumption, the quantization error δ follows a uniform distribution with a probability density function $p(\delta) = 1/\Delta$. Accordingly, the lower bound on the estimation error is commonly regarded as the variance of the grid quantization noise:

$$\varepsilon_{\text{LB}} = E[\delta^2] = \int_{-\Delta/2}^{\Delta/2} \delta^2 \cdot \frac{1}{\Delta} d\delta = \frac{\Delta^2}{12}. \quad (33)$$

To mitigate this quantization limit, the off-grid model in Ref. [7] expands the actual steering vector $\mathbf{a}(\theta_{\text{true}})$ around the nearest grid point θ_{grid} :

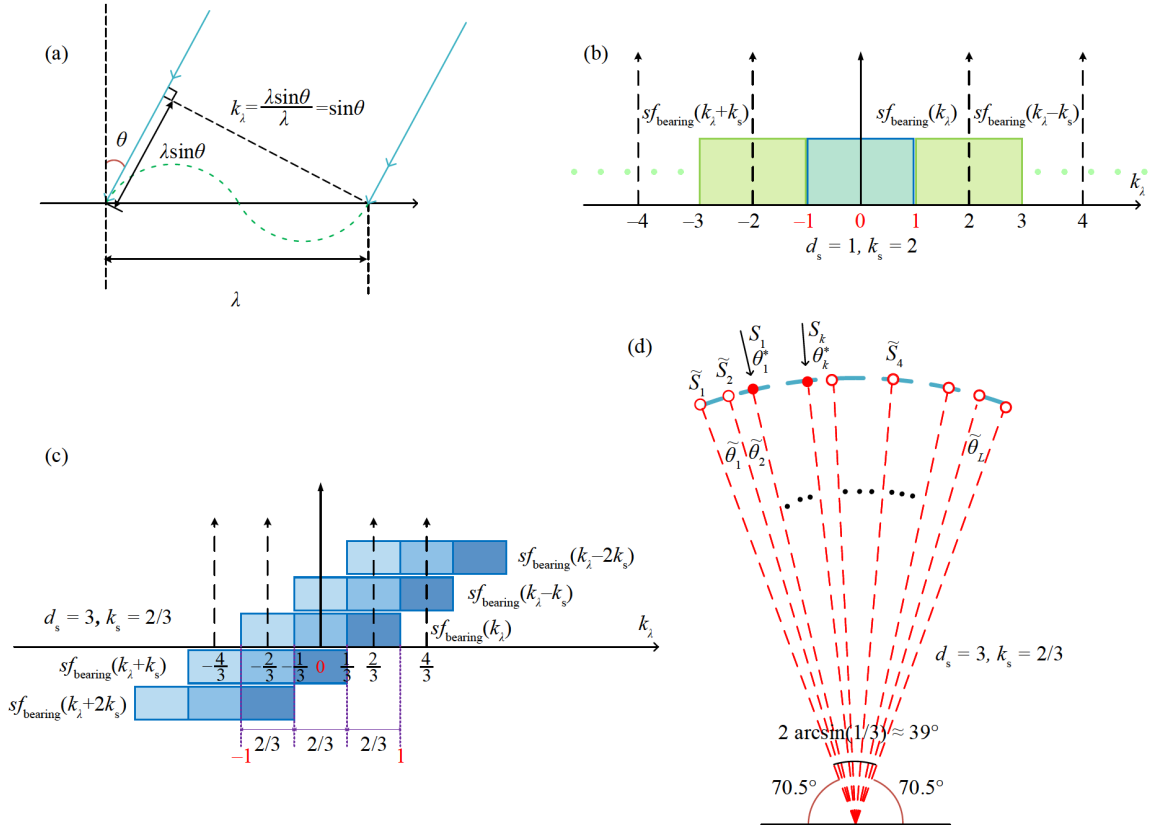


Fig. 4 Diagram of the compressed-grid concept: (a) Normalized spatial frequency; (b) Spatial sampling; (c) Spectral overlap; (d) Compressed-grid.

$$\mathbf{a}(\theta_{\text{true}}) = \mathbf{a}(\theta_{\text{grid}}) + \mathbf{a}'(\theta_{\text{grid}})(\theta_{\text{true}} - \theta_{\text{grid}}) + \mathbf{r}_{\text{res}}, \quad (34)$$

where $\mathbf{r}_{\text{res}}$ denotes the linearization residual. Based on the Lagrange remainder theorem, the upper bound of the residual norm is determined by the second-order derivative:

$$\|\mathbf{r}_{\text{res}}\|_2 \leq (1/2)C_{\max}(\theta_{\text{true}} - \theta_{\text{grid}})^2, \quad (35)$$

where $C_{\max} = \max_{\xi \in [-90^\circ, 90^\circ]} \|\partial^2 \mathbf{a}(\xi) / \partial \theta^2\|_2$ represents the maximum magnitude of the array manifold over the entire bearing range of -90° – 90° . Considering the worst-case quantization error $|\theta_{\text{true}} - \theta_{\text{grid}}| \leq \Delta/2$, the modeling error is

$$\|\mathbf{r}_{\text{res}}\|_2 \leq \frac{C_{\max}}{8}\Delta^2 \propto \Delta^2. \quad (36)$$

This demonstrates that the modeling error bound scales quadratically with the grid interval Δ . Consequently, given the same number of grid points, by narrowing the search range with a compression ratio $\eta \ll 1$, the effective grid spacing is reduced to $\Delta_{\text{CG}} \leq \eta \Delta_{\text{full}}$. Substituting this into Eq. (36), the modeling error bound decreases quadratically by a factor of η^2 , which directly underpins the improved estimation accuracy of the proposed method.

As discussed in the previous section, the sub-ULA 3 of the TCA-I, which is the dense subarray, has an inter-element spacing of $(M+N)(\lambda/2)$. In light of the above analysis, by applying this dense subarray, the grid system can be established in the spatial range corresponding to the subintervals rather than across the full -90° – 90° . This compresses the estimation interval to achieve higher angular resolution, allowing the same grid count to yield higher bearing estimation accuracy. Although any sub-interval theoretically ensures that no source is omitted, the proposed CG-SBL typically selects the sub-range near 90° (specifically $\arcsin(1 - 2/d_s) - n/2$), which spans the widest physical angular range, to ensure that the effective grid spacing remains dense enough when mapped to narrower central regions. Accordingly, this selected interval is uniformly divided into L grids, and the bearing set is denoted as $\{\tilde{\theta}_1, \tilde{\theta}_2, \dots, \tilde{\theta}_L\}_{d_s}$. The K signal directions can be found within the compressed angular range: $\{\theta_1^*, \theta_2^*, \dots, \theta_K^*\} \subset \{\tilde{\theta}_1, \tilde{\theta}_2, \dots, \tilde{\theta}_L\}_{d_s}$, but some of them correspond to alias bearings. Fortunately, TCA-I also includes a PCA; although the estimate is of lower precision, it is free of aliasing: $\{\theta_1', \theta_2', \dots, \theta_K'\} \subset \{\tilde{\theta}_1, \tilde{\theta}_2, \dots, \tilde{\theta}_L\}$. Next, according to Eq. (32), the d_s alias bearings of θ_k^* are computed, then denoted as θ_{k-i}^* for $i = 1, 2, \dots, d_s$. Then, using Euclidean-distance nearest-neighbor (NN) matching, identify their corresponding true bearings:

$$(\theta_{k-i(k)}^*, \theta_j') = \underset{i,j}{\operatorname{argmin}} \|\theta_{k-i(k)}^* - \theta_j'\|_F^2, \quad (37)$$

where $k, j \in \{1, 2, \dots, K\}$, $k = 1, 2, \dots, K$. Finally, K high-precision estimates are obtained, represented as $\{\theta_{1-i(1)}^*, \theta_{2-i(2)}^*, \dots, \theta_{K-i(K)}^*\}$, in which $i(k)$ denotes the index i selected for k . Given the TCA-I configuration, the sparse and dense subarrays consist of Q and $M+N-1$ physical sensors, respectively. In the absence of difference co-array vectorization, where the estimation relies directly on the physical array manifold, the identifiability limits are naturally constrained by the physical aperture dimensions. Specifically, the limits are derived as $K_{\text{sparse}} = Q-1$ and $K_{\text{dense}} = M+N-2$. Consequently, under ideal noise-free conditions, the maximum number of estimable sources for the proposed system is theoretically bounded by $K_{\max} = \min(K_{\text{sparse}}, K_{\text{dense}})$, which guarantees the uniqueness of the solution and confirms that the system capacity is determined by the subarray with the more restrictive limit. Figure 5 illustrates the flowchart of the proposed TCA-I-based CG-SBL algorithm.

It should be noted that the B-SBL, V-SBL, and CG-SBL algorithms employed in Secs. 5 and 6 incorporated an off-grid refinement stage. Specifically, the original off-grid optimization procedure from Ref. [7] without modification is adopted. For B-SBL and V-SBL, this refinement is applied to the models in Eqs. (20) and (26), respectively, by iteratively

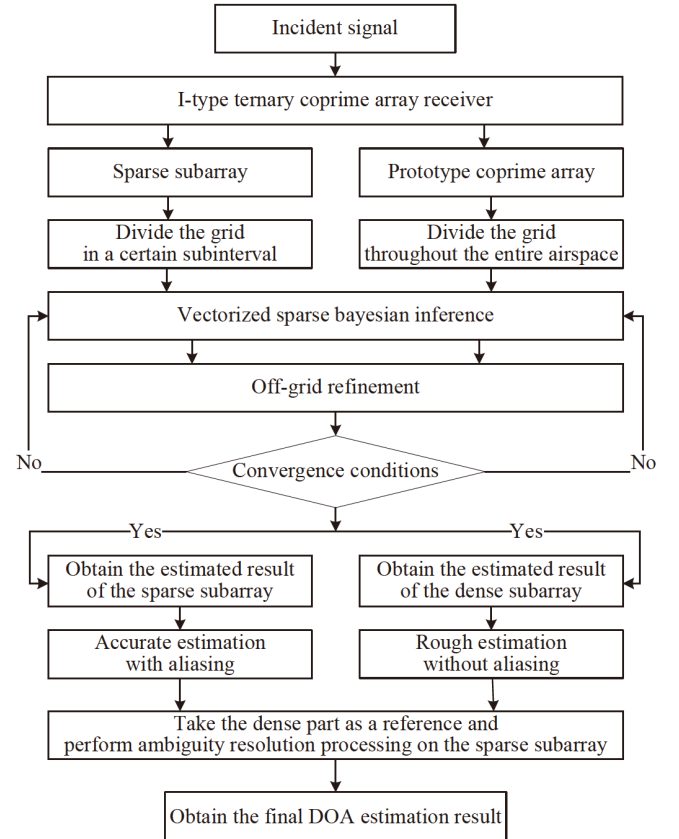


Fig. 5 The CG-SBL algorithm flowchart.

updating the grid offset parameters. For CG-SBL, the same procedure is applied separately to the two subarrays.

5 Simulation results and analysis

The first simulation scenario evaluates the performance of TCA-I, NA, PCA, and the conventional ULA using the V-SBL algorithm in the multiple-snapshot case. With the element count fixed at 15 for all configurations, mutual coupling is considered with a coefficient of $c = 0.3$ and $G = 100$, as defined in Eq. (8). Table 1 summarizes the corresponding equivalent array configurations, DOFs, and coupling leakages. The bearings of the 11 sources are uniformly distributed within the interval -50° – 50° , and all sources have the same SNR. As shown in Figs. 6(a) and 6(b), TCA-I demonstrates superior accuracy due to its higher DOFs, even in the absence of mutual coupling. Furthermore, when mutual coupling is

taken into account, this performance advantage becomes even more pronounced, as illustrated in Figs. 6(c) and 6(d).

The second simulation scenario investigates the few-snapshot case, validating the performance of the joint CG-SBL and TCA-I framework against NA, PCA, and the conventional ULA under the S-SBL algorithm. In Figs. 7(a)–7(c), the element count is fixed at 10, with two sources located at 20° and 50.5° , respectively. Additionally, both sources share the same SNR. Table 2 summarizes the corresponding equivalent array configurations, DOFs, and coupling leakages. As illustrated, the proposed CG-SBL-TCA-I demonstrates a distinct advantage. This superiority is attributed to two factors: the higher angular resolution provided by the compressed grid, and the significantly reduced mutual coupling within the sparse subarray of the TCA-I. In Fig. 7(c), the varying grid numbers, 46, 61, 91, and 181, correspond to coarse angular resolutions, 4° , 3° , 2° , 1° , over the full bearing interval. In this specific instance, the sparse subarray of the TCA-I has an inter-element

Table 1 Comparison of array indicators with the element count fixed at 15

Array configuration	DOFs	Coupling leakage
$\mathbf{P}_{\text{TCA-I}} = \{0, 11, 22, 33, 44, 55, 60, 61, 65, 67, 70, 73, 75, 79, 80\}$	$\text{Card}(\mathbf{D}_{\text{TCA-I}}) = 139$	$\gamma_{\text{TCA-I}} = 0.1996$
$\mathbf{P}_{\text{ULA}} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$	$\text{Card}(\mathbf{D}_{\text{ULA}}) = 29$	$\gamma_{\text{ULA}} = 0.4434$
$\mathbf{P}_{\text{NA}} = \{0, 1, 2, 3, 4, 5, 6, 13, 20, 27, 34, 41, 48, 55, 62\}$	$\text{Card}(\mathbf{D}_{\text{NA}}) = 127$	$\gamma_{\text{NA}} = 0.3013$
$\mathbf{P}_{\text{PCA}} = \{0, 7, 9, 14, 18, 21, 27, 28, 35, 36, 42, 45, 49, 54, 56\}$	$\text{Card}(\mathbf{D}_{\text{PCA}}) = 77$	$\gamma_{\text{PCA}} = 0.1993$

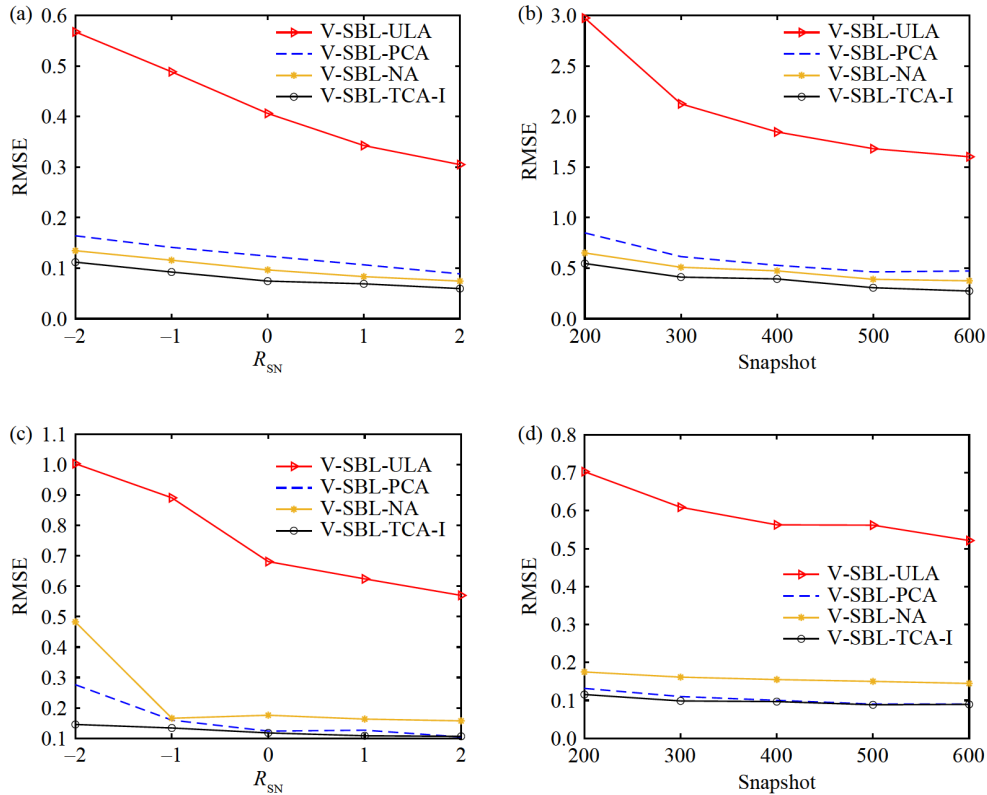


Fig. 6 The root mean square error (RMSE) for TCA-I and other arrays: (a) RMSE versus SNR at snapshot = 200 (coupling free); (b) RMSE versus snapshot at $R_{\text{SN}} = 0$ (coupling free); (c) RMSE versus SNR at snapshot = 200 and $c = 0.3$; (d) RMSE versus snapshot at $R_{\text{SN}} = 0$ and $c = 0.3$.

spacing d_s of 7, which establishes a grid subinterval of 46° – 90° . Consequently, even with a minimum of 46 grids, a fine resolution of 0.96° is achieved for the widest angular sub-range, thereby surpassing the angular resolutions of B-SBL with 181 grids for full -90° – 90° .

Given the inherent scalability of the TCA-I architecture, Figs. 7(d)–7(f) present the comparative results obtained by extending the sparse subarray of the 10-element TCA-I with five additional elements. This extension further enhances its relative superiority, achieving a $\text{Card}(\mathbf{D}_{\text{TCA-I}})$ with $\gamma_{\text{TCA-I}} = 0.1992$. Consequently, the TCA-I can be flexibly configured to adapt to complex and dynamic sea environments, effectively balancing the performance requirements of both V-SBL and CG-SBL algorithms.

6 Sea trial data processing and analysis

The V-SBL-TCA-I algorithm proposed in this paper is used to process and analyze the sea test data of the actual

passive sonar. The towed linear array sonar adopts 96-element uniform array, the array element spacing is 0.4 m, the working frequency band is 500–1875 Hz, and the sampling frequency is 12 kHz. There are two groups of test data: one is multi-target, the other is cross-target. Each group of data has a total of 540 frames. In the data pre-processing stage, each received frame consists of 4096 samples. We partition the frame into 32 non-overlapping segments, each containing 128 samples, to ensure statistical independence among snapshots. A 256-point FFT with zero-padding is then applied to each segment to improve spectral resolution, and the coefficients within the signal bandwidth are extracted for DOA estimation. To facilitate comparison and analysis, the conventional beamforming (CBF) algorithm of the whole array element is used as a reference, and the number of array elements of other algorithms is 15. The iRVM-DOA and B-SBL algorithms use ULA, that is, uniformly spaced sampling on the whole array element. The V-SBL-NA and the V-SBL-TCA-I algorithms are vectorized.

Figure 8 shows the time-direction evolution diagram of

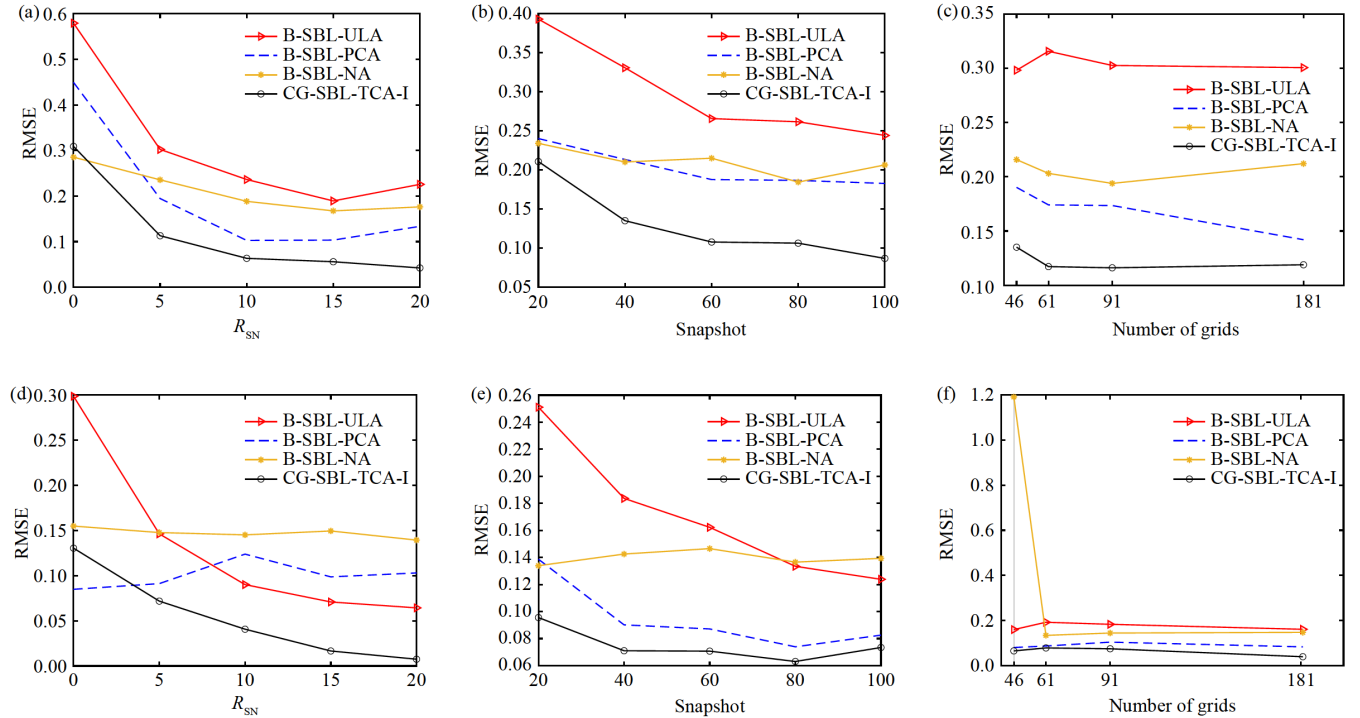


Fig. 7 RMSE for TCA-I and other arrays at $c = 0.3$. For $V = 10$: (a) RMSE versus SNR at snapshot = 50 and grid = 61; (b) RMSE versus snapshot at $R_{\text{SN}} = 5$ and grid = 61; (c) RMSE versus grid at $R_{\text{SN}} = 5$ and snapshot = 50; For $V = 15$: (d) RMSE versus SNR at snapshot = 50 and grid = 61; (e) RMSE versus snapshot at $R_{\text{SN}} = 5$ and grid = 61; (f) RMSE versus grid at $R_{\text{SN}} = 5$ and snapshot = 50.

Table 2 Comparison of array indicators with the element count fixed at 10

Array configuration	DOFs	Coupling leakage
$\mathbf{P}_{\text{TCA-I}} = \{0, 7, 14, 21, 28, 31, 32, 34, 36, 37\}$	$\text{Card}(\mathbf{D}_{\text{TCA-I}}) = 65$	$\gamma_{\text{TCA-I}} = 0.2362$
$\mathbf{P}_{\text{ULA}} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$	$\text{Card}(\mathbf{D}_{\text{ULA}}) = 19$	$\gamma_{\text{ULA}} = 0.4295$
$\mathbf{P}_{\text{NA}} = \{0, 1, 2, 3, 4, 9, 14, 19, 24, 29\}$	$\text{Card}(\mathbf{D}_{\text{NA}}) = 59$	$\gamma_{\text{NA}} = 0.2982$
$\mathbf{P}_{\text{PCA}} = \{0, 5, 6, 10, 12, 15, 18, 20, 24, 25\}$	$\text{Card}(\mathbf{D}_{\text{PCA}}) = 39$	$\gamma_{\text{PCA}} = 0.2392$

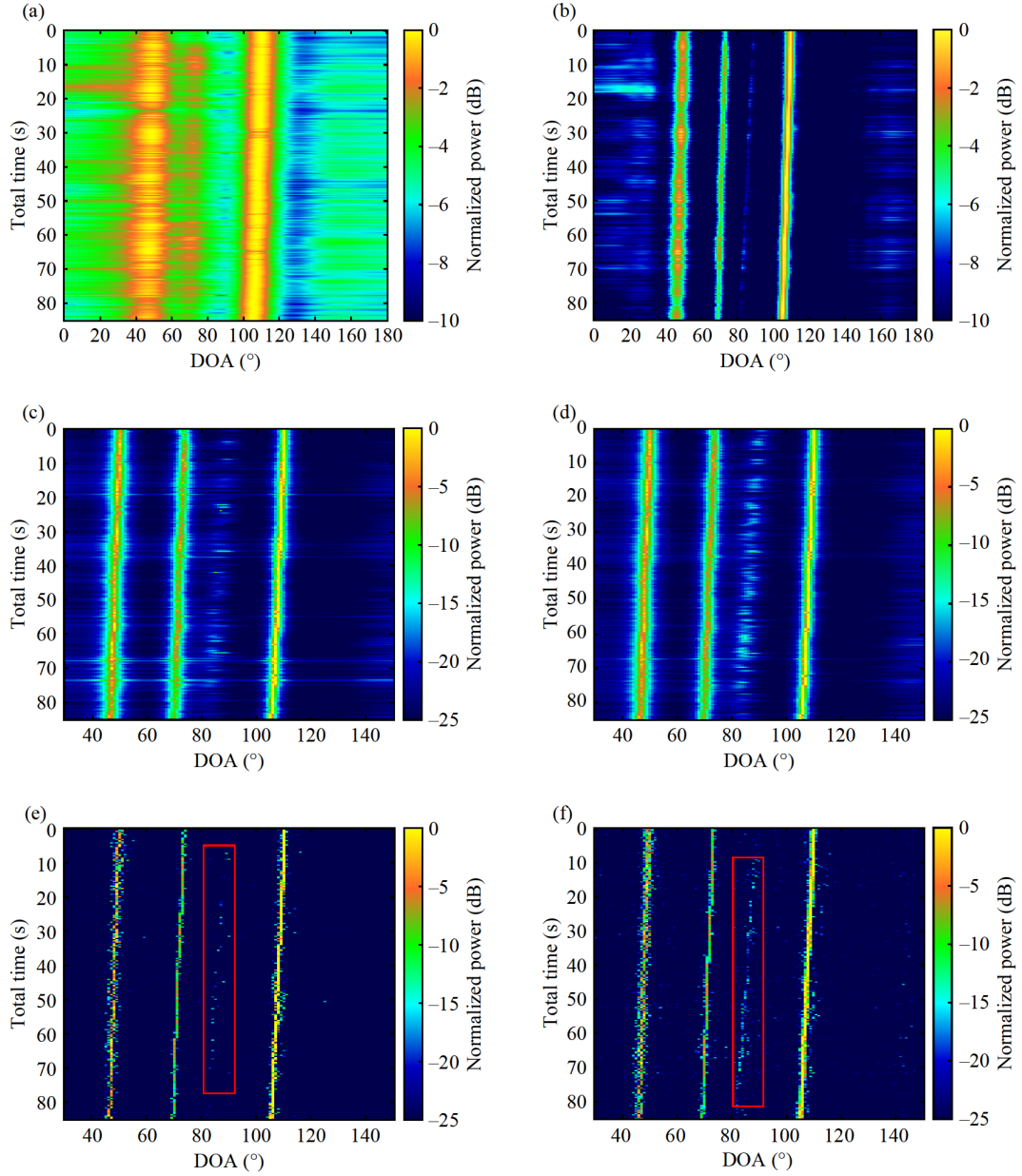


Fig. 8 Time-direction evolution diagram of various algorithms with multiple targets: (a) CBF, $V = 15$, ULA; (b) CBF, $V = 96$, ULA (Reference); (c) iRVM-DOA, $V = 15$, ULA; (d) B-SBL, $V = 15$, ULA; (e) V-SBL-NA, $V = 15$; (f) V-SBL-TCA-I, $V = 153$.

CBF, iRVM-DOA, V-SBL-NA, and V-SBL-TCA-I with multiple targets. For the convenience of analysis, the 150th–400th frames are selected for comparison, and the angle range is from 30° to 150° . The colorbars are normalized to the maximum intensity of each respective subplot. First, as can be seen in Figs. 8(a) and 8(b), compared to the CBF with 15 array elements, CBF using 96 array elements exhibits higher resolution. Through the CBF of the whole array element, it can be seen that there are four targets in total, including three strong targets and one weak target. Compared with iRVM-DOA, B-SBL can detect weak target contour in the middle, but their spectral lines are thicker and their spatial resolution is weaker. The latter V-SBL-NA and

V-SBL-TCA-I obviously have sharper spectral lines and a lower spatial background level. The reason is that the sparse array SBL adopts vectorization processing, which significantly expands the spatial aperture, possesses more information than ULA, and has superior DOA estimation performance. Through further comparison, it can be seen that V-SBL-NA can not effectively detect weak targets in the middle, while V-SBL-TCA-I has clearer spectral lines for weak targets in the middle than V-SBL-NA, as shown in the red rectangle box in Figs. 8(e) and 8(f). It can be seen that compared with the traditional sparse array, TCA-I, combined with SBL processing method, has higher spatial resolution. The reason is that TCA-I has more different DOFs than the

traditional sparse array, which has a better effect on DOA estimation for the sparse reconstruction algorithm.

Figures 9 and 10 show the average number of iterations and the spatial spectrum of various algorithms under multi-objective conditions. It is obvious that the SBL algorithm under a sparse array has fewer iterations than the SBL algorithm under traditional ULA, while the V-SBL-TCA-I algorithm and V-SBL-NA algorithm have the same number of iterations. By observing the spatial spectrum, it can be seen that the proposed V-SBL-TCA-I algorithm has a lower sidelobe level than the traditional SBL algorithm, and can identify the weak target in the middle compared with the V-SBL-NA algorithm, as shown in the black rectangular box in Fig. 10.

Figure 11 shows the time-direction evolution diagram of CBF, iRVM-DOA, V-SBL-NA, and V-SBL-TCA-I with cross targets. Here, the 150th–400th frames are selected for comparison, and the angle range is from 80° to 130° . First, through the CBF of the whole array element, it can be seen that there are three targets in total, including one weak target and two adjacent cross targets, while CBF with 15 elements can hardly be identified. For the convenience of analysis, cross-targets are selected for comparison. It can be seen from the figure that iRVM-DOA can hardly detect effectively. Compared with the former, B-SBL is significantly improved, but it can not distinguish between two targets. The latter V-SBL-NA and V-SBL-TCA-I can obviously distinguish two targets. Through further comparison, it can be found that V-SBL-TCA-I has fewer pseudo peaks than V-SBL-NA, and its time-direction evolution diagram is cleaner and clearer, as shown in the red rectangular box in Figs. 11(e) and 11(f). It is important to note that although the low-frequency towed sonar exhibits weaker acoustic coupling than high-frequency rigid sonar arrays, the physical interactions between closely spaced elements driven by hydro-mechanical noise, specifically mechanical “bulge waves” and “flow-induced noise”, can not be ignored^[55,56]. Since the decimated TCA-I subset operates within the high-interference environment of the full 96-element dense host array, the observed performance

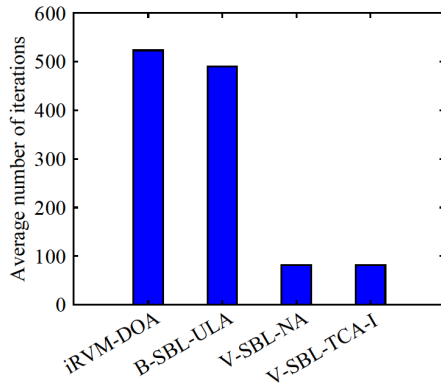


Fig. 9 Average iteration times of various algorithms with multiple targets.

advantage of V-SBL-TCA-I is solely attributed to the increased DOFs. Consequently, we plan to deploy a physical TCA-I array to experimentally verify the further performance improvements expected from reduced physical interactions due to higher sparsity, thereby addressing the limitations of the subset selection method.

Figures 12 and 13 show the average number of iterations and spatial spectrum of various algorithms under cross targets. It can be seen that although iRVM-DOA has the least number of iterations, its DOA estimation performance is too poor, so the actual situation is not considered. Compared with B-SBL, the iteration times of V-SBL-NA and V-SBL-TCA-I are significantly reduced, and the specific reasons have been described in the previous article. Further comparison shows that the number of iterations of V-SBL-TCA-I is slightly lower than that of V-SBL-NA. Considering that the time of single iteration is roughly the same, V-SBL-TCA-I has lower time complexity. By observing the spatial spectrum, it can be seen that both V-SBL-TCA-I and V-SBL-NA can accurately identify the two crossed targets, but the sidelobe level and the number of pseudo peaks of V-SBL-TCA-I are much lower than those of V-SBL-NA, and the pseudo peaks of V-SBL-NA are shown in the black rectangular box in Fig. 13.

7 Conclusion

To address the limitations of traditional sparse arrays in complex marine environments, a comprehensive bearing estimation framework integrating the TCA-I with SBL was presented in this paper. The TCA-I architecture demonstrates superior modular scalability and reduced mutual coupling, maintaining closed-form DOFs essential for flexible deployment.

A key practical contribution of this work is the CG-SBL algorithm. By innovatively exploiting the aliasing ambiguity of sparse subarrays rather than treating it as a defect, this method compresses the search space to specific sub-intervals.

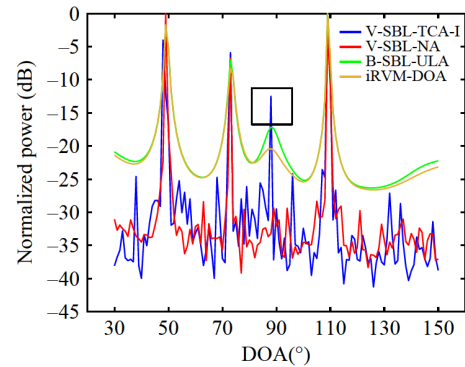


Fig. 10 Spatial spectrum of a frame with multiple targets in various algorithms.

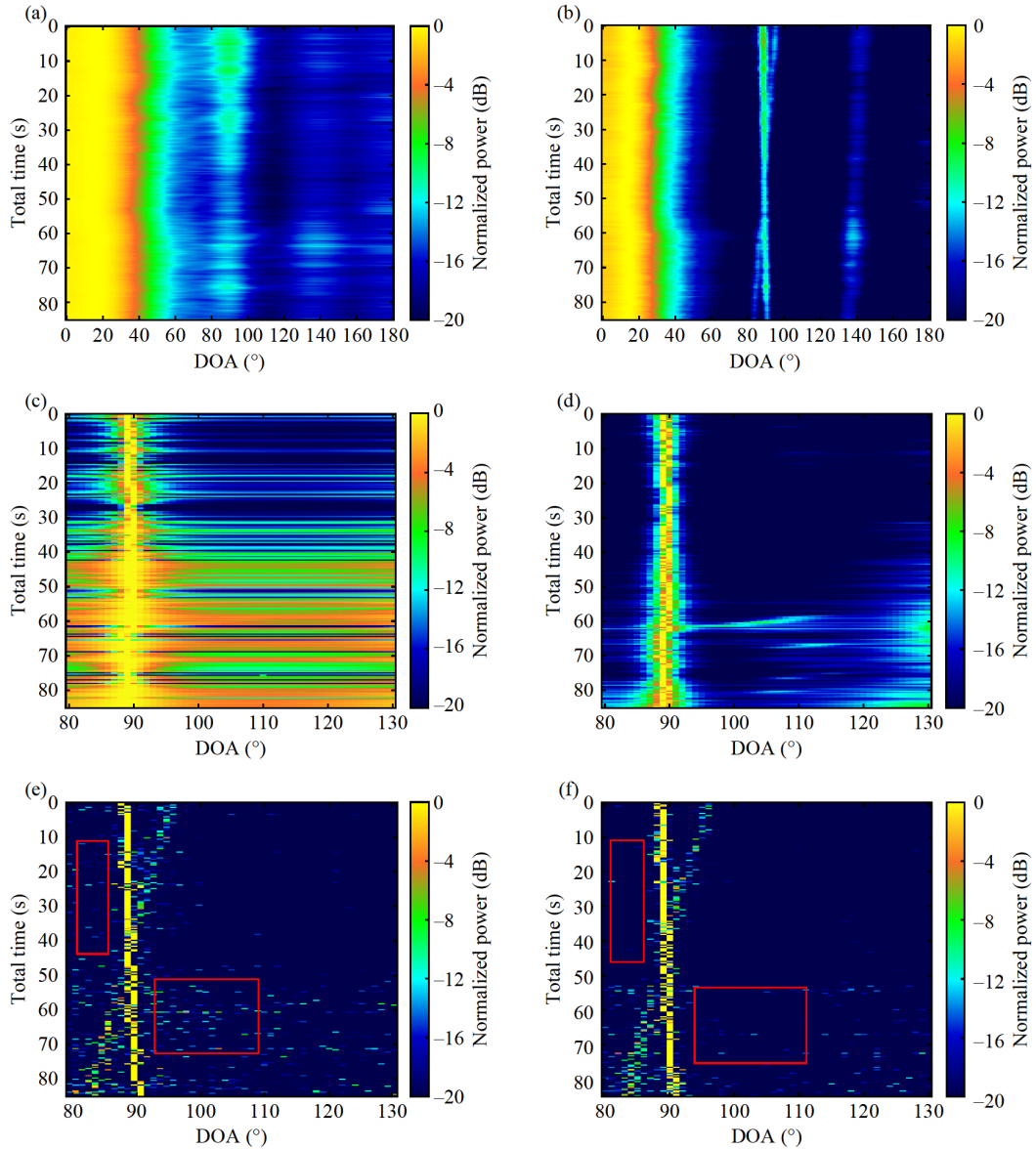


Fig. 11 Time-direction evolution diagram of various algorithms with cross targets: (a) CBF, $V = 15$, ULA; (b) CBF, $V = 96$, ULA (Reference); (c) iRVM-DOA, $V = 15$, ULA; (d) B-SBL, $V = 15$, ULA; (e) V-SBL-NA, $V = 15$; (f) V-SBL-TCA-I, $V = 15$.

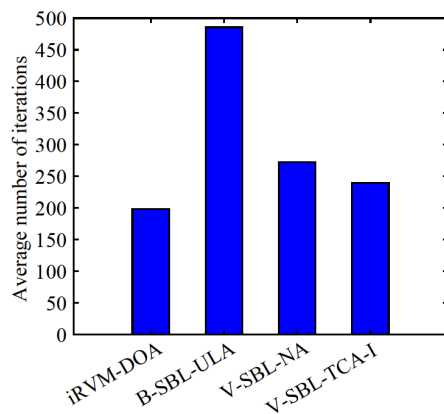


Fig. 12 Average iteration times of various algorithms with cross targets.

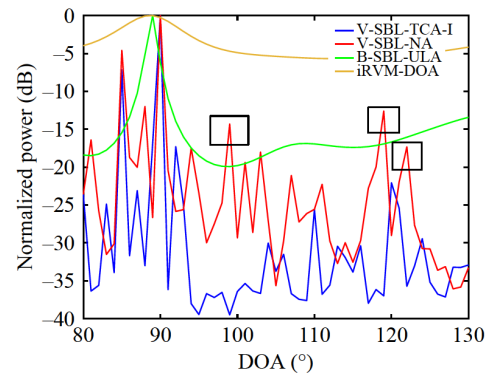


Fig. 13 Spatial spectrum of a frame with cross targets in various algorithms.

This approach offers a novel perspective for sparse reconstruction, achieving high-resolution estimation in snapshot-deficient scenarios without structural hardware changes.

Performance validations were conducted through both simulations and sea trials. Simulations confirmed that CG-SBL significantly outperforms existing methods in few-snapshot cases. Meanwhile, sea-trial data processing using the V-SBL verified that the TCA-I configuration effectively detects weak and crossing targets with sharper spatial spectra compared to NA, PCA, and ULA. Future work will focus on applying the proposed CG-SBL algorithm to experimental sea-trial data to further assess its practical capability in highly dynamic and snapshot-limited underwater environments.

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