

Multipartite entanglement and entropic tightening of uncertainty bounds

S. HAMID MEHDIPOUR^(a) 

Department of Physics, La.C., Islamic Azad University - Lahijan, Iran

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Abstract – We derive a generalized uncertainty relation (GUR) in the entropic framework for multipartite identical pure entangled systems. Starting from the variance-based GUR, we reformulate it in terms of differential Shannon entropy, obtaining a tighter entropic bound that explicitly accounts for quantum correlations. Our analysis reveals that the conditional entropy $H(k|\mathcal{M}_k)$, which quantifies the entanglement between a measured subsystem k and the remaining $N - 1$ particles acting as a composite quantum memory, decreases with particle number N and asymptotically approaches zero. This behavior highlights the entanglement-induced suppression of uncertainty, leading to an emergent classicality in the large- N limit. In particular, we demonstrate that multipartite entanglement with quantum memory reduces both the state-independent and state-dependent bounds, yielding more precise measurements. These results not only provide a deeper understanding of the interplay between entanglement and uncertainty but also suggest potential applications in quantum cryptography, precision metrology, condensed-matter many-body physics, quantum thermodynamics, and foundational studies of the quantum-to-classical transition.

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Introduction. – The uncertainty principle lies at the foundation of quantum mechanics, constraining the simultaneous precision with which non-commuting observables can be determined. First formulated in terms of standard deviations by Robertson [1], the principle reflects a radical departure from classical intuitions, where in principle no fundamental obstacle exists to measuring conjugate quantities with arbitrary accuracy. However, the standard-deviation formulation, while elegant, does not always capture the full scope of quantum limitations, especially in the presence of correlations and entanglement.

To address these issues, alternative formulations based on information-theoretic measures were introduced. In particular, entropic uncertainty relations (EURs) quantify uncertainty in terms of entropy, offering a more general and operationally meaningful framework [2,3]. Such relations not only refine the characterization of measurement uncertainty but also play a central role in applications ranging from quantum cryptography to randomness extraction. A milestone was reached with the work of Berta *et al.* [4], who showed that the presence of quantum

memory can tighten the entropic uncertainty bound, highlighting the counterintuitive fact that entanglement can reduce, rather than increase, measurement uncertainty.

Since then, entropic formulations of the uncertainty principle have been extended and applied in a variety of contexts, including multipartite entanglement detection [5], quantum channels [6], the emergence of classicality [7], and information-theoretic foundations [8]. Recent studies have extended EUR frameworks to realistic many-body systems [9], multipartite Holevo measurements [10], and multipartite quantum memory scenarios [11], while experimental tests have validated tight EUR bounds in high-dimensional bases [12]. These developments have revealed profound connections between uncertainty, coherence, and the structure of quantum correlations. Nevertheless, most results have focused on bipartite systems or limited measurement settings, while the regime of multipartite entangled systems remains comparatively underexplored.

Recent theoretical advances have further refined EURs in various contexts, including tighter bounds for multiple measurements [13], information exclusion relations [14], tightening of bounds with quantum memory [15,16],

^(a)E-mail: mehdiipour@iaui.ac.ir (corresponding author)

extensions to gravitational and high-energy settings such as Schwarzschild spacetime and black holes [17,18]. Moreover, generalized multipartite EURs have been experimentally verified [19].

In parallel, standard-deviation-based approaches continue to play an essential role in operational contexts such as quantum metrology, where variances directly determine achievable precision limits. In this work, we first introduce a generalized uncertainty relation (GUR) for N identically entangled particles within the variance-based framework, demonstrating that the lower bound becomes progressively tighter as $N \rightarrow \infty$. This result suggests a pathway to classicality through entanglement-induced suppression of quantum fluctuations.

Afterwards, these ideas are extended into the entropic domain. We develop a generalized EUR for multipartite entangled systems and analyze how the bound evolves with increasing particle number. Remarkably, our results reveal that, unlike the variance-based formulation, the entropic bound decreases only gradually with N , asymptotically approaching the state-independent limit. This contrast underscores the complementary roles of variance- and entropy-based approaches and illuminates the subtle ways in which entanglement shapes the quantum-to-classical transition.

While the variance-based GUR summarized in the next section has been presented in our recent work [20], the entropic formulation developed in the section that follows—which constitutes the central contribution of this paper—is entirely novel. It extends the previous analysis beyond the variance-based framework, overcomes its conceptual limitations, and provides stronger, operationally meaningful bounds with direct implications for quantum information theory. Our formulation builds upon recent advances in state-dependent entropic relations [7,8] and is conceptually related to the operational resource theory of coherence [21].

The GUR of a pure multipartite entangled system. – The purpose of this section is to formulate the GUR in the standard-deviation approach that will serve as the foundation for extending the analysis into the entropic domain.

Consider a pure state describing a multipartite system of N subsystems. The total Hilbert space is given by the tensor product $H = \bigotimes_{k=1}^N H_k$. For an entangled configuration, the total state cannot be factorized into a product of subsystem states, *i.e.*, [8,20,22,23],

$$|\psi\rangle \neq \bigotimes_{k=1}^N |\psi_k\rangle. \quad (1)$$

Following the framework developed in ref. [20], where a detailed analysis of the variance-based GUR for multipartite entangled systems has been presented, we assume that the position and momentum operators for an ensemble of

N identical pure entangled particles are defined as

$$\hat{x} = \hat{x}_1 + \dots + \hat{x}_N, \quad \hat{p} = \hat{p}_1 + \dots + \hat{p}_N, \quad (2)$$

where each pair satisfies the canonical commutation relation $[\hat{x}_k, \hat{p}_l] = i\hbar \delta_{kl}$ with $k, l = 1, 2, \dots, N$. Consequently, the collective operators obey

$$[\hat{x}, \hat{p}] = [\hat{x}_1, \hat{p}_1] + \dots + [\hat{x}_N, \hat{p}_N] = Ni\hbar. \quad (3)$$

To properly capture correlations between particles, we introduce the quantum covariance functions (QCFs) for the position and momentum operators as

$$\begin{cases} C_x(k, l) = \langle \hat{x}_k \hat{x}_l \rangle - \langle \hat{x}_k \rangle \langle \hat{x}_l \rangle, \\ C_p(k, l) = \langle \hat{p}_k \hat{p}_l \rangle - \langle \hat{p}_k \rangle \langle \hat{p}_l \rangle, \end{cases} \quad (4)$$

which quantify the statistical dependence between different subsystems. These QCFs play a central role in generalizing the standard-deviation-based uncertainty principle to the multipartite entangled case.

The QCFs introduced above are, in general, non-zero for pure non-factorizable (entangled) states. In this case, the GUR for the collective operators takes the form

$$(\Delta \hat{x})^2 (\Delta \hat{p})^2 \geq \frac{N^2 \hbar^2}{4}. \quad (5)$$

Equation (5) follows directly from the Robertson uncertainty relation $\Delta \hat{A} \Delta \hat{B} \geq \frac{1}{2} |\langle [\hat{A}, \hat{B}] \rangle|$ applied to the collective operators $\hat{x} = \sum_k \hat{x}_k$ and $\hat{p} = \sum_k \hat{p}_k$, using the commutation relation $[\hat{x}, \hat{p}] = Ni\hbar$. Here, the total variance of the position operator can be expressed in terms of the QCFs as

$$(\Delta \hat{x})^2 = \sum_{k,l=1}^N C_x(k, l) = \sum_{k=1}^N (\Delta \hat{x}_k)^2 + \sum_{k \neq l=1}^N C_x(k, l), \quad (6)$$

where the first term accounts for local uncertainties of individual particles, while the second captures inter-particle correlations. An analogous expression holds for the momentum variance.

To proceed, let us define the following general states in position space:

$$|\psi_{1x}\rangle = \left(\sum_{k=1}^N a_k \hat{x}_k \right) |\psi_x\rangle, \quad |\psi_{2x}\rangle = |\psi_x\rangle, \quad (7)$$

with coefficients $a_k = \pm 1$. These auxiliary states are the multi-particle generalization of the construction introduced in ref. [22] for $N = 2, 3$ and extended to arbitrary N in ref. [20]. They are introduced for obtaining explicit bounds on the QCFs by applying the Cauchy-Schwarz inequality, $|\langle \psi_1 | \psi_2 \rangle|^2 \leq \langle \psi_1 | \psi_1 \rangle \langle \psi_2 | \psi_2 \rangle$, in a form that directly generates the cross-terms $\langle \hat{x}_k \hat{x}_l \rangle$, thereby connecting the operator formalism to the QCFs defined in eq. (4). Applying the Cauchy-Schwarz inequality to these states yields

$$\left\langle \left(\sum_{k=1}^N a_k \hat{x}_k \right)^2 \right\rangle \geq \left\langle \sum_{k=1}^N a_k \hat{x}_k \right\rangle^2. \quad (8)$$

This inequality establishes a lower bound on the quadratic form involving correlated particle positions and sets the stage for expressing the GUR explicitly in terms of subsystem variances and covariances.

For the choice $a_1 = a_2 = \dots = a_N = 1$, the lower bounds of the QCFs lead to

$$\sum_{k \neq l=1}^N C_x(k, l) \geq - \sum_{k=1}^N (\Delta \hat{x}_k)^2. \quad (9)$$

By manipulating inequality (8) and using the fact that $a_k^2 = 1$, one obtains the following upper bound for the QCFs in a multipartite system:

$$\sum_{k \neq l=1}^N C_x(k, l) \leq (2^{N-1} - 1) \sum_{k=1}^N (\Delta \hat{x}_k)^2. \quad (10)$$

Equation (10) is obtained by summing the Cauchy-Schwarz inequality of eq. (8) over all 2^N possible sign choices $a_k = \pm 1$. For each configuration, eq. (8) gives $\sum_{k=1}^N (\Delta \hat{x}_k)^2 \geq - \sum_{k \neq l} a_k a_l C_x(k, l)$. Summing over all sign configurations and using the combinatorial identity $\sum_{\{a_k = \pm 1\}} a_k a_l = 0$ for $k \neq l$ (since each pair contributes equally with both signs) isolates the covariance sum. A systematic combinatorial argument —explicitly demonstrated for $N = 3, 4, 5$ in ref. [20]— shows that the optimal bound is given by eq. (10), where the factor $2^{N-1} - 1$ counts the number of independent sign configurations that yield non-canceling contributions. An analogous bound holds for the QCFs of the momentum operator. Substituting these bounds into eq. (5) and performing some algebra, we arrive at

$$\left(\sum_{k=1}^N (\Delta \hat{x}_k)^2 \right) \left(\sum_{k=1}^N (\Delta \hat{p}_k)^2 \right) \geq \frac{N^2 \hbar^2}{2^{2N}}. \quad (11)$$

Equation (11) represents the GUR for a system of N identical and entangled particles. In the special case where all particles share the same uncertainty in position and momentum, *i.e.*, $\Delta \hat{x}_1 = \dots = \Delta \hat{x}_N$ and $\Delta \hat{p}_1 = \dots = \Delta \hat{p}_N$, the GUR simplifies to

$$\Delta \hat{x}_k \Delta \hat{p}_k \geq \frac{\hbar}{2^N}. \quad (12)$$

The inequalities (11) and (12) clearly demonstrate that correlations among particles systematically reduce the lower bound on the product of uncertainties in position and momentum. As the number of particles N increases, this lower bound decreases, signaling a gradual suppression of quantum uncertainty and a corresponding drift toward classical behavior. Thus, the presence of quantum correlations significantly enhances the tendency of multipartite systems toward emergent classicality.

While this approach captures the essential role of multipartite entanglement, it still inherits certain conceptual limitations of the conventional variance-based framework.

To overcome these limitations and to obtain stronger, information-theoretically motivated bounds, we now turn in the next section to an *entropic formulation* of the GUR, where uncertainty is quantified in terms of Shannon entropy rather than standard deviation.

The GUR in an entropic scenario. — The uncertainty principle is a cornerstone of quantum mechanics, expressing the fact that measurement outcomes of incompatible (non-commuting) observables are subject to unavoidable uncertainty. However, the conventional uncertainty relation based on the standard deviation exhibits several deficiencies. First, the variance-based formulation depends not only on the probability distribution of measurement outcomes but also on the numerical values assigned to these outcomes; as a result, the uncertainty measure can be skewed by outcome values rather than reflecting purely probabilistic features. Second, the lower bound of the variance-based uncertainty relation is state dependent, and it can even vanish for certain states, despite the corresponding measurement statistics exhibiting genuine uncertainty [8]. Ideally, one would like to separate the bound into two contributions: one reflecting only the degree of measurement incompatibility and another determined by the quantum state. Third, and perhaps most importantly, the standard deviation lacks a clear operational interpretation in information theory.

Entropy, in contrast, provides a natural measure of uncertainty with direct operational significance. For example, entropic quantities govern fundamental tasks such as randomness extraction (deriving uniform random bits from imperfectly random sources) and data compression (minimizing the information required to reproduce random data) [24]. These operational interpretations underpin the usefulness of EURs in applications such as the security of quantum key distribution and cryptographic protocols [25]. For this reason, entropy is widely regarded as the uncertainty measure of choice, while the variance-based approach plays at best a limited role in modern information-theoretic contexts.

Thus, a revision of the uncertainty principle is clearly required in order to overcome the aforementioned deficiencies. The idea of expressing the uncertainty principle in terms of entropy was first suggested by Everett in 1957 [26] and shortly thereafter realized by Hirschman for the case of position and momentum observables [27]. This result was subsequently sharpened by Beckner [28] and independently by Biaynicki-Birula and Mycielski [29], who established the EUR

$$h(\hat{x}) + h(\hat{p}) \geq \log_2(e\pi), \quad (13)$$

valid for all quantum states, where hereafter in this article we set $\hbar = 1$. Here, h denotes the differential Shannon entropy for continuous probability distributions, whose precise definition will be introduced later.

Importantly, inequality (13) provides a purely information-theoretic quantification of uncertainty, replacing

variance with entropy. Moreover, it is strictly stronger than the conventional variance-based uncertainty relation $\Delta\hat{x}\Delta\hat{p} \geq 1/2$ [29], and thus implies the latter as a special case. This reformulation not only avoids the shortcomings of the standard-deviation approach but also paves the way for EURs with direct applications in quantum information theory.

In this section, our goal is to establish an entropic counterpart of the GUR. We begin with eq. (12), *i.e.*, $\Delta\hat{x}_k\Delta\hat{p}_k \geq 1/2^N$, where $\Delta\hat{x}_k$ denotes the standard deviation of the position distribution $\Gamma(x_k)$, and $\Delta\hat{p}_k$ is defined analogously for momentum.

To proceed toward an entropic formulation, let us consider Gaussian wave packets with the position-space probability density

$$\Gamma(x_k) = \frac{1}{\sqrt{2\pi(\Delta\hat{x}_k)^2}} \exp\left[-\frac{(x_k - \bar{x}_k)^2}{2(\Delta\hat{x}_k)^2}\right], \quad (14)$$

where $\bar{x}_k$ denotes the mean position. Gaussian distributions are special because, for a fixed standard deviation $\Delta\hat{x}_k$, they maximize the differential Shannon entropy. This property can be rigorously proven using variational calculus with Lagrange multipliers.

The differential Shannon entropy associated with a position measurement is defined as

$$h(\hat{x}_k) = - \int_{-\infty}^{\infty} \Gamma(x_k) \log_2 \Gamma(x_k) dx_k. \quad (15)$$

Note that we assume this quantity to be finite. In complete analogy, the Gaussian probability distribution for momentum takes the form

$$\Gamma(p_k) = \sqrt{\frac{2(\Delta\hat{p}_k)^2}{\pi}} \exp[-2(p_k - \bar{p}_k)^2(\Delta\hat{p}_k)^2], \quad (16)$$

with corresponding differential entropy

$$h(\hat{p}_k) = - \int_{-\infty}^{\infty} \Gamma(p_k) \log_2 \Gamma(p_k) dp_k. \quad (17)$$

It is straightforward to verify that the Gaussian distributions (14) and (16) saturate the bound in eq. (12), thereby minimizing the uncertainty in terms of the product of the standard deviations. Moreover, inserting (14) into (15) yields the well-known expression for the entropy of a Gaussian position distribution. Since Gaussian distributions maximize the differential entropy for a fixed variance, the following inequalities for position and momentum hold in general:

$$h(\hat{x}_k) \leq \frac{1}{2} \log_2(2\pi e (\Delta\hat{x}_k)^2), \quad (18)$$

$$h(\hat{p}_k) \leq \frac{1}{2} \log_2\left(\frac{\pi e}{2(\Delta\hat{p}_k)^2}\right). \quad (19)$$

Now consider an arbitrary quantum state for a particle's translational degree of freedom, which gives rise to the

random variables $\hat{x}_k$ and $\hat{p}_k$ for position and momentum, respectively. By combining the bounds (18) and (19) and substituting into the entropic uncertainty relation (13), we obtain

$$\log_2(2\pi e \Delta\hat{x}_k \Delta\hat{p}_k) = \frac{1}{2} \log_2(2\pi e (\Delta\hat{x}_k)^2) + \frac{1}{2} \log_2(2\pi e (\Delta\hat{p}_k)^2), \quad (20)$$

which implies

$$\log_2(2\pi e \Delta\hat{x}_k \Delta\hat{p}_k) \geq h(\hat{x}_k) + h(\hat{p}_k). \quad (21)$$

Furthermore, using the generalized N -particle bound (12) yields

$$h(\hat{x}_k) + h(\hat{p}_k) \geq \log_2\left(\frac{e\pi}{2^{N-1}}\right). \quad (22)$$

By comparing the left- and right-hand sides of (20) and noting that the logarithm is a monotonic function, one finds that (20) is consistent with the generalized variance-based inequality (12). Altogether, this establishes the GUR in an entropic scenario:

$$h(\hat{x}_k) + h(\hat{p}_k) \geq \log_2(e\pi) + 1 - N. \quad (23)$$

It is evident that the EUR (13) is valid only for $N = 1$. In this case, there is no violation of the conventional uncertainty principle, since its original derivation did not account for quantum correlations between identical particles [20,22,23]. However, for a pure multipartite entangled system with $N \geq 2$, the lower bound of the generalized EUR (23) decreases with increasing N .

We emphasize that the bound in eq. (23) is derived specifically for Gaussian wave packets that saturate the variance-based GUR. For general quantum states—in particular non-Gaussian states—the inequalities (18) and (19) become strict, meaning that the actual entropy sum $h(\hat{x}_k) + h(\hat{p}_k)$ lies substantially higher than the lower bound. Consequently, while the bound $\log_2(e\pi) + 1 - N$ may not be globally “tight” (*i.e.*, saturable) for all states when $N \geq 2$, it correctly captures the *maximally allowed entanglement-induced suppression* of uncertainty attainable with Gaussian resources. For $N \geq 2$, the bound is smaller than the standard single-particle EUR (13); this reduction is not a deficiency but rather the central physical result of our analysis: it quantifies how multipartite quantum correlations can progressively overcome the single-particle uncertainty limit. In other words, the bound sets the ultimate lower threshold that uncertainty can reach when the full N -particle entanglement is harnessed. This behavior is a hallmark of emergent classicality in collective observables of the ensemble.

For sufficiently large N , the bound even becomes negative. This behavior is not paradoxical: in quantum information theory, negative entropies naturally arise as a signature of entanglement. In fact, negative conditional quantum entropy has a well-established operational interpretation, both in quantum information processing [30]

and in thermodynamics [31]. For example, the conditional entropy of an entangled bipartite state equals the difference between the joint entropy and the marginal entropy, and its negativity corresponds to the coherent information, which quantifies the distillable entanglement of the system [32]. Moreover, the fully quantum entropy obeys the data-processing inequality, which states that the uncertainty of a system X conditioned on another system Y can never decrease by processing Y [33]. This ensures that negative conditional entropies remain consistent within the operational framework of quantum theory.

It is also worth emphasizing that eqs. (18) and (19) are strict inequalities whenever the distributions are non-Gaussian. As a result, the generalized UR (23) is strictly stronger than the variance-based GUR (12) for non-Gaussian states. This highlights the robustness of the entropic formulation, which naturally extends to scenarios involving correlations among particles.

To illustrate this connection, let us first consider a bipartite entangled system and formulate the EUR in the presence of quantum memory for continuous position and momentum measurements [34]. In this setting, the uncertainty in the measurement outcomes of particle j depends explicitly on the amount of entanglement with the memory system (particle 2). The corresponding relation takes the form [4]

$$h(\hat{x}_j|2) + h(\hat{p}_j|2) \geq \log_2(2\pi) + H(1|2), \quad j = 1, 2, \quad (24)$$

where $H(1|2)$ denotes the conditional von Neumann entropy of subsystem 1 given access to subsystem 2 and explicitly quantifies the amount of entanglement between the two particles. This conditional entropy can become negative in the presence of strong entanglement, which tightens the entropic bound and further illustrates how quantum correlations suppress uncertainty in multipartite systems.

In eq. (24), the uncertainty about the outcome of measuring $\hat{x}_j$, given access to quantum information stored in particle 2, is quantified by the differential quantum conditional entropy $h(\hat{x}_j|2)$. We note that a finite-dimensional approximation has been employed so that all terms in the conditional entropy $H(1|2)$ —defined as the difference between the joint entropy and the marginal entropy—remain finite.

The state-independent bound in (24) is $\log_2(2\pi)$, which is strictly smaller than the corresponding bound $\log_2(e\pi)$ in the memoryless scenario (13). This indicates that the presence of a quantum memory reduces the state-independent uncertainty limit. The physical reason is that in a finite-dimensional approximation, a gap persists between accessible classical and quantum correlations, and the quantum memory exploits the latter to tighten the uncertainty bound.

It is instructive to evaluate $H(1|2)$ in the light of our generalized EUR (23). We find $H(1|2) \approx \log_2(\frac{e}{4})$, which is smaller than the value obtained from the standard EUR (13), namely $H(1|2) \approx \log_2(\frac{e}{2})$. Thus, when applying the generalized EUR (23), the state-dependent term

$H(1|2)$ is reduced, leading to a tighter overall lower bound in (24). In operational terms, this reflects a smaller gap between classical and quantum correlations and consequently a more precise characterization of measurement uncertainty.

We now extend this analysis to a genuinely multipartite entangled ensemble of N identical particles. For such a system, one can designate a specific particle k as the system to be measured, while treating the *entire set of the remaining $N - 1$ particles* as a composite quantum memory, which we denote collectively by $\mathcal{M}_k \equiv \{1, \dots, N\} \setminus \{k\}$. The conditional entropy $H(k|\mathcal{M}_k)$ then quantifies the total entanglement between the measured particle and the rest of the ensemble. The corresponding multipartite EUR with quantum memory generalizes eq. (24) to

$$h(\hat{x}_k|\mathcal{M}_k) + h(\hat{p}_k|\mathcal{M}_k) \geq \log_2(2\pi) + H(k|\mathcal{M}_k). \quad (25)$$

Remarkably, using the scaling behavior derived from our entropic GUR, the conditional entropy for this multipartite configuration scales as $H(k|\mathcal{M}_k) \approx \log_2(\frac{e}{2^N})$, which tends monotonically to zero as $N \rightarrow \infty$. This result establishes that in highly entangled multipartite systems, the presence of many quantum memories (*i.e.*, the collective action of the remaining $N - 1$ particles) progressively suppresses conditional entropies, driving the uncertainty bound to its state-independent minimum $\log_2(2\pi)$. Physically, this signifies that the measured particle becomes so strongly correlated with its environment (the rest of the ensemble) that the uncertainty in its individual observables is maximally reduced, leading to near-classical precision in measurement outcomes. We emphasize that the relation (25) is distinct from a simple bipartite scenario: here the memory is not a single particle but rather the collective reservoir of $N - 1$ entangled subsystems, which explains the enhanced suppression of uncertainty compared to eq. (24).

Before proceeding to the implications of these results, it is important to address the physical validity of employing differential Shannon entropy $h(\cdot)$ for continuous variables such as position and momentum. In realistic experimental scenarios and quantum information protocols, measurements are inherently discrete due to finite detector resolution or coarse-graining of phase space. Nevertheless, the continuous-variable formulation presented here captures the essential scaling behavior in the high-resolution limit or for macroscopic ensembles where N is large.

To establish a rigorous connection to discrete measurements, consider a discretization of the position and momentum axes into intervals of size ϵ_x and ϵ_p , respectively, with the phase-space cell satisfying $\epsilon_x \epsilon_p \sim 1$. The discrete Shannon entropy $H^\Delta(\hat{x}_k)$ is related to the differential entropy via $H^\Delta(\hat{x}_k) \approx h(\hat{x}_k) - \log_2 \epsilon_x$ [35]. Consequently, the relative tightening of the entropic bound with increasing particle number N —namely, the term $1 - N$ in eq. (23)—remains independent of the resolution parameters ϵ_x and

ϵ_p . Therefore, the suppression of uncertainty due to multipartite entanglement persists unaltered in the discrete-measurement scenario, validating the continuous-variable analysis as a reliable proxy for the underlying physics. For a fully rigorous discrete treatment, one may invoke the EUR for finite-dimensional systems [3,4], which converges to the continuous bound in the appropriate limit.

Having established the theoretical framework, it is natural to ask how such entropic GURs may find applications across physics. In the following section, we discuss several prospective domains—including quantum information protocols, condensed-matter systems, early-universe cosmology, and black-hole physics—where our results may provide valuable new insights.

Potential applications and implications. – The generalized EUR derived in the previous section is not only of foundational significance but also has far-reaching implications across several domains of quantum physics and information theory. In the context of quantum information and cryptography, EURs have long served as key tools in establishing the security of quantum communication protocols. The presence of quantum memory allows an eavesdropper to correlate with the system and thereby reduce uncertainty, as emphasized in the seminal work of Berta *et al.* [4]. Our generalized relation, which incorporates multipartite entanglement, naturally extends this framework. In particular, the reduction of conditional entropy $H(k|\mathcal{M}_k)$ with increasing N suggests that multi-party quantum key distribution and distributed quantum communication may benefit from stronger security bounds.

In quantum metrology, multipartite entanglement is known to enhance measurement precision beyond the standard quantum limit [36]. The entropic GUR developed here, and especially the scaling behavior of $H(k|\mathcal{M}_k)$ as $N \rightarrow \infty$, provides a refined constraint on measurement uncertainty. Large ensembles of entangled particles can therefore, at least in principle, achieve more accurate estimation of physical parameters, with the conditional entropy approaching zero as a signature of metrological advantage.

The implications extend further to many-body and condensed-matter systems, where collective entanglement underlies a variety of emergent macroscopic phenomena. Strongly correlated systems such as spin chains, superconductors, and Bose-Einstein condensates exhibit entanglement-driven behaviors that often resemble classical collective order [37]. The suppression of entropic uncertainty per particle offers a new information-theoretic perspective on why such systems display quasi-classical features, despite their fundamentally quantum nature. Thus, our results may serve as a conceptual bridge between entanglement theory and macroscopic emergence.

The appearance of negative conditional entropy in our framework also bears direct relevance to quantum thermodynamics and resource theories of information.

Negative entropy is closely associated with advantages in state merging, work extraction, and reversibility in quantum processes [38,39]. Our generalized EUR provides explicit conditions under which negative conditional entropy arises or becomes more pronounced as N increases, thereby illuminating how multipartite entanglement can enhance the efficiency of thermodynamic tasks.

Finally, these considerations touch on foundational questions concerning the quantum-to-classical transition. In early-universe cosmology, entanglement between quantum field modes across the inflationary horizon has been suggested as a mechanism for producing effectively classical fluctuations observed in the cosmic microwave background [40]. Similarly, in black-hole physics, correlations between near-horizon modes may influence uncertainty suppression and the information content of Hawking radiation [41]. Taken together, our entropic GUR framework offers a unifying lens through which such phenomena may be examined, potentially shedding light on some of the most profound challenges in theoretical physics.

These diverse applications underscore that the entropic GUR is not merely a mathematical reformulation of the uncertainty principle but also a versatile tool with broad conceptual and practical consequences for quantum science and technology.

Conclusion. – In this work, we have developed an entropic formulation of the GUR for multipartite identical pure entangled systems. Our central analytical result is the inequality $h(\hat{x}_k) + h(\hat{p}_k) \geq \log_2(e\pi) + 1 - N$, which establishes that the entropic lower bound decreases linearly with the number of entangled particles N . This reduction is a direct manifestation of entanglement-induced suppression of quantum uncertainty and signifies a pathway toward emergent classicality in macroscopic ensembles.

We further extended this analysis to the conditional entropic uncertainty relation in the presence of quantum memory. In the bipartite case, we demonstrated that the conditional entropy takes the reduced value $H(1|2) \approx \log_2(e/4)$, compared to $\log_2(e/2)$ obtained from the standard EUR. More importantly, for a genuine multipartite configuration where the remaining $N - 1$ particles collectively act as a composite quantum memory, the conditional entropy scales as $H(k|\mathcal{M}_k) \approx \log_2(e/2^N)$, which asymptotically approaches zero in the large- N limit. This result rigorously quantifies how the collective entanglement of a many-body system can maximally suppress the measurement uncertainty of its individual constituents, driving the bound toward its state-independent minimum $\log_2(2\pi)$.

The entropic formulation presented here resolves conceptual limitations inherent in variance-based approaches, provides strictly stronger bounds for non-Gaussian states, and remains valid—up to resolution-dependent offsets—in physically realistic discrete-measurement scenarios.

Beyond foundational significance, our findings open several promising avenues for application, as previewed in the

previous section. The scaling behavior of the conditional entropy may inform improved security analyses in multipartite quantum key distribution protocols, guide precision enhancements in entanglement-assisted quantum metrology, and offer a novel information-theoretic perspective on the emergence of classical collective behavior in strongly correlated many-body systems. Furthermore, the explicit connection between multipartite entanglement and negative conditional entropy reinforces the operational interpretation of entanglement as a thermodynamic resource.

In conclusion, the entropic GUR developed in this work provides a robust and versatile framework for exploring the deep interplay between quantum correlations, measurement uncertainty, and the quantum-to-classical transition, with significant implications for both foundational physics and emerging quantum technologies.

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