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UNIFIED ANALYSIS OF OBSERVATION DATES FOR ANCIENT STAR MAPS AND CATALOGUES IN ASIA

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Abstract: It is an intriguing subject in the history of astronomy to determine scientifically observation dates of stars recorded in ancient star maps and catalogues of East Asia. Most of previous studies used least-squares fitting as a function of time between observation and calculation for a group of selected stars, and they simply took the resulting mean residuals as the dating errors. In 2014, the author of this paper was given an opportunity to perform a dating analysis of a star map drawn on the ceiling of the stone room of the Kitora tumulus discovered in 1998, located at Nara in Japan. The construction time of the tumulus had been inferred by archaeologists to be around the end of the 7th century. The main difficulty in treating stellar coordinates of Chinese origin was that their longitudes had not been measured from a common cardinal point, thus preventing us from directly using the precession theory. After trial-and-error experiments, we developed a method for analyzing the observation epoch of stars exclusively using the 28-Xiu constellations based on an interval estimation in modern statistics. This method is applicable to many historical star maps and catalogues in a unified way. Furthermore, to reduce the error range of dating, the bootstrap method for small sample sizes was introduced. As a result, we obtained 81 BC ± 42 years (for a 90% confidence level) as the observation date of the Kitora star map. The same procedure was applied, with successful outcomes, to data in the Shi shi xingjing, the Almagest catalogue, the famous stone-inscribed Suzhou tianwentu, the 28-Xiu observations by Guo Shoujing, the Japanese paper star map Koshi Gesshin-zu, and Ulugh Beg' s star catalogue. Thus, it is certain that our approach can be effectively used for other old star maps and catalogues yet unexplored.

Keywords: Star map and catalogue, constellation, precession, dating analysis, interval estimation

1 INTRODUCTION

Astronomy has often been regarded as one of the earliest natural sciences in human history. Astronomy is also characterized by the fact that it had already been developed as an exact mathematical science from ancient times in both the Western and Eastern worlds. Its representative aspects are calendrical astronomy based on the motion theories of celestial bodies and star catalogues and maps compiled and drawn from positional observations of stars on the celestial sphere.

Regarding the second aspect, this paper reports our recently devised method for statistically estimating observation dates of stars, which can be widely applied to historical star catalogues and maps.[1] This method uses, in a unified way, exclusively star positions for the Chinese fundamental system of asterisms that originated approximately 2,500 years ago, namely the 28-Xiu constellations (Xiu 宿 literally means a lodge) (e.g., Needham, 1959; Sun and Kistemaker, 1997). Such an approach that adopts strict interval estimation in modern statistics has never been attempted before in this field.

This research was initially motivated by the Kitora whole-sky star map discovered some 20 years ago at Nara, an ancient capital of Japan, to examine the observational epoch of its stars. In general, the dating analysis of star catalogues and maps utilizes gradual time variations of celestial coordinates of stars caused by the precessional motion of the Earth's rotation axis. However, since the ancient traditional Chinese coordinates of stars are unique and did not allow us to compare them directly with the coordinates calculated from modern precession theory, we had to use a variety of trial-and-error approaches to achieve our goal.

First, by applying our proposed method to a certain old star catalogues and maps whose observation epochs have already been established, we could successfully reproduce their values. Thus, we decided to apply the same method to stars in the Kitora star map; we obtained their reasonable observation periods and found them to be consistent with relevant historical materials. Section 2 below provides an overview of mural-painted burial mounds in East Asia and their characteristics, including the Kitora tumulus in Japan. Section 3 explains the principle and practical processes of how our statistical dating analysis for ancient star catalogues and maps was conducted relating to the traditional Chinese stellar coordinate system and its observation instruments. Section 4 presents the estimated observational epoch for the 28-Xiu constellations in the Kitora star map.

To assess the applicability of our dating analysis method, the results applied to the star

Figure 1: Map of Korea, China and Japan and their present capitals. Many mural-painting or decorative tumuli are found over the Jilin province of China, the northern half of Korean peninsula and the western part of Japan. The diamond mark near Osaka is Nara city where the Kitora tumulus is located (map: Tsuko Nakamura).

catalogue contained in the *Almagest* by Ptolemy are provided in Section 5 in comparison with the results from the previous section. Section 6 discusses dating of several historical star catalogues and maps produced in China, the Middle East, and Japan during the medieval and pre-modern eras. Through these sections, we could validate our proposed dating method.

2 MURAL PAINTED TUMULI OF EAST ASIA AND DISCOVERY OF THE STAR MAP IN THE KITORA TUMULUS

In Korea, China, and Japan, there exist numerous special kinds of ancient burial mounds for the dead (Figure 1). They are called the mural painting or decorative tumuli whose stone walls of an interior room are decorated commonly with colorful paintings. Such tumuli are densely distributed from the Jilin Province of north-eastern China down to the northern part of the Korean Peninsula that used to be under the reign of the Koguryeo Kingdom (37 BC–AD 668). To date, about one hundred of these sites have been excavated by archaeologists, revealing their construction era to be the third to the seventh centuries AD (Chon, 2005). Most of their inside walls were painted with colorful designs of people wearing ethnic clothing or abstract geometric patterns. Although some of these paintings appeared to express constellations in the sky, the star configurations were so imprecise that none of them could be used for scientific analysis—from an astronomical viewpoint.

In 1973, a decorated tumulus named Takamatsu-dzuka, located at Asuka village, Nara (the earliest capital of Japan), was under archaeological investigation, and a neat 28-Xiu drawing was found on the ceiling of the stone room. As the shape of each constellation was realistic, this tumulus attracted considerable public attention. Yabuuchi (1975) reported that the arrangement of the Takamatsu-dzuka's 28-Xiu constellations closely resembled a star map investigated in 1963–1965 on the ceiling of the Astana tumulus at Turfan, in the Xinjiang province of China (Institute of Archeology, Chinese Academy of Social Sciences, 1978: Figure 66), suggesting that the star map origin-

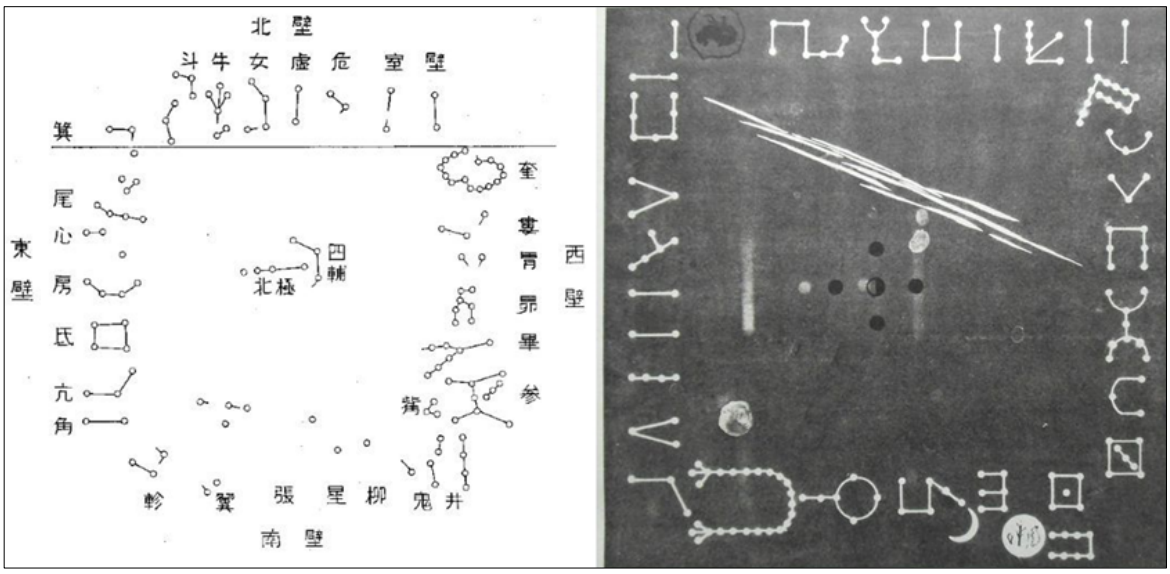


Figure 2: The 28-Xiu constellations drawn on the ceilings of the Takamatsu-dzuka tumulus (left, after Yabuuchi, 1975) and the Astana one (right, Institute of Archeology, Chinese Academy of Social Sciences,1978). The shapes of the 28-Xiu asterisms for the Takamatsu-dzuka look close to the reality, while those for the Astana seem to be fairly stylized. The band-like pattern running from the upper left to the lower right seen in the Astana drawing could be the Milky Way.

ated in central China and was later transmitted to both Middle Eastern and Far Eastern countries (see Figure 2).

From 1983, researchers started unearthing another small burial mound in the same village, called the Kitora tumulus, and in 1998, they discovered an elaborate-looking painting of the hemispherical star map using an endoscope camera. Since this star map was so similar to a stone-inscribed map made in the middle of the thirteenth century in China (see Figure 12 and Section 6), it had a strong impact on both historians and astronomers. Archaeologists suggest that the two Japanese tumuli were constructed between the end of the seventh century and the beginning of the eighth century, and the dead buried inside might be members of the Emperor's family or local ruling clans (e.g., Aboshi, 2006).

Miyajima (1999) first tried to estimate the observation epoch of the Kitora tumulus stars using positional data measured by an endoscope—we will come back to this matter again in Section 4. In 2004, the Nara National Research Institute for Cultural Properties removed all the Kitora paintings, including the star map, from walls to protect them from damage by erosion and mold and to preserve them in an air-conditioned building. In this occasion, the institute produced very precise digital images of the star map by correcting distortion effects caused by, for example, camera lenses. Thanks to the Kitora image files made available by the Nara institute, the author of this paper could develop a unified dating method for ancient star catalogues and maps (Nakamura, 2015).

3 STATISTICAL METHODS FOR DATING ANALYSIS OF HISTORICAL STAR CATALOGUES AND MAPS USING 28-XIU CONSTELLATIONS

Our initial interest in the Kitora star map was to know whether the shape of each asterism and the arrangement of the 28-Xiu constellations were correct from an astronomical viewpoint. If so, we can expect to statistically estimate the observation epoch of these stars by measuring their positions on the star map and combining them with modern precision theory. Having said that, it is difficult to directly compare measurements with theories because the traditional Chinese celestial coordinates are quite different from modern ones, that is, the right ascension (α) and declination (δ). Before discussing this situation in concrete terms, however, it will be appropriate for us to explain first the system of celestial coordinates and observational instruments developed in ancient China as a background to the subsequent sections.

3.1 The 28-Xiu Constellations as a Celestial Framework

From around the Tang Dynasty (seventh century), the Chinese had divided all the constellations into a few sky zones such as the circumpolar region called Ziwei Yuan (Purple Palace city wall). Among them, the system of the 28-Xiu constellations, which lies roughly along the equator and the ecliptic, had already been established in as early as the fifth century BC and was one of the most fundamental concepts in the ancient Chinese astronomy. In fact, the

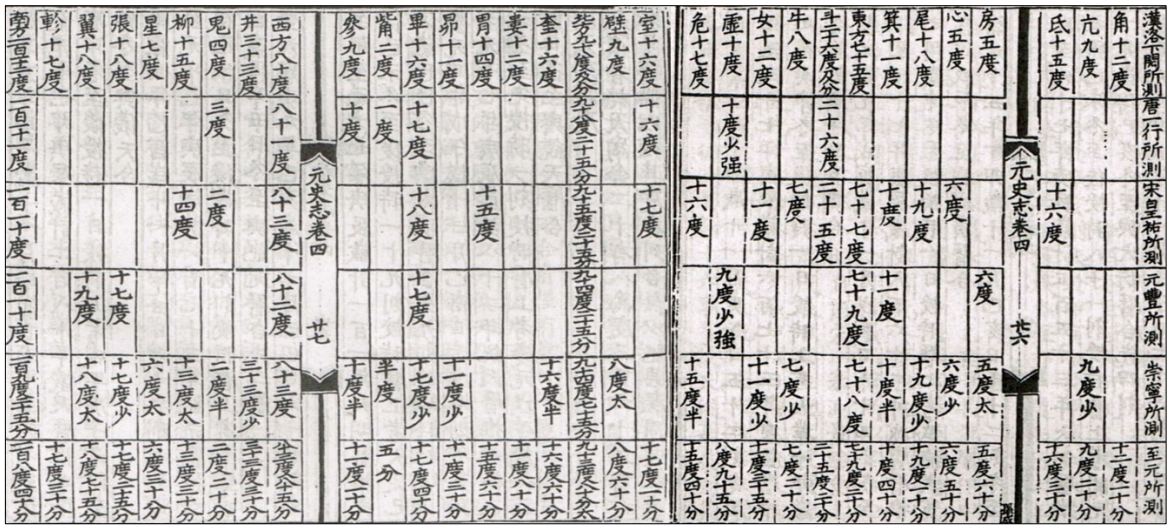


Figure 3: Historical Xiudu observations of Juxing stars for the 28-Xiu constellations cited in the *Yuan shi* (1369). Empty cells of the table mean that their values are the same as those in the upper corresponding cells (based on Bo, ca. 1999).

earliest complete set of the 28-Xiu constellation names was recorded on the cover of a lacquered costume box as one of the burial remains in the tumulus of King Yi (乙), who ruled over the Zeng(曾) province during the Warring States period and died in 433 BC (e.g., Sun and Kistemaker, 1997).

It is likely that the 28-Xiu constellations were invented at their origin to make astrological predictions depending on where the Moon lodged in the sky, since the number 28 is close to the sidereal period of the Moon, 27.3 days. Later on, the 28-Xiu constellations served as a framework to measure celestial coordinates of stars and planets, as well as season indicators in the Chinese calendar, even though the significance of the latter role was gradually lost towards pre-modern times (Nakamura, 2017).

The way in which traditional Chinese celestial coordinates were expressed has intimate relations with their observation instruments. In ancient China, the equatorial armillary sphere was used to observe the latitudes and longitudes of celestial bodies (e.g. Needham, 1959a). According to *Han shu*, *Lulizhi* (History of the Former Han Dynasty), the Chinese armillary sphere was invented by an astronomer Luoxia Hong (落下閎) at the time of the Taichu Calendar reform (104 BC) during the Former Han era, and he and his colleagues used it to measure celestial positions of the 28-Xiu constellation stars for the first time. For example, these values are cited in *Yuan shi* (History of Yuan Dynasty) as observations made by Luoxia Hong (see the uppermost part of Figure 3).

The rightmost column reads the observation times, from the top downwards, respectively, measurements in 104 BC of the Former Hang Dynasty by Luoxia Hong (落下閎), in the Kai-yuan (開元) period (713–741) of the Tang Dynasty by the monk astronomer Yi Xing (一行), in the Huangyou (皇祐) period (1049–1051) of the Song Dynasty, in the Yuanfeng (元豐) period (1078–1085), in the Chongning (崇寧) period (1102–1106), and in the early times of the Zhiyuan (至元) period (1264–1294) of the Yuan Dynasty by Guo Shoujing.

For each of the 28-Xiu constellations, a positional reference star called Juxing (距星) was defined from which relative longitudes and latitudes of other nearby stars were measured. On the other hand, the location of each Juxing star itself in the sky was indicated by the two quantities named Xiudu (宿度) and Qujidu (去極度). The former is the difference in right ascension (α) between two neighboring Juxing stars (star $i + 1$ and star i), $\alpha_{i+1} - \alpha_i$, and the latter means co-declination of each star, namely, $90° - \delta$. For easy recognition, the following notation will be used hereafter whenever appropriate:

$$dRA = \alpha_{i+1} - \alpha_i \quad (1)$$

and

$$cDC = 90° - \delta \quad (2)$$

From the viewpoint of modern astronomy, Xiudu (dRA), in particular, is a very strange quantity to show a position because its value does not give us any information where the star is located on the celestial sphere. Given the whole set of the 28 Juxing stars, the mutual configuration of the 28-Xiu constellations can be determined, but their absolute arrangement in the sky is still unknown. The reason why the ancient Chinese adopted such an unusual measure of longitude is, perhaps, that dRA and

cDC can easily be observed with an early equatorial armillary sphere (see the section on the armillary sphere in Needham, 1959a), while it was difficult for them to measure α and δ, the arc lengths from cardinal points like the vernal or autumnal equinox and the equator, since they are both invisible.

3.2 Unified Use of the 28-Xiu Constellations

As all the 28 Juxing stars are reasonably bright and widely distributed throughout the sky, we decided to adopt them solely for the dating analysis of the Kitora star map and other sources because it was expected that this approach would allow us to obtain an unbiased observation epoch of the star map even with data points as small as 28.

In statistical dating analysis of historical star catalogues and star maps, the first thing we need to do in preparation is to identify the stars in the material in question. This identification process involves two steps. The first is to know which Juxing star corresponds to the one in a modern star catalogue by using information such as the Bayer designation, which was first adopted in his star atlas *Uranometria* (1603).

This process was performed in 1911 by Jesuit Fathers Yachita Tsuchihashi (土橋八代太) and S. Chevalier at the Yushan Astronomical Observatory in Shanghai (Tsuchihashi and Chevalier, 1911). They collated approximately 1,300 Chinese star names with corresponding European ones based on the star catalogue in *Yixiang kaocheng* (*Treatise on Astronomical Instruments*; the stars were observed in 1744), which were published in 1755 by the German Jesuit astronomer Dai Jinxian (I. Kögler) and his collaborators. We also make use of this result in this paper.

The second identification step is to know which star in the Kitora star map corresponds to the one named in the star table of *Yixiang kaocheng*. This step took more time and effort than we had initially anticipated because every star on the original Kitora star map had no written literal information, and most drawings of the 28-Xiu constellation shapes were more or less exaggerated or distorted. As a result, we inevitably compromised with subjective or unconfident identifications of several Juxing stars (see Figure 7). After this process, we can proceed to the next research phase: dating analysis using precession theory.

3.3 Plan of Statistical Dating Analysis

The basic idea of dating analysis is to look for the year that is closest to a corresponding one (C) calculated from the precession theory (a contribution from proper motion is neglected because of its insignificance) for the measured position (O) of each Juxing star. In other words, we require that the absolute value of (O–C) becomes minimum. However, since the measured position of each star generally includes various errors, to increase the robustness of obtained results, it is common to process data for all the Juxing stars as a whole, namely using the criterion $\Sigma_i(O–C)_i^2/n$ = minimum (where the summation is taken over the data points of n). This is a well-known statistical approach called the least squares method—we also follow the same approach in this paper.

Of course, the least squares method can be applied directly to the Kitora dRA and/or cDC data. However, regarding the dRA data, it will be shown in the next section that dating analysis using this quantity does not provide a generally substantially useful estimation of the observation epoch in general because the dRA is a particularly insensitive variable to the time variation due to precession. Owing to this drawback, we had to search for other approaches in the following subsection.

As mentioned earlier in Section 3.1, the largest obstacle to the adoption of the Xiudu (dRA = $\alpha_{i+1} - \alpha$) for the dating analysis is that the quantity is not measured from the equinoxes, so each Xiudu cannot be located in the sky. This situation can be visually illustrated using a schematic diagram.

For simple explanation, a three-star system is assumed here (Figure 4) instead of the actual 28 Juxing stars, which is not shown on a sphere but on a plane. P denotes the north pole, G is the vernal equinox, and O_i and C_i are measured and theoretical positions of each star, respectively. The point of each C_i on the sky plane can be determined unambiguously from precession theory. Each arc PO_i can also be drawn from the Qujidu value since cDC = 90° – δ. On the other hand, Xiudu such as arc O_1O_2 (or angle O_1PO_2) can never be drawn clearly in Figure 4 because it is not measured from the cardinal direction PG. If we consider all of O_is as a whole, we can specify the shape of the triangle $O_1O_2O_3$ around P, but its orientation is not yet defined. This is the main reason why Xiudu is not appropriate to our dating criterion, $\Sigma_i(O–C)_i^2/n$ = minimum.

3.4 Analysis Using 'Positional Shift'

Before addressing the above problem, we introduce and discuss a new observable, 'positional shift' (hereafter abbreviated as PS). This quantity is defined as

$$PS^2 = (\alpha_O - \alpha_C)^2 \cos^2\delta_C + (\delta_O - \delta_C)^2 \quad (3)$$

for each Juxing star, where the subscripts O and C represent measured and theoretical values. In Figure 4, PS corresponds to the arc O_iC_i, denoted by a thick and short bar.

The introduction of PS has two purposes. One is intended to improve the dating precision of star catalogues and maps. By using PS, we can statistically estimate observation epochs more precisely rather than analyzing α or δ separately due to increased information. However, it is obviously impossible to calculate ($\alpha_O - \alpha_C$) from the Xiudu value. As for this problem, a plan to overcome this difficulty is discussed in Subsection 3.5 below.

The other purpose to adopt PS is to take into account the drawing quality of the star map. Examining the Kitora star map in detail, we noticed that lines connecting stars in the asterisms are often not straight, and sometimes doubly drawn—this is good evidence of freehand drawing without the use of a scale or a protractor. In dating analysis of such a star map, we believe that it is more reasonable to measure each Juxing star as a point (namely as a PS value) than as dRA or cDC data independently. This idea for mathematically treating freehand-drawn images is consistent with the psychological notion that human brains are likely to recognize the shape of things not as one-dimensional information such as longitude and latitude but as two-dimensional information like PS.

These are the reasons why we introduce PS as a new observation quantity. Furthermore, it is shown in Section 4 that adopting PS data is useful in enabling us to perform more precise dating, as long as the dating results obtained by using the dRA or cDC data separately do not contradict each other.

3.5 Juxing Stars Taken as the Longitudinal Origin

Through trial-and-error attempts to utilize the dRA effectively, an idea came to our mind: to perform statistical dating analysis by regarding each Juxing star as the longitudinal origin. Figure 5 shows a conceptual explanation of the idea. First, we assume that the positions of all the Juxing stars in the Kitora star map were observed in a hypothetical year (AD 300) and were perfectly accurate without any errors. In this case, the origin of longitude can be arbitrary, and we may take each Juxing star as the origin. Then, we can calculate the longitudes of all the Juxing stars, and their PSs in Equation (3) become zero only in AD 300. As a result, the square root of the corresponding value $\Sigma_i(O-C)_i^2/n$ for each Juxing star will behave as a function of time like each left-hand panel of Figure 5, in which for simplicity, the number of the Juxing star system is taken to be three and the precession theory is approximated by a linear expression.

Next, we add a small random error to PS data of each Juxing star. This time, the square root of $\Sigma_i(O-C)_i^2/n$ for each Juxing star is expected to behave as shown in the right-hand panel of Figure 5, and the estimated observation epoch for each Juxing star will be T_1, T_2, and T_3, which are slightly different from AD 300. Nevertheless, their average value should still be close to AD 300, especially if the number of data points is close to that of the real Juxing stars. Note that this procedure can be effective in processing not only PS data but also the dRA.

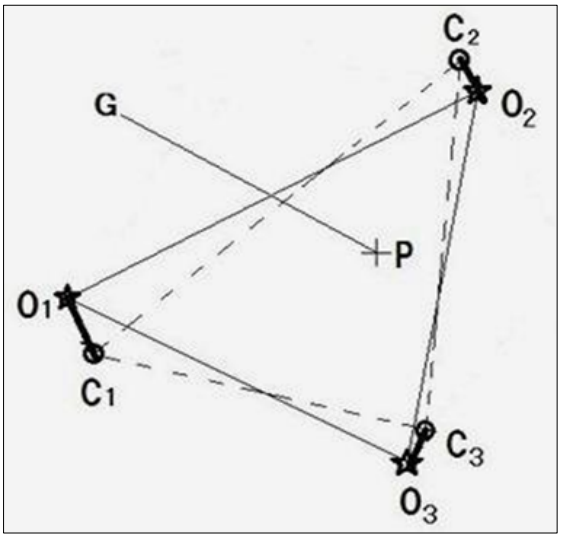


Figure 4: A schematic diagram of the dating analysis using the Chinese Xiudu and Qujidu when n = 3. For the meaning of each alphabetical symbol see the text (diagram: Tsuko Nakamura)

That said, we were not 100% sure whether or not this approach was logically correct in a strict sense. However, guided by intuition, we attempted to apply this procedure to a few historical star catalogues and star maps whose observation dates were known and, eventually, could obtain satisfactory results—their respective outcomes are discussed in Section 6. Hence, we decided to apply the above method to the Kitora data as well.

3.6 Dating Estimation by Simulation

In most of the past dating analyses of star catalogue and star maps, the standard deviation (SD) at the minimum value of $\Sigma_i(O-C)_i^2/n$ was considered to be the error range of an estimated date. However, this is justified only if the number of data points is large enough for the residual distribution to become a Gaussian (or normal) distribution. Meanwhile, the Juxing

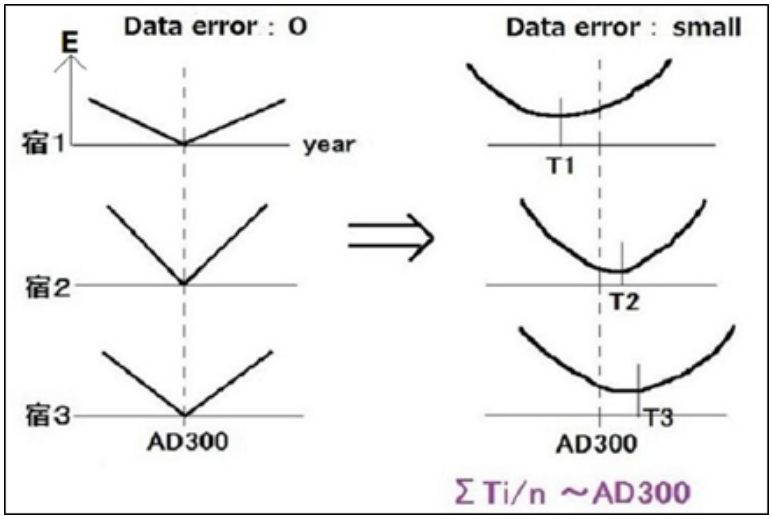


Figure 5: Conceptual graphs explaining the behavior of the square root (E) of $\Sigma_i (O–C)_i^2/n$ (n = 3) for zero error (left) and small non-zero error (right) data. It is assumed that the observation date is AD 300 and the precession theory is approximated by a linear equation (plots: Tsuko Nakamura).

stars provide at most 28 data points and often a fewer than that owing to some deficit of data. In this situation a blindly calculated SD generally gives no more than a meaningless measure as an error range of the estimated date, because the residual distribution for a star map such as the Kitora one containing large errors is often far from the Gaussian distribution, or its theoretical distribution property is unknown.

In such cases, it is common to calculate a practical error range by simulation. This method infers an error range by numerical simulation from about a hundred artificial residual distributions made from the histogram of the original residuals $(O–C)_i$ using computer-generated pseudo-random numbers. This error range estimation by simulation was adopted also for the Kitora data and other star catalogues and maps in the following sections.

There is a promising means to confirm the validity of the above procedure, called model analysis. This approach uses, as model data, theoretical positions of the Juxing stars at an assumed date such as AD 300.0 with artificial errors (generally of random nature) added. Then, after having analyzed these hypothetical observation data by the above procedure, we examine how precisely the original date AD 300.0 is recovered. This result will be explained in Section 4, along with that for the Kitora star map.

3.7 Interval Estimation Using the Bootstrap Method

In modern statistics (e.g., see Conover, 1971), the estimation of a statistical quantity is conducted as follows: first, specify a confidence level (β) such as 90% or 95%, and thereby calculate a corresponding confidence interval (in other words, an uncertainty width of estimation) for the mean value of the quantity, for example.[2] This approach is usually called interval estimation. An interval can be obtained analytically or numerically; the simulation method described in the previous subsection is one of the numerical procedures.

Although the simulation method mentioned above can be applied to a wide variety of problems, it generally takes a lot of work and time to produce a sufficient number of simulated observation data—a more refined interval needs much more simulation data.

To overcome this inconvenience we searched for numerical techniques that facilitate calculations with reduced labor and time and, eventually found the bootstrap method. The bootstrap method (or bootstrapping) is a new interval estimation algorithm, recently proposed by a US statistician Bradley Efron (1979). This method allows the processing of a wide variety of statistical variables, including nonparametric variables. It can also provide a reasonably precise estimation for noisy data and a sparse sample size, as small as 20–30, (Efron and Tibshirani, 1994). Because of these benefits, the bootstrap method has recently become one of the most popular statistical techniques widely used in various fields of science and technology, economics, psychology, etc. (e.g., Chernick, 1999).

In this paragraph, we briefly describe an outline and characteristics of the bootstrap method. The basic concept of this method is to estimate the statistics of a population by random resampling. Usually, a hundred or a little more artificial datasets are produced from orig-

inal observed data, allowing multiple sampling of the same data value and using all the datasets synthetically, necessary statistical quantities such as a mean (average) and the corresponding confidence intervals are calculated for a pre-specified confidence level β. Concerning the theoretical details, the readers should refer to relevant references (e.g., Chernick, 1999; Efron, 1979). The concrete steps of the bootstrap calculation for the Kitora data are given in Section 4.

Regarding the dating estimation of ancient star catalogues and maps, we have so far discussed a few methods above. In the analyses of Section 4–6, these methods will selectively be used depending on their necessity and intended purposes. However, since we found that the approach mentioned below could generally achieve the most reliable observation epochs, we have called it the '28-Xiu BS method', and it is summarized in the following paragraph.

The 28-Xiu BS method first obtains by a least squares fitting the 28 observation epochs for each Juxing star taken as the longitudinal origin. Statistical quantities to be analyzed can be right ascension, PS, or even insensitive Xiudu (dRA) if useful. Then, by using all the obtained epochs we calculate a synthetic epoch in combination with the bootstrap method—this value is the final observation epoch that we need. The 28-Xiu BS method will mainly be applied to estimation of the observation dates of historical star maps and catalogues, including the Kitora star map.

4 ESTIMATION OF THE OBSERVATION EPOCH FOR THE KITORA STAR MAP

4.1 The Kitora Tumulus Star Map and its Measurements

This star map was drawn on the ceiling of a stone room (size: 2.40m deep × 1.04m wide × 1.24m high) in the Kitora burial mound. Figure 6 is a photograph of the main part of the map with the north pole as the center, painted on the plaster wall (Nara National Research Institute for Cultural Properties, 2016). The equator (diameter 40cm), ecliptic,[3] inner (17cm), and outer (61cm) circles (called 規 Gui)[4] and star-connecting lines for each asterism are all drawn in red ink. The total number of confirmed asterisms and stars amounted to 68 and approximately 300, respectively. A magnifying glass revealed that most of the star images were represented by a tiny coin-like gold with a diameter of 6mm and a thickness of approximately 0.3mm. Although Figure 6 shows horizontal exfoliation of plaster in the upper middle, this did not affect the dating result of this star map.

After printing the Kitora digital image of Figure 6 in the same size as the original one, we measured the Xiudu (cDC) with a steel scale and the Qujidu (dRA) with a protractor for each of the 28 Juxing stars relative to the north pole as the center of the equatorial circle. Since in China there existed no mathematical projection concept such as the stereographic one that was widely used by ancient Greek scholars, in Figure 6 the Qujidu simply represents the radial distance from the north pole to a Juxing star, and the Xiudu is an angle between two neighboring stars seen at the north pole.[5] Measuring accuracy for a distance and an angle were 0.5mm (corresponding to 0.23°) and 0.5°, respectively.

Figure 7 represents a graphical summary of the above measurements. To each of the 28-Xiu constellations, its name by a single Chinese character and the pronunciation is attached. Our identified Juxing stars are shown by red dots with a sequential Arabic number starting from Jiao (角). We could not identify three Juxing stars, Niu (牛), Nu (女) and Xu (虛) due to the damage of the plaster surface so that neither the Xiudu nor the Qujidu of them were obtained.

All of the measured data for the Juxing stars are given in Table 1. Note that names of other asterisms are indicated only by Chinese characters in Figure 7; all of those identifications were made by the Nara National Research Institute for Cultural Properties (2016). Since our main interest of this paper is to do the dating analysis by exclusive use of the 28-Xiu Juxing stars, we do not discuss here whether the identification of other stars is correct or not.

4.2 Calculations of Precession

To estimate the observation date of the Kitora star map, first, we have to calculate the exact positions (α and δ) of the Juxing stars for more than ten centuries at an interval of 50 or 100 years. In doing so, it is common to use the precession theory by S. Newcomb (Nautical Almanac Office, 1961) and the standard of modern star catalogues, the *Yale Bright Star Catalogue* (Hoffleit 1981). As these mathematical forms are expressed most elegantly by rotation matrices but their actual operations are somewhat complicated, they are not shown here. Interested readers should refer to Mueller (1969), Seidelmann (1992), or the Appendix in Nakamura (2018).

Figure 6: The central part of a photograph of the Kitora ceiling star map (courtesy: Nara National Research Institute for Cultural Properties), cited from *Kitora Tumulus Star Atlas Constellations Photo Book* (2016). The equator, ecliptic and inner and outer Gui circles are drawn with red ink, and each star image is expressed by a tiny coin-like gold.

4.3 Dating Analysis of the Xiudu and Qujidu

In this paper, we performed almost all calculations manually using Microsoft Excel. In such circumstances, it is practically impossible to apply the exact procedure mentioned above to the interval estimation of the Kitora's observation epoch, in which very laborious repeated similar computations are required.

This problem can be alleviated by adopting approximated equations for the precession theory instead of the exact method above. For this purpose, we constructed linear and quadratic equations to obtain the α and δ of the Juxing stars as a function of time. These coefficients are given in Tables A1 and A2 of the Appendix. In the following analyses of Sections 4–6, the theoretical positions of the 28 Juxing stars are calculated using the two tables. We confirmed that the linear approximation provided us with mean errors of 0.5°–0.8° from the rigorous values between 200 BC and AD 1300, thus this approximation was sufficiently accurate for the dating analysis of the Kitora star map whose (O–C) errors were often more than 3°–4°. Similarly, mean errors of the quadratic approximation were less than 0.1° during the period 200 BC–AD 1800 (Nakamura, 2018), so this approximation can effectively be applied to most observational data of pre-modern times.

4.3.1 Results Using the Qujidu Data

For the interval estimation, we first specify a confidence level (β). For large sample statistics, it is common to use β = 95% or 99%. However,

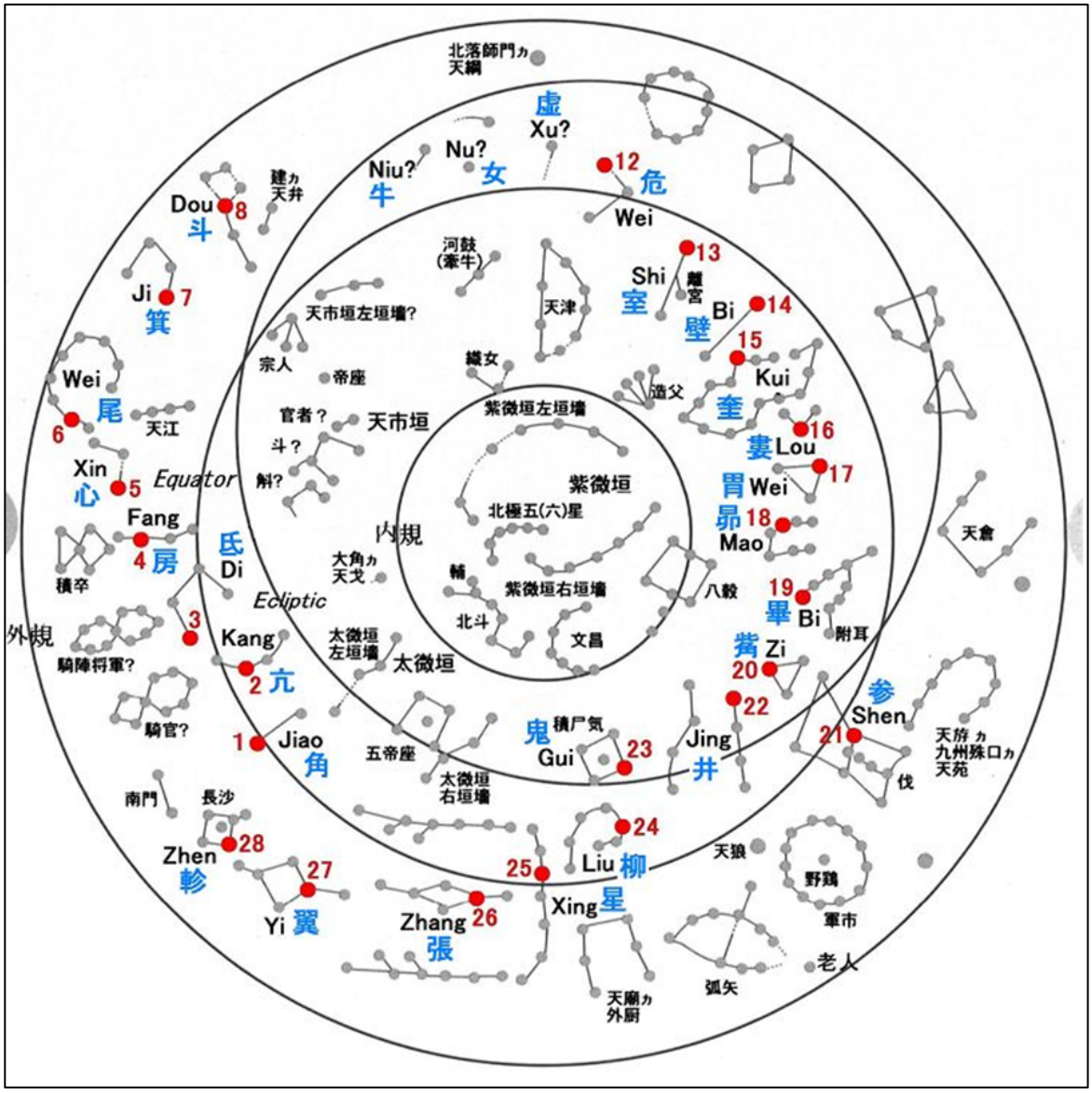


Figure 7: A graphical representation of the Kitora star map. Each red dot with an Arabic number indicates the Juxing star identified by the author. Note that the orientation of the ecliptic circle is totally wrongly drawn (see Note No. 3). The fundamental structure of the graph was borrowed from a figure shown in *Kitora Tumulus Star Atlas Constellations Photo Book* (Nara National Research Institute for Cultural Properties, 2016).

the data points of the Juxing stars in ancient star maps are limited to 28 at maximum and their raw positions often comprise large errors, thus β was set to a modest value, 90%, throughout in this paper.

At intervals of a century, using the linear precession formula for declination in Table A1, we calculated the $SD^2 \equiv \Sigma_i (O-C)_i^2/n$, where SD corresponds to the mean error (or the standard deviation), and n is the sample number used in this analysis. In the case of the Kitora star map, the data for three Juxing stars (Niu, Nu, and Xu) were missing, and a few (O–C)s reached unacceptably large values, 8–10°, due to imprecise freehand drawing. Such outlier data must be removed before analysis generally with a limit of 2SD or 3SD, otherwise a reliable dating cannot be expected.

Eventually, we selected 17 sample data (Table 1) with their (O–C)s of less than 5° at 100 BC and drew a Year versus SD graph, which is shown in Figure 8(b). After fitting a quadratic curve to the SD data points by least squares, we obtained 73 BC as the most probable estimate ('point estimation') of the observation year for declination. Then, to estimate its error range, we made up 100 pseudo data sets of (O–C)s for simulation, which were constructed by adding to theoretical values artificial random errors based on the histogram of the original (O–C)s (whose SD was 2.8°) and calculated the error range ('interval estimation') for β = 90% by repeatedly drawing graphs, such as those in Figure 8(b), a hundred times. This

Table 1: Measured values for the Xiudu and Qujidu of the Juxing stars in the Kitora star map.

No.	28-Xiu	Juxing	Qujidu	DC	Xiudu	rel-RA	Year
1	角	α Vir	92.2	−2.2	11.0	0.0	−64
2	亢	κ Vir	84.6	5.4	9.0	11.0	−23
3	氐	α Lib	95.5	−5.5	15.0	20.0	−82
4	房	π Sco	104.3	−14.3	6.8	35.0	−255
5	心	σ Sco	112.2	−22.2	7.8	41.8	−76
6	尾	μ Sco	126.3	−36.3	19.1	49.6	8
7	箕	γ Sgr	117.4	−27.4	12.9	68.7	19
8	斗	φ Sgr	119.5	−29.5	54.4	81.6	150
9	牛	β Cap					
10	女	ε Aqr					
11	虚	β Aqr					
12	危	α Aqr	98.1	−8.1	16.1	136.0	−79
13	室	α Peg	83.6	6.4	16.1	152.1	−137
14	壁	γ Peg	81.7	8.3	5.1	168.2	−118
15	奎	ζ And	67.3	22.7	19.8	173.3	42
16	婁	β Ari	71.4	18.6	7.9	193.1	−102
17	胃	35 Ari	73.1	16.9	11.9	201.0	−61
18	昴	17 Tau	61.7	28.3	15.7	212.9	−274
19	畢	ε Tau	68.3	21.7	17.2	228.6	−105
20	觜	λ Ori	67.7	22.3	1.9	245.8	−2
21	参	δ Ori	94.6	−4.6	8.0	247.7	−69
22	井	μ Gem	64.8	25.2	29.6	255.7	165
23	鬼	θ Cnc	63.7	26.3	4.3	285.3	−185
24	柳	δ Hya	77.7	12.3	16.1	289.6	−88
25	星	α Hya	86.5	3.5	10.3	305.7	−9
26	張	υ Hya	94.6	−4.6	23.4	316.0	−313
27	翼	α Crt	109.1	−19.1	11.9	339.4	
28	軫	γ Crv	114.4	−24.4	8.7	351.3	−279
						mean	−80.7
						SD	121.5

range (confidence interval) was found to be [350 BC, AD 210] or 73 BC ± approximately 280 years, and it is displayed as a horizontal bar at the bottom of Figure 8(b) panel.

4.3.2 Results for the Xiudu Data

Next, we applied the same procedure to the Kitora Xiudu (dRA) data in Table 1. Figure 8(a) represents a Year versus SD plot, to which a quadratic curve was fitted by the method of least-squares, and we obtained 79 BC as the most probable observation date using 20 sample data with their (O−C)s less than 3° (about 2SD). Moreover, the corresponding confidence interval was found to be [550 BC, 370 AD] or 79 BC ± approximately 450 years using the same simulation method as the one adopted in the previous subsection. Comparing this interval estimate with that for the Qujidu data, one can see that the two ranges overlap well with each other, and the central values, 73 BC and 79 BC, are very close.

However, this is no more than showing that the two interval estimates are not inconsistent, and both dates are far from precise. Particularlarly, the total width of the Xiudu interval amounted to 900 years, indicating that we could not obtain any meaningful observation date from the Xiudu data. Note that the error ranges still remained ±150−200 years even when β was reduced to 50%−60%.

The large uncertainty width produced by the Xiudu data can be easily explained qualitatively by examining Table A1 in the Appendix. In fact, one can see that the time rate of the Xiudu, dΔRA/dt, is approximately one hundredth of that for RA, thus variations per millennium are only ≤1°. The reason for such a slow change is that the Xiudu is defined as the difference in the right ascension between two neighboring stars (see Equation 1), while the RA of each Juxing star increases nearly at a similar rate due to the precession. As we tried to detect such a small change in the Xiudu data with a few degrees of noise, it was inevitable for us to have had the uncertainty range as large as ±450 years (also see the long horizontal bar at the bottom of Figure 8(a)). In other words, the Xiudu is a very insensitive observation variable with respect to time (this can also be understood if we compare the ordinate spacing of Figure 8(a) with those of two other panels), and thus it is inappropriate to use the Xiudu in the dating analysis of star maps and catalogues except for otherwise inevitable cases.

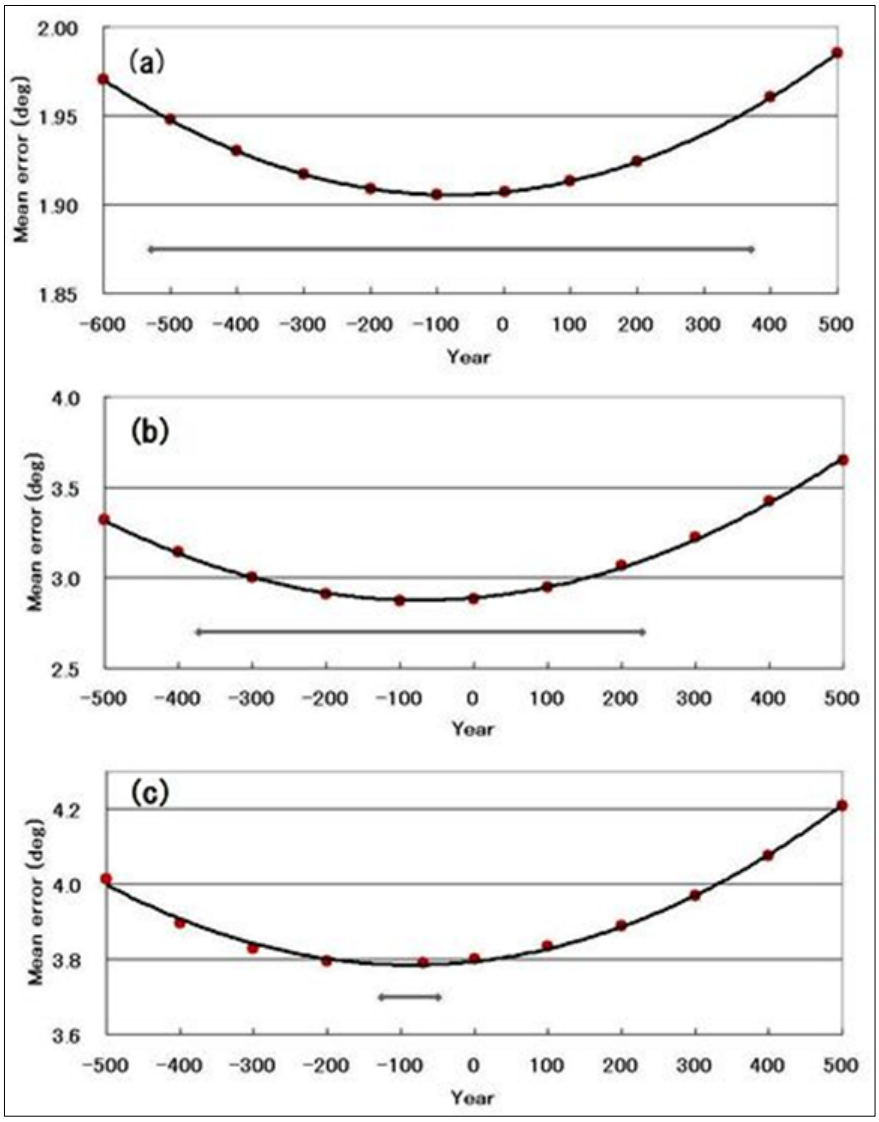


Figure 8: Year vs. Mean errors (SD) for three types of analyses: (a) Xiudu data (section 4.3.2), (b) Qujidu data (section 4.3.1) and (c) the 28-Xiu BS method for PS data. A horizontal bar below each panel represents its full width of the confidence interval (β=90%).

4.4 Dating Analysis by the Bootstrap Method Using Positional Shift

Next, let us consider the dating analysis using both the Qujidu and Xiudu data together. In this approach, we can expect more precise dating results (here 'more precise' means that we can reduce the confidence interval of estimation) unless results from the Qujidu and Xiudu data contradict each other.

For this purpose, a new quantity, namely PS, was introduced in Section 3.4 (Equation 3), and its significance was explained there. Then, in Section 3.5, we proposed a new method that uses PS data effectively. As the first step, this method calculates 28 dates from PS values by taking each Juxing star as the longitudinal origin, whose results are summarized in the last column of Table 1 (for the Kitora only 24 PS data were available). For example, the most probable observation date for No. 1 star (角 Jiao, α Vir) was found to be 64 BC, for No. 8 (斗 Dou, φ Sgr) it was AD 150, and so on. For No. 27 star (翼 Yi, α Crt), we could not detect any date when $\Sigma_i(O-C)_i^2/n$ = minimum between 500 BC and AD 1000.

The second step is to obtain the most probable observation date of the Kitora star map by synthetically combining all the years listed in Table 1. There may be a few choices to achieve the goal. The most primitive method will be to calculate the average of 24 dates shown in Table 1, resulting in 81 BC with the SD of 122 years. It is worth noting that the 81 BC is very close to 73 BC and 79 BC obtained from the Qujidu and Xiudu analyses—this is not just a coincidence but an anticipated outcome from the discussion in Section 3.5.

The final step is to improve the dating estimation precision, specifically to reduce the width of the confidence interval using sound statistical techniques. Following the trial-and-error experiments, we recognized that the 28-Xiu BS method mentioned in Section 3.7 was one of the best dating methods in terms of both precision and labor, as far as we use Microsoft Excel to perform manual calculations (Yoshihara, 2009).[7] Then, we obtained [123 BC, 39 BC] or 81 BC ± 42 years (β = 90%) as the final estimate of the observation date for the Kitora star map. This result is shown in Figure 8(c), along with the corresponding confidence interval as a horizontal bar at the bottom of the panel.

4.5 Summary of Results and Model Analysis

Figure 8 is intended as a summary to show a comparison of the three dating estimates so far made in Section 4. On the basis of the lengths of the three horizontal bars in the figure, we understand that the 28-Xiu BS method on PS data achieved the most precise estimation. Therefore, we regard [123 BC, 39 BC] or 81 BC ± 42 years (β = 90%) as our final estimated observation date for the Kitora star map. Furthermore, the fact that we could succeed in statistical estimation with the uncertainty range of as small as 42 years also means that the Kitora star map is one of the oldest scientific products.

Meanwhile, a question may arise as to whether the 28-Xiu BS method can really be effective in dating analysis of other ancient star maps and catalogues. The best way to answer this question is to perform a model analysis.[8]

For this analysis, we first prepared a few sets of hypothetical model data for the 28 Juxing stars to be analyzed; the data were generated by adding pseudo-random errors to the theoretical right ascensions and declinations of the stars at an assumed date AD 300.0. Then, our goal is to ascertain how precisely the originally assumed date can be recovered by the 28-Xiu BS method.

As shown at the top row of Table A3 in the Appendix, we generated three kinds of model data, including pseudo-random errors whose SDs are 1.5°, 1.0°, and 0.7°, similar to real error sizes of the Kitora data, and analyzed them using the 28-Xiu BS method. These artificial random errors were added to the right ascension and declination separately. Entries in each line are least-squares-fitted observation dates when the corresponding Juxing star is taken as the origin of longitude. The last line contains the resulting final confidence intervals: [282, 337] or 310 ± 28 years for SD = 1.5°, [282, 319] or 301 ± 19 years for SD = 1.0°, and [261, 298] or 280 ± 19 years for SD = 0.7° (β = 90%).

These findings indicate that the 28-Xiu BS method could recover the assumed original observation date AD 300.0 with error intervals of 20–30 years, and the intervals decreased approximately in proportion to SD values. Therefore, we conclude that the 28-Xiu BS method is one of the most prospective approaches to perform dating analysis of old star maps and catalogues such as the Kitota star map.

At the end of this Subsection, let us briefly mention the geographic latitude calculated from the sizes of the equatorial and the inner Gui circles in an old star map, which some researchers have claimed to be that of the observation place of stars. In the case of the Kitora star map, the latitude was found to be 37.6°–38.1°, the same as the Miyajima's value (1999).

However, Nakamura (2018) discusses another possibility. That is, from an extensive survey of Chinese literature and the dating analysis of historical star maps such as the one in Section 6.3, he suggests that it is more suitable to regard the inner Gui circle as the sky area drawn mainly for the convenience of users of a star map, rather than expressing the latitude of the observing place of stars.[4]

4.6 Past Estimates and Historical Background on the Construction of the Kitora Tumulus

4.6.1 Previous Dating Estimates

Shortly after the discovery of the Kitora star map in 1998 (see Section 2), Miyajima attempted for the first time to estimate the observation date of the map by using several measured stars and reported approximately 65 BC without providing any error range (1999). Although his date is seemingly close to our estimate 81 BC ± 42 years (β = 90%), this should be taken as a coincidence.

The biggest problem of Miyajima's estimation is that he claims to have obtained the observation date by analyzing the right ascension data. However, as explained in Section 2, since the ancient star map like the Kitora cannot provide the right ascension values in principle, how Miyajima actually measured the right ascension of the analyzed stars is unclear. Besides, both Miyajima's data points and quality were fairly limited compared with our case. Thus, it will be reasonable to understand that Miyajima's result is just a preliminary inference, although it was eventually not far off the point.

Soma (2016) independently performed a dating of the Kitora star map using its digital data provided by the Nara National Research Institute for Cultural Properties. He adopted a unique approach of analysis, inverse to the orthodox ones taken by Miyajima (1999) and Nakamura (2015) to measure stellar positions. Using only declinations of 11 stars (5 near the equator and 6 near the inner Gui circle), Soma first determined the center and the size of the inner Gui circle, without referring to the one drawn in the Kitora star map. Then, by recalculating coordinates of those stars, he concluded that the observation epoch of the Kitora stars was AD 300 ± 90 years (the standard deviation), and the geographic latitude of the observing site was 33°.9 ± 0°.7. However, these results contain a few non-negligible problems.

First, as mentioned in Section 3.2, most asterism shapes including the 28-Xius were freehand drawn, so more or less exaggerated or distorted. In such a situation, the reliability of the position and the size for the inner circle obtained from only six stars (and the resulting geographic location of the observation site) is highly doubtful. The second problem is that we cannot find out any clues about the estimated observation year of AD 300 among the official historical records of the successive Chinese dynasties.[6] Thirdly, since the uncertainty range of the estimated date, ± 90 years, is the classical standard deviation blindly calculated from the (O–C) residuals using as small as about ten stars, the value itself is almost meaningless as a practical measure of errors from the viewpoint of the small sample statistics. In order for Soma's conclusion to be sufficiently persuasive, elaborate simulation procedures and model analysis based on such statistics will be inevitable, as was made extensively in this paper.

4.6.2 Historical Background on the Construction of the Kitora Tumulus

Lastly, we discuss briefly the historical background on the construction of the Kitora tumulus. It is mentioned in Section 2 that archaeologists infer that the tumulus was completed between the end of the seventh century and the beginning of the eighth century. So, around this time, what was the political situation in Japan?

During the last half of the seventh century, two brother Emperors Tenji and Tenmu first built up the basic framework of Japan as a centralized state, called the Ritsuryo system under legal codes having the same name. The structure and function of the Japanese Government was a scaled-down model of the Chinese Tang Dynasty. Regarding astronomical matters in the Government, Tenmu played a leading role because he was interested in astrological administration.

According to *Nihon Shoki*, the first officially compiled national history of Japan in AD 675, the Bureau of Onmyo (Yinyang) was established by Tenmu, as a scaled-downed institution model corresponding to that of the Tang Dynasty.

This Bureau consisted of departments of astrological divination, Yinyang philosophy, water clock (clepsydra), and calendar publication. It is also reported that in the same year, an astrological observatory named the Senseidai (占星台) was constructed. This was probably similar to the Korean ancient astronomical observatory in Kyeongju close to Busan (see Figure 9), which was founded in the first half of the seventh century and still exists (Nha et al., 2017). Moreover, it is worth noting that only during the reign of Tenmu (673–686), the number of records on astronomical events written in *Nihon Shoki* increased significantly, demonstrating Tenmu's strong interest in astronomical divination.

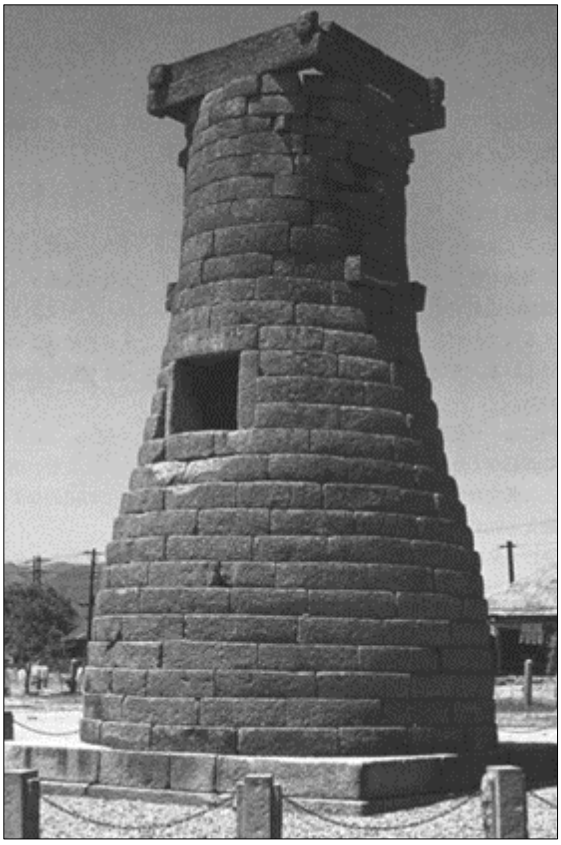


Figure 9: The Korean ancient astronomical observatory called Silla Observatory (瞻星台) near Busan. The Korean historical literature states that this was founded during the reign period (AD 632–647) of the Queen Seondeok 善德女王 of the Silla Kingdom, 57 BC–AD 935.

This fact suggests that responding to Tenmu's intention, observing staff of the Sensei-dai and the Bureau of Onmyo worked hard to watch the sky. For the purpose, they must have needed a detailed whole-sky star map that the Japanese at that time had no other choice than to import from China. Because of these historical backgrounds, we imagine the possibility that the unknown person buried in the Kitora tumulus might have been a high-ranking officer or a patron related to the Bureau of Onmyo; whose members and subordinates might have drawn the ceiling star map to express their respect for and gratitude to the dead.

The main reason for this speculation stems from the fact that as found in almost all the ancient tumuli excavated thus far in East Asia, the inside rooms of tumuli were never places where a scientific star map like the Kitora one was drawn. Such places were mostly decorated with paintings showing objects praying for the peaceful soul of the dead, or a heavenly afterlife. In this sense, the Kitora star map was an exceptional tumulus drawing. Hence, we predict that there is little chance that a similar star map will be discovered on the inside of another tumuli in the future.

4.7 The Original Sources of the Kitora Star Map

Considering both the dating analysis result of the Kitora star map and its historical background, it is almost certain that the original source of the star map was brought to Japan directly from China or via the Korean Peninsula. If so, it will be historically meaningful to explore possible sources in the ancient Chinese literature. The star map is likely to have been drawn as paper copies from the original, since *Nihon Shoki* reported that in AD 610 the Buddhist priest Donchou (曇徴) from the Koguryeo Kingdom came to Japan and taught the Japanese the technique of papermaking, together with Chinese ink and color paints. At that time, the Japanese lacked the ability to draw star maps using numerical data such as those found in star catalogues.

Chinese history of astronomy tells us (e.g., Sun and Kistemaker, 1997) that the astronomer of the fourth century Chen Zhuo (陳卓) combined past constellation systems invented by three ancient astronomical schools into one and, thereby for the first time, probably produced a circular star map similar to the Kitora star map. However, no such ancient star maps have survived in China, with the earliest-known existing maps dating to the eleventh and twelfth centuries. Therefore, we have no other option than to look for clues of the original source of the Kitora star map in Chinese historical documents, where the 28-Xiu data would have been recorded in the form of descriptive sentences or star catalogues, because star maps usually were drawn based on star catalogues.

4.7.1 Analysis of the Shi Shi Xingjing Data

As can be seen in the table of Figure 3, the only positional data on the 28-Xiu Juxing stars known prior to the construction date of the Kitora tumulus (approximately AD 700) were those observed by Luoxia Hong during the Former Han era (Section 3.1). Historical records report that these data were written in the book *Shi shi xingjing* (石氏星経 *Star Manual of Master Shi*), which is attributed to the legendary astronomer Shi Shen (石申). However, the book was lost a long time ago and is only cited in the *Kaiyuang zhanjing* (*Treatise on Astrology of the Kaiyuang Times*) authored by the astronomer of Indian origin Qutan Xida during the Kaiyuang period (713–741) of the Tang Dynasty. Hence, in this Subsection, we carry out a dating analysis of the 28-Xiu data cited in the *Kaiyuang zhanjing*.

The *Shi shi xingjing* data of Juxing stars taken from the *Kaiyuang zhanjing* are summarized in Table A4 in the Appendix. Note that the numbers are shown in the traditional Chinese degree unit (365.25°, instead of ordinary 360°).

Applying the simulation method to the Xiudu data, we obtained AD 134 ± ~270 years (unless specifically mentioned, β is always equal to 90%) with a mean residual of approximately 0.5°. The comparison of this mean residual with that for the Kitora star map (which is approximately 3°) indicates that the *Shi shi xingjing* is a very reliable data source. Analysis of the Qujidu data using the same method gave AD 21 ± ~250 years.

On the other hand, the 28-Xiu BS method applied to the PS data provided us with the date [65 BC, 43 BC] or 54 BC ± 12 years—a much more precise value than the previous two values; hence, we take this estimate as our final dating of the *Shi shi xingjing* data. This finding also reveals that the observation epoch of the Juxing stars in the *Shi shi xingjing* is substantially newer than the traditional explanations in historical documents that the book *Shi shi xingjing* was written in the Chunqiu-Zhangguo (春秋・戦国, the eighth to third centuries BC) era. Our obtained date of 54 BC ± 12 years is close to the time of Luoxia Hong (see Figure 3), the inventor of the armillary sphere, or seemingly closer to the time of Geng Shouchang (耿寿昌) who observed stellar positions

using an armillary sphere developed by him in 52 BC (Yabuuchi 1969: 68; Sun and Kistemaker 1997: 38).

Since the *Shi shi xingjing* is one of the very important classics in Chinese astronomy, there is a long history of research on this literature, particularly regarding the estimation of the observation dates of stars recorded in the book. Thus, we briefly review these studies in the context of our results.

Professor Ueda (1929) of Kyoto University was the first to perform a dating analysis of stars by a graphical method using the Qujidu data in the *Shi shi xingjing*. Most researchers mentioned below, including Ueda, used many other stars in addition to the 28-Xiu Juxing stars. Ueda concluded that the stars in the *Shi shi xingjing* consisted of two separate groups, whose observation dates were approximately 300 BC and AD 200. Although Ueda's results were supported by some later historians (e.g., see Pan, 1989), other scholars (e.g., Yabuuchi, 1969) opposed Ueda's conclusions.

Yabuuchi (*ibid.*) inferred the observation date of the *Shi shi xingjing* to be approximately 70 BC (without providing any error range) from a paragraph in the book that reported that the Sun's ecliptic longitude was 20° from the No. 8 Juxing star Dou (斗) on the winter solstice day. Other scholars (Maeyama, 1977, and Sun and Kistemaker, 1997) estimated the date to be 70 BC ± 30 years and 78 BC ± 20 years, respectively. Thus, we find that the two most recent estimates are consistent with our results, and the observation date of the *Shi shi xingjing* has now been established. However, it will be worth emphasizing that the uncertainty ranges of all the past estimates, other than ours, were not derived from the interval estimation theory of modern statistics but simply were the SDs calculated using residuals after analysis.

4.7.2 The Cheonsang Yeolcha Bunyajido Korean Stone-inscribed Star Map

It is well known that Korea has an historically famous hemispheric stone star map entitled Cheon-sang Yeolcha Bunyajido (天象列次分野之圖), that was inscribed in 1396 (201cm × 123cm), which is very similar to the Kitora star map. The upper part of Cheonsang Yeolcha Bunyajido draws the star map, and the lower part provides relevant astronomical explanations, including information on the 28-Xiu constellations (Figure 10).

According to sentences written in the lower part, when the Koguryeo Kingdom was conquered by the Tang-Shilla powers in 668, they abandoned in a river the original stone-inscribed star map made by the Kingdom. Fortunately, however, a rubbed print copy had been preserved by a Korean family for centuries, and its descendant offered it to the King Lee Seiji, the founder of the Lee Dynasty around the end of the fourteenth century. The King soon ordered Court astronomers to inscribe a new stone star map in 1395, based on the original one, that is shown here in Figure 10. Taking account of this historical situation, we conducted dating analysis of the Juxing stars shown on this star map.

We first attempted to measure star positions on the star map but could not identify several Juxing stars, perhaps, due to the inaccuracy of the inscription. Hence, we analyzed the map with an emphasis on the data provided in the table of the lower part. As values of the Xiudu data were essentially the same as those in the *Shi shi xingjing*, we did not analyze the Xiudu data. Eventually, we estimated the observation dates to be 53 BC ± ~100 years from the Qujidu data measured from the star map using the simulation method, 51 BC ± ~100 years (with a mean residual of 1.2°) from the same stars listed in the table, and [78 BC, 54 BC] or 66 BC ± 13 years from PS data using the 28-Xiu BS method, respectively. Hence, as usual, we consider 66 BC ± 13 years to be our final date for the Korean star map.

From the dating estimates obtained so far on the Kitora star map (81 BC ± 42 years), the *Shi shi xingjing* data (54 BC ± 12 years), and the Korean stone-inscribed star map (66 BC ± 13 years), one can see that the confidence intervals of each dating result overlap well. This strongly suggests that there must have existed a common source from which the three sources mentioned above originated.

Meanwhile, since there are no known historical star positions older than those of the *Shi shi xingjing*, it will be reasonable for us to conclude at the moment that as far as the 28-Xiu Juxing stars concerned, their data in the Japanese and Korean star maps were inherited from the *Shi shi xingjing*. This situation is graphically summarized in the upper part of Figure 11. Regarding mention of the *Almagest* in the Figure, refer to Section 5 below.

5 ANALYSIS OF THE STAR CATALOGUE IN THE *ALMAGEST* AND THE AUTHENTICITY PROBLEM OF PTOLEMY'S OBSERVATIONS

The dating estimation method so far discussed using the 28-Xiu constellations as a 'common probe' was initially planned for analyses of old star maps and catalogues in East Asia, but it is no problem at all to apply it to those of the West-

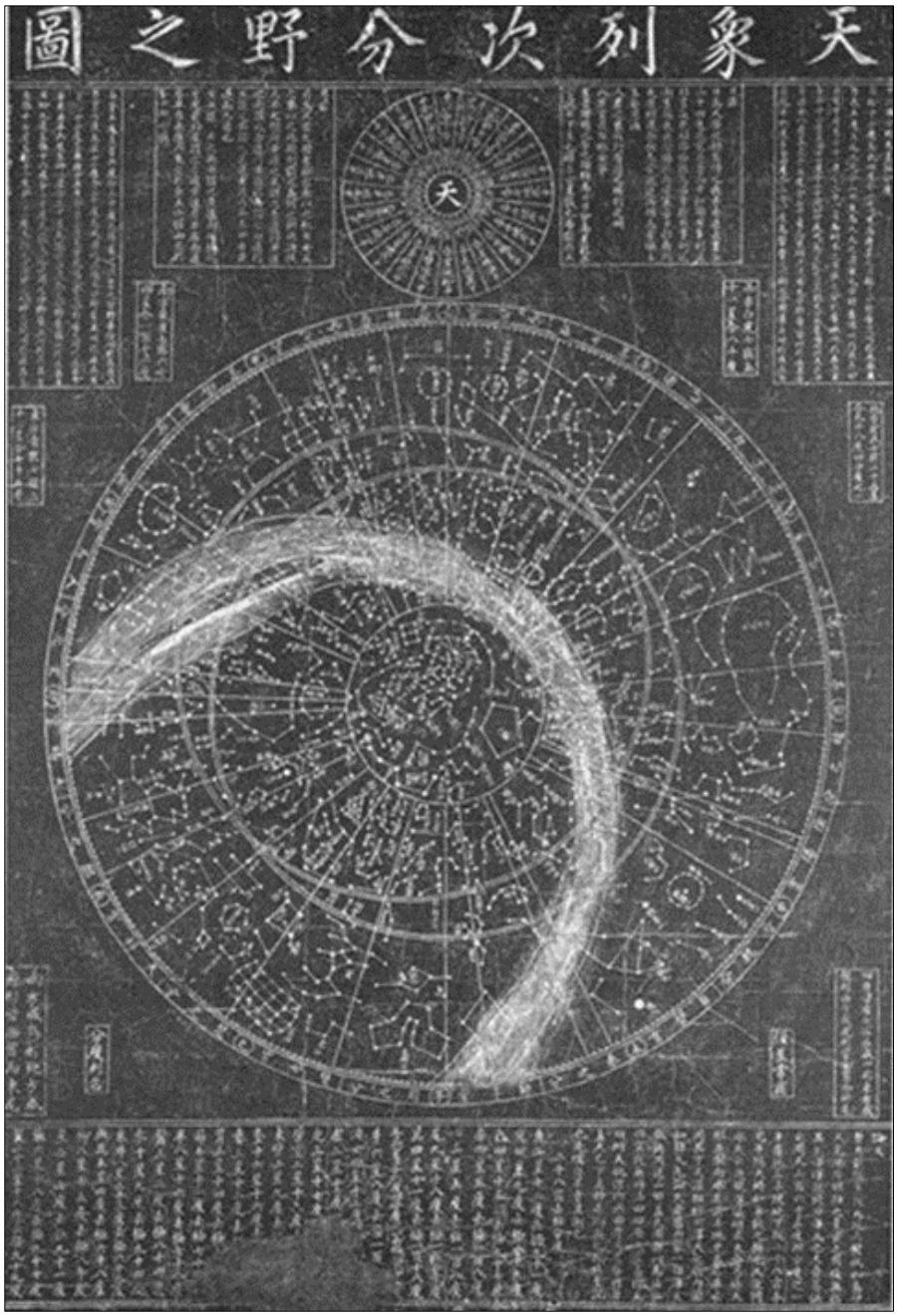


Figure 10: A rubbed print of the stone-inscribed star map Cheonsang Yeolcha Bunyajido in 1395.

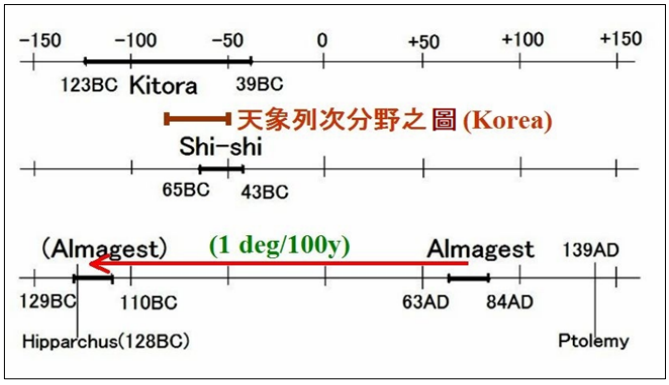


Figure 11: A graphical summary of dating estimates for the Kirota star map, the Shi shi xingjing, the Korean stone-inscibed star map, and the Ptolemy's star catalogue in the *Almagest* (diagram: Tsuko Nakamura).

ern world. In this Section we treat the star positions contained in the *Almagest* (*Mathematike Syntaxis*) authored by the Greek astronomer Ptolemy around 145 AD (Claudius Ptolemaeus, ~AD 90–ca. AD 168?). In fact it is intriguing to compare Ptolemy's data with those in the *Shi shi xingjing* by the 28-Xiu BS method, since both are the representative ancient star catalogues of the West and the East compiled in almost the same historical era. At the same time, this comparison is also useful to evaluate objectively the performance ability of our dating procedure.

As is well known, the *Almagest* is the culminating achievement of Greek astronomy, with emphasis on the theory of planetary motion, and the 48 constellations recorded in the *Almagest* are direct ancestral ones of the modern constellation system now in worldwide use, which contains 88 members. Ptolemy's star catalogue is included at the end of Book VII (the 27 northern constellations) and at the beginning of Book VIII (the 21 southern ones) in the *Almagest*, and lists 1022 stars in total (Grasshoff, 1990; Toomer, 1984; and Yabuuchi, 1958). After the decline of Hellenistic Civilization, this star catalogue has been respected for more than ten centuries in the Western world as an authority on stellar positions.

5.1 Did Ptolemy Observe the Stars in his Star Catalogue?

In Chapter 2 of Book VII Ptolemy claims that star positions of his catalogue were observed by himself from Alexandria, Egypt, in AD 139. However, from the late sixteenth century on doubts have been raised about whether Ptolemy's star catalogue in the *Almagest* really was based on his own observations. It has been suggested that Ptolemy did not measure the positions of the stars in his catalogue, but simply modified an earlier star list by Hipparchus, adopting an assumed precession rate in order to pretend that the star positions actually were based on Ptolemy's own observations.

The Danish astronomer Tycho Brahe (1546–1601) was the first to discuss this authenticity problem. It is well known that at this observatory on the island of Hven in Denmark, Brahe continued astronomical observations during 1575–1597 by improving on or inventing many superior instruments and securing a large body of measured data, having the highest accuracy attainable with the unaided eye. Tycho first determined a precession constant (the variation rate in the ecliptic longitude) from his own high-precision data covering many years, and with this constant he calculated back the stellar positions to Ptolemy's times and compared them with those in the *Almagest*. The result was that the ecliptic longitudes in Ptolemy's list were systematically about 1° too small (see Thurston, 1994). Thus, Tycho suspected that Ptolemy's catalogue could have been a product of manipulated calculation, not of Ptolemy's direct observations.

Ever since, although there had been 'pros' (for example Laplace (1787), cited in Section 2.2 in Grasshoff (1990)) and 'cons' on the validity of Ptolemy's star catalogue among reputed astronomers, some of them—such as J.J. Lalande (1732–1807) and J.B.J. Delambre (1749–1822)—have been strongly criticical of Ptolemy. They concluded that as Ptolemy adhered too much to the precession rate of one degree per century, which had originally been obtained by Hipparchus, Ptolemy intentionally removed the observational data inconsistent with this rate, or modified Hipparchus' star list. Perhaps the most severe critic of the *Almagest* was the American astronomer and historian of science Robert R. Newton (1918–1991).

In his book *The Crime of Claudius Ptolemy*, Newton (1977) queried not only the star catalogue in the *Almagest* but also other chapters, on motions of the Sun, Moon and five planets, and theories of luni-solar eclipses. A main cause of Newton's overall distrust of the *Almagest* came from the fact that Ptolemy's theories and observations in his book were in accordance with each other too perfectly. This meant that Ptolemy must have deliberately ignored observations that ran counter to his theories. Regarding Newton's accusation (1977) of Ptolemy's star catalogue, in particular, various opposing arguments (e.g., see Gingerich, 1981; 1993) and defending ones (Dambis and Efremov, 2000; Evans, 1987; Rawlins, 1982), have continued to be published. Therefore, we decided to apply our 28-Xiu BS method to the star catalogue in the *Almagest* as well, in an attempt to ascertain whether the long-standing accusation against Ptolemy are justified or not.

5.2 Analysis of the 28-Xiu Stars in Ptolemy's *Almagest* Catalogue

Section 3 explained that traditional Chinese astronomy used stellar positions in the equatorial coordinate system. On the other hand, in ancient Greece, astronomers adopted those in the ecliptic coordinate system; star catalogue data in the *Almagest* also were written with ecliptic longitudes and latitudes referred to the equinoxes. In the equatorial coordinate system, we saw in Tables A1 and A2 that the precessional motion changes both in right ascension and declination of stars in a compli-

cated way as a function of time. However, in the ecliptic coordinate system only the longitudes of all stars increase nearly at the same rate, while their ecliptic latitudes stay almost fixed (Thurston, 1994: 150).

Therefore, it is simpler to treat positions in the ecliptic coordinate system directly using the 28-Xiu BS method. Nevertheless, we wished to analyze the *Almagest* catalogue in the same way as the *Shi shi xingjing* data from a standpoint of impartial comparison. Fortunately, in Appendix B Grasshoff (1990) provided positions in both coordinate systems, so we could easily calculate the Xiudu and Qujidu values for the 28 Juxing stars (Table A5) via the star designations in the Bayer Atlas, *Uranometria* (1603). Because the *Almagest* catalogue gives angular values down to the unit of 10′, it was expected that we could estimate a more precise observation date than that for the *Shi shi xingjing* data.

First, from the analysis of the Xiudu data by the simulation method, the estimated date was AD 13 ± approximately 200 years. Thus, here again, one can understand that the Xiudu is a very insensitive quantity for the dating analysis. Next, applying the 28-Xiu BS method to PS data of the *Almagest,* we obtained [63, 84] or AD 74 ± 10 years (β = 90% and the mean residual of 0.7°). We regard this result as our final estimation for Ptolemy's alleged observation date and it is indicated in the lower right-hand part of Figure 11 as a short horizontal bar. As anticipated, one can see that a slightly more precise dating is achieved, in terms of the confidence interval, than that for the *Shi shi xingjing* data.

By the way, we soon notice from Figure 11 that our obtained date for the *Almagest* catalogue is very different from the one Ptolemy claimed, AD 139, so that some interpretation of the contradiction is required. Ptolemy mentioned that the precession rate discovered by Hipparchus was one degree per century for the ecliptic longitude (the true value is 1° every 72 years). Thus, we attempted to bring back our obtained date [AD63, AD84] to the past using Ptolemy's wrong precession rate during the time span between Ptolemy and Hipparchus (139+128 = 267 years). That is, we should go back by 267 × 72/100 = 193 years. This resulted in [129BC, 110BC]; this new confidence interval is shown at the left-hand side of the bottom in Figure 11. Then we see that the range surely overlaps with the observation time by Hipparchus, 128 BC.

This result seems to support the long-time suspicion, at least as far as the 28 Juxing stars concerned, that Ptolemy did not make the observations himself, but simply modified the positions in Hipparchus' star list by calculing the wrong precession correction of 1° per century. Although our finding is no more than a confirmation of past suspicions, at the same time it shows that analysis by the 28-Xiu BS method can give proper dating estimations of ancient star maps and catalogues in general.

Finally, we finish this section by presenting below our views on the authenticity problem of Ptolemy's star catalogue. In many chapters of the *Almagest* Ptolemy improved on or refined his theories and astronomical constants by incorporating the achievements of earlier astronomers, including Hipparchus. These works by Ptolemy covered the fields of solar and lunar motions, luni-solar eclipses, and the orbits of the five known planets. Surely, if Ptolemy had measured star positions himself he would have obtained a more refined precession rate than 1° per century by combing his own observations with those provided by Hipparchus.

In fact, had a precession rate been recalculated using the whole set of declination data for the 18 stars that Ptolemy gave in Chapter 3 of Book VII as his own observations, it would have provided about 50 arcsec per year, a value close to the true one (Delambre, 1817; Grasshoff, 1997: 101–104). Nevertheless, Ptolemy stubbornly adhered to the rate of 1° per century by picking only six stars whose positions did not contradict his assumed rate, and neglected 12 other stars—otherwise, his adopted precession rate was not in accordance with the his catalogue as a whole. Ptolemy's irrational choice disclose by itself that his catalogue in the *Almagest* was compiled not by observation but by modified Hipparchus' list.

6 DATING ANALYSIS OF OTHER HISTORICAL ASIAN STAR MAPS AND CATALOGUES

As noted in Section 4, the original or paper copies of the Kitora tumulus star map drawn around the end of the seventh century did not exist in neither China nor Japan at the time. The earliest known surviving star maps and catalogues date after the tenth century. In this Section, therefore, we try to apply our 28-Xiu BS method to historical star maps and catalogues from these later times. However here, our primary interest in analyzing those records is not the dating itself but rather to confirm how precisely the 28-Xiu BS method can estimate their observation dates, because those dates are more or less known from the official historical literature. Below we present dating results for a few representative star maps and catalogues from China, Japan and the Islamic world.

6.1 The Suzhou tianwen tu Start Map

The oldest Chinese whole sky star map is the one attached to the book *Xin yixiang fayao* (新儀象法要, *New Design for Astronomical Instruments*), which was published in 1094 by Su Song (蘇頌, 1020–1101). Su was a very talented mechanical engineer from the Quan Zhou Province and famous for his construction around 1092 of the gigantic astronomical clock tower mechanically driven by water power. *Xin yixiang fayao* explains the detailed structure and function of this tower clock with varied figures.

The star map in it consists of two rectangular charts with the horizontal center line as the equator and two circular parts with the centers as the north and south poles. This is also the earliest printed star map expressed in a rectangular format like the Mercator's geographical map. At the upper edge of each rectangular chart, the Xiudus for the 28 Juxing stars are written by Chinese letters and all their values coincide completely with those inscribed on the stone star map *Suzhou tianwen tu* (蘇州天文図). However, as the *Xin yixiang fayao* is of an ordinary book size in woodblock print, shapes and positions of many constellations seemed far from precise. Therefore, we gave up analyzing this star map.

In the medieval era of China, the most well-known star map is so-called *Suzhou tianwen tu*, which is preserved in a Confucius temple in the Suzhou district. It is also called *Zhunyou tianwen tu*, taken from a Chinese chronological name, though its formal title reads as *Tianwen tu* (天文圖, *Astronomical Chart*). The stone size measures 2.2m × 1.1m, with a circular star map inscribed in the upper part and long explanatory sentences in the lower half. Another stone monument placed next to this star map describes that Wang Zhiyuan (王致遠) constructed it in 1247, citing from the original text authored by the court scholar Huan Chang (黃裳) about a half century ago.

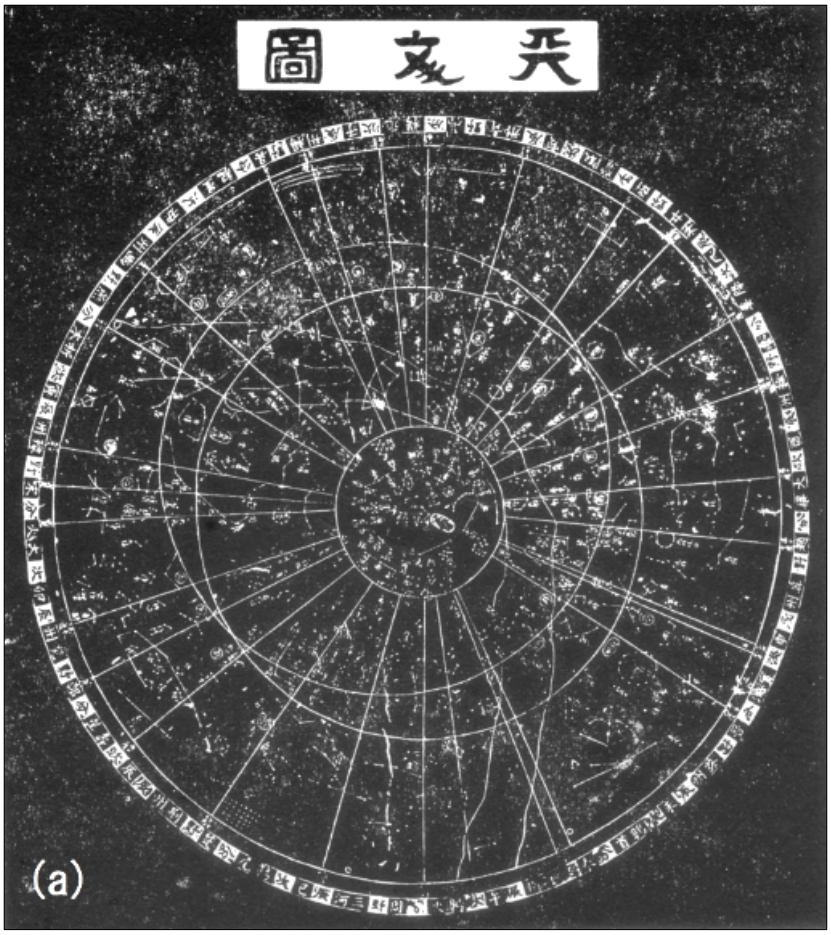
Figure 12(a) shows an ink-rubbed print of the star map in the *Suzhou tianwen tu*. Radial lines corresponding to the Xiudu of each Juxing star emanate from the center (North Pole), and at the outer ends of them are written the Xiudu values in Chinese characters (Figure 12(b)). The innermost and outermost circles represent the neigui (内規, the inner circle) and weigui (外規, the outer circle) respectively, which indicate the visible sky area at a location, depending on the terrestrial latitude (φ). As the latitude calculated from sizes of the neigui and the equator was found to be 34.4 ± 0.2°, it seems that the designer of this star map intended it to be used near Kaifeng (φ = 34.8°), the capital of the Song Dynasty at that time. Each name of the 28-Xiu constellations is surrounded by an oval (see Figure 12(b)). Contours of the Milky Way are also drawn.

Measurements of the Xiudu and Qujidu for the 28 Juxing stars were made with an enlarged copy of the *Suzhou tianwen tu* included in the book, *Photographic Catalogue of Chinese Astronomical Relics* (Institute of Archeology, Chinese Academy of Social Sciences, 1978). In the case of dating analysis of premodern star maps like this, an approach different from that for ancient star maps and catalogues is required. That is, we need to use a more precise (namely, quadratic) approximation for precession theory (Table A2) instead of a linear one adopted for the Kitora, because, in general, observational errors of stars decreased in modern times.

Since results of our dating analysis for the *Suzhou tianwen tu* are somewhat complicated, we first summarize them in Table 2 along with those of other studies. For the Xiudu data, the simulation method provided us with [740, 1060] or 900 ± 160 years, while the 28-Xiu BS method gave [875, 995] or 935 ± 60 years. Hence following the rule so far used in this paper, we take 935 ± 60 years as our final estimate for the Xiudu.

Regarding the Qujidu analysis, we have two data sources (see Table 2): one measured from the *Suzhou tianwen tu* and the other cited in *Wenxian tongkao* (文献通考, *History of Political Institutions in China*, 1317). The latter mentions that during the North Song Dynasty (960–1127) systematic star observations were performed as many as four times (Yabuuchi, 1969). As observations for the Jingyou (景祐) period (AD 1034–1038) were most extensive and elaborate (see the table of Figure 3), here we analyzed them as well. From the data measured in the *Suzhou tianwen tu*, we obtained [1035, 1165] or 1100 ± 65 years (the mean residual: about 1.2°) by the simulation method. Meanwhile, the dating of the *Wenxian tongkao* data was [1013, 1077] or 1045 ± 32 years. This is the expected outcome, that the confidence interval of the latter estimate is smaller, since star maps had commonly been drawn based on the star catalogue data.

From Table 2 one can notice that our dating results for the Xiudu and Qujidu data, 935 ± 60 years and 1045 ± 32 years are obviously inconsistent each other, showing that both the data are very likely to be from different observation times. Hence we did not analyze the PS data in this case.



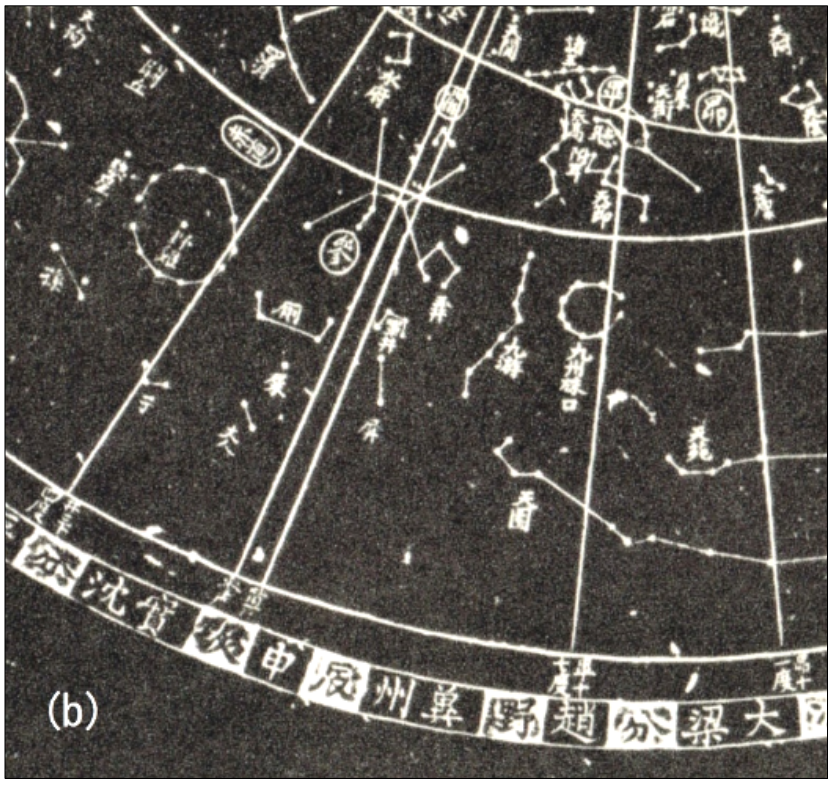


Figure 12(a): The star map of the *Tianwen tu*, or the so-called *Suzhou tianwen tu* construced by Wang Zhiyuan in 1247.
Figure 12(b): A part of the star map including constellations Mao (No.18 昴, Pleiades) and Shen (No. 21, 參, Orion).

On the other hand, Yabuuchi (1969) studied in detail observations of the 28-Xiu constellations conducted during the Song Dynasty, to check whether or not the Juxing stars at that time were the same as those in the late Han era. Comparing theory (C) with observations (O) of the Qujidu and Xiudu for the assumed year of 1050, he judged that, because of both the (O−C)s being sufficiently small, the Qujidus and Xiudus in the literature were surely from the Song Dynasty era. In particular, he noticed that all the Xiudu values inscribed along the outer circumference of the *Suzhou tianwen tu* completely matched the records of observations for the Yuanfeng (元豊) period (1078–1085).

Then, from our results and Yabuuchi's work, what can we learn about the dating of the *Suzhou tianwen tu*? One important point is that our statistical dating analysis demonstrates that the Xiudu data inscribed in the *Suzhou tianwen tu* were not from the North Song Dynasty era but actually from much earlier times (the first half of the tenth century). In other words, the Xiudu and Qujidu observations shown in the *Suzhou tianwen tu* were made during two different periods separated by more than a century (Table 2). This means that even official historical records by the national Government sometimes were wrong, presumably because scholars and astronomers at that time often had a tendency to traditionally trust more the data in the ancient literature rather than new observations.

6.2 The 28-Xiu Observations by Guo Shoujing

Guo Shoujing (郭守敬, 1231–1316) was the astronomer from the Hebei province who played the most important role in the calendar reform of the Yuan Dynasty. As is well known, the Shoushi Calendar was completed in 1280 by Guo and his contemporary colleagues and was the culminating point of achievements in Chinese calendrical history.

Soon after Khubilai, the first Emperor of the Yuan Dynasty, ordered Guo and other Chinese astronomers to compile a new calendar in 1276, Guo started designing and constructing as many as thirteen astronomical instruments, and he made precise observations of the Sun, Moon and stars with them for about five years. It is likely that star positions including those of the 28-Xiu constellations were observed by the instrument named Jianyi (the Simplified Instrument), which was used to measure the Xiudu and Qujidu of stars independently using separate graduated circles, revealing Guo's outstanding talent (e.g., see Needham, 1959a). The Xiudu data of the 28 Juxing stars by the instrument are cited as observations during the Zhiyuan (至元) period (1264–1294) in the calendrical part of the official history of the Yuan Dynasty (the *Yuanli*, see Figure 3). From their values shown in the last row of the table in Figure 3, we recognize that Guo's instrument could measure angles down to the unit of one twentieth of the Chinese degree, an unprecedented accuracy.

We first analysed the above Xiudu data for the Zhiyuan period by the simulation method. The obtained date [1247, 1307] or 1277 ± 30 years (mean residual: 0.13°) suggests that Guo's Juxing star observations were surely performed within just a few years after the order of calendar reform by the Emperor. Moreover, the smallness of both the confidence interval and the mean residual also confirm that Guo's measurements of star positions were really very precise, as was repeatedly mentioned in the history of Chinese astronomy.

Table 2: Dating summary of the *Suzhou tianwen tu*.

	Xiudu	Qujidu-1*	Qujidu-2*
Simulation method	[740, 1060] or 900±160	[1035, 1165] or 1100±65	[1013, 1077] or 1045±32
28-Xiu BS method	[875, 995] or 935±60		
Yabuuchi (1969)	1078–1085		1050 (assumed)

(*) Qujidu-1 and Qujidu-2 mean the data measured from the star map and those cited in the *Wenxian tongkao* (1317), respectively. In these analyses theoretical values of precession are calculated based on a quadratic approximation (Table A2).

As for the Qujidu data observed by Guo, there seems to be no record among documents in the official Chinese history. In the 1980s the book entitled *Sanyuan lieshe ruxiu qujiji* (三垣列舎入宿去極集, *Collection of the Xiudu and Qujidu for Celestial constellations*) was discovered, which summarized the Xiudu and Qujidu of about 740 stars including the 28-Xiu Juxing stars, along with a diagram for each asterism (Chen, 2003: 538). This was one of a series of books by an unknown author compiled during the Ming Dynasty. Although it was once supposed that the Qujidu data were based on Guo's observations in 1280 (Pan, 1989), later they were found to be actually from around 1380 (Sun, 1996). As our 28-Xiu BS method applied to the same data also gave the date [1354, 1386] or 1370 ± 16 years, this result certainly negates the argument that the Qujidu data were observed ones made by Guo.

6.3 The Ceiling Star Map in King Qian Yuan-guan's Tumulus

There still exist some notable circular star maps

in the pre-modern era of China other than those already mentioned (e.g., see Chen et al., 1996). However, we will not treat them any further, since observation dates for most of them in this period are more or less known from historical documents and literature, thus seemingly not much worthy of the analysis. Nevertheless, here is one very interesting exception, the ceiling star map of the King Qian Yuanguan's tumulus. Below we describe the outline of the star map, followed by its dating analysis.

In the 1960s and 1970s in the Hangzhou district of Zhehong Province, burial mounds for the ruler and his family of the Wuyue state were excavated. They were constructed for King Qian Yuanguan (錢元瓘) and his second wife, who died in 941 and 952 respectively. On the stone ceiling inside each tumulus an elaborately designed star map was inscribed. These ceilings measured as large as 4.7 m × 2.6 m × 0.3 m (Institute of Archeology, Chinese Academy of Social Sciences, 1978). The unique point about the star maps was the number of constellations shown there. In such a wide stone area, only a few circumpolar asterisms, the Big Dipper and the 28-Xiu constellations were drawn, in addition to the equator and the inner and outer Gui circles (see Figure 13(a)). Obviously, this was not due to the star map being unfinished, since the stone ceiling of the second wife's tumulus also had the same configuration of asterisms. Neither star names nor the circle graduation were written anywhere. We attempt to do dating analysis of this Qian Yuanguan star map in the following paragraph.

We measured the Xiudu and Qujidu of the 28-Xiu Juxing stars on an enlarged copy of the ink-rubbed print (Institute of Archeology, Chinese Academy of Social Sciences, 1978) using a linear scale and a protractor. According to Yi Shitong 伊世同 (1989), real sizes of the equator diameter and the inner circle were 123.1 cm and 51.1 cm. Figure 13(b) shows a part of the star map near the Shen xiu (Orion) with Arabic numerals for our identified Juxing stars. Table A6 represents their measured Xiudu and declination values. The data for four Juxing stars are missing because of image defects.

For the declination data, the simulation method using a hundred curves like Figure 8 provided us with the date [833, 1233] or 1033 ± 200 years, in which 15 Juxing stars with residuals less than 5° were adopted. Although a simple least-squares estimate of the Xiudu data gave us the year 835, we did not attempt the simulation method for the data. The reason is that experience has taught us that the confidence interval for the Xiudu data is generally larger than that for the declination.

Next, we made a least-squares dating estimation for PS values by taking each Juxing star as the longitudinal origin. The results for 23 Juxing stars are summarized in the last column of Table A6,[9] in which the linear precession theory of Table A1 was used. Then we applied the 28-Xiu BS method to all the years in the table, thus getting our final estimation date [897, 987] or 942 ± 45 years. From this dating one can see that the 28-Xiu BS method for PS data gives a much better result also in this case. Nevertheless, since the mean residual during this period was as large as 5°, we may conclude that King Qian Yuanguan's star map was far from accurate, contrary to its impressive appearance; remember that the mean residual for the Kitora star map was about 4° (refer to Section 4).

Now let us consider the background as to why such an unusual stone star map was constructed. A clue to answering this question is the interval 942 ± 45 years itself. This period overlaps with the lifetime of King Qian Yuanguan and there is no observation record of the 28 Xiu constellations in the Chinese national history during this era, so his astronomers may have made measurements of these constellations in response to his order for some unknown but important purpose. Then after the deaths of the King and his wife, it is likely that his successor, vassals, and astronomers inscribed the observations as the special star map to express appreciation of and respect for the King. In any case, the problem of King Qian Yuanguan's star map will be an interesting target for future studies.

At the end of this Subsection we make some comments about the geographic latitude calculated from the equator and the inner (or outer) Gui circle of the star map. Such a latitude has often been understood as that of the observed location. But, as the obtained value for the star map was 37.1–37.9°, this is quite different from the latitude of the Wuyue state which is around 30°. Yi (1989) also worried about this contradiction. However, we have already discussed how the latitude calculated from the Gui circles of a star map generally has nothing to do with the place where the stars actually were observed from.

6.4 Dating Analysis of the Koshi Gesshin-zu Japanese Astrological Star Map

Once there existed in Japan an old star map which was drawn by a court astronomer of the fourteenth century for astrological purposes, but was lost during the Tokyo bombardment fire

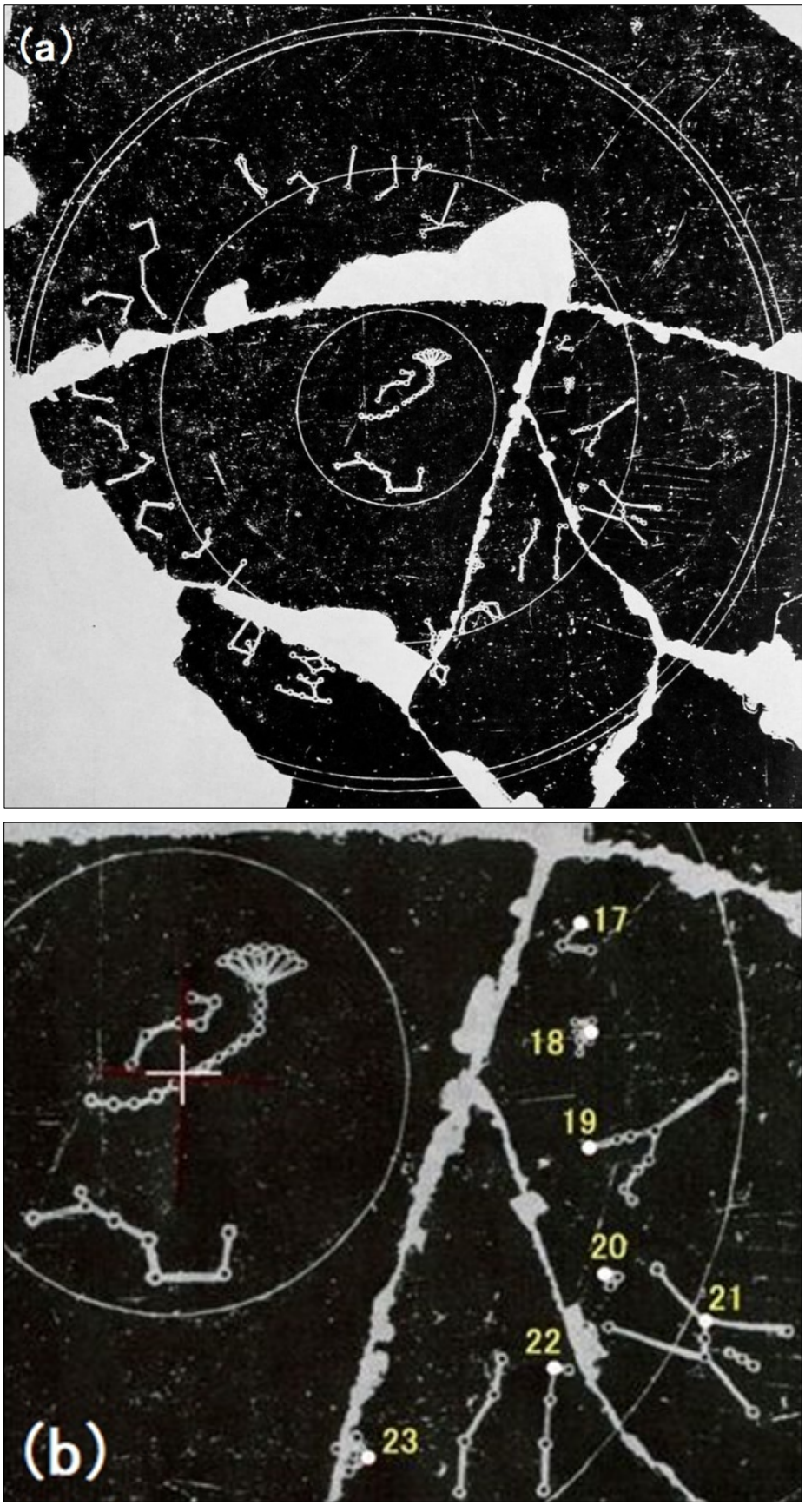


Figure 13(a): An ink-rubbed print of the King Qian Yuanguan's star map in his tumulus.
Figure 13(b): A part of the star map including constellations from Wei (胃) through Gui (鬼). Numbered white dots are the Juxing stars identified by the author, and the plus-mark on the left was determined as the North Pole, the center of the equatorial circle.

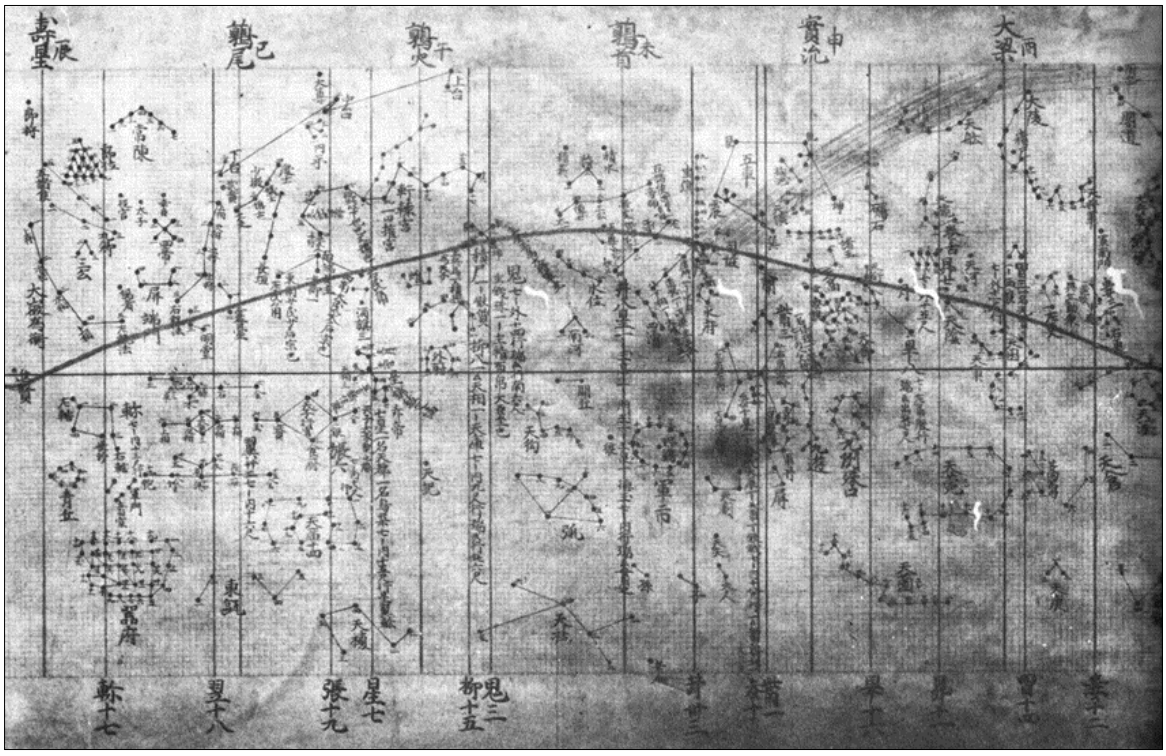


Figure 14: A photographic copy of the Koshi Gesshinzu star map. (Courtesy of the National Astronomical Observatory of Japan). The vertical size of the star map copy measures 28cm.

of WW II in 1945. A photographic copy is now preserved at the library of the National Astronomical Observatory of Japan. It is titled *Koshi Gesshin-zu* (*Latticed Lunar Motion Star Map*), which consists of a long rectangular map of Mercator type with the Equator as the central horizontal line and a circular circumpolar part centered at the North Pole. Figure 14 shows an eastern half of the rectangular part including 13 of 28-Xiu constellations from No. 16 to No. 28 (see Table 1). Explanatory notices attached to the star map say that its author Yasuyo Abe produced this around AD 1320 based on a much earlier star map that had been handed down from his ancestors. Because the Abe family (later called the Tsuchimikado) provided the Chief Court Astrologer to the Emperor from the tenth century, the original star map must have been used for the purpose of Yinyang astrological divination, depending on which constellation the Moon was located at. Although the Chinese origin of the star map is unknown, as the following analysis verifies, this was surely the earliest scientific star map drawn on paper in Japan.

The most conspicuous characteristic of the star map is that constellations are shown on a latticed (or grid) graph paper,[10] which is also reflected in the title of the star map. In modern sciences latticed papers are commonly used to express numerical data exactly in graphical form. Examining the star map in detail, however, we see that the positions and shapes of many of the asterisms are far from precise and seem to have been drawn free-hand, suggesting that the author of this star map did not recognize the correct usage of latticed graph paper. Moreover, a sinusoidal curve crossing the Equator in the map also looks so inaccurate that some researchers have regarded it as the ecliptic, and others as the lunar path (Ohsaki, 1987)—although the latter interpretation seems more consistent with the title of the star map.

We measured positions of Juxing stars using an enlarged copy owned by the NAOJ. As the total number of lattices in the horizontal direction amounted to 365, this means that the unit of the longitude is the traditional Chinese one. Regarding the Qujidu data we counted the number of the latitudinal lattices for each Juxing star from the Equator, while the Xiudu values written at the bottom of the rectangular map (see Figure 14) were adopted for analysis because of inaccuracy in asterism shapes.

As for dating of the Xiudu data, 485 ± about 25 years (the mean residual: 0.8°) was obtained by the simulation method (we got nearly the same result for both the linear and the quadratic approximations of the precession theory). For unknown reasons this confidence interval was unusually small compared with other cases.

On the other hand, from the Qujidu values

measured for 21 Juxing stars we got 545 ± 90 years (the mean residual: 2.1°) by the simulation method with the quadratic precession theory (Table A2). As the confidence interval for the Xiudu overlaps with that for the Qujidu, both the dating results do not contradict each other. But one can easily notice that the Qujidu estimate is worse than that for the Xiudu, probably because of free-hand drawing of the 28-Xiu constellations. This indicates that this star map was not as precise as its latticed graphical appearance would suggest.

In the past Ohsaki (1987) analyzed the observation date of this star map by adopting the declination data of 191 stars. His result was 500 ± 50 years, which agrees with our estimate of 545 ± 90 years within each uncertainty range. At the same time, since the uncertainty range by Ohsaki was an ordinary standard deviation, corresponding to a theoretical probability of about 68% for the Gaussian error distribution, it is understood that our method (β = 90%) can provide a date that is as precise as Ohsaki's but with a much smaller sample size, of 21.

In the Chinese official histories, we could not locate observations of the 28-Xiu constellations between the latter half of the fifth century and the first half of the sixth century. However, it may be worth noting that this period overlaps with the lifetime of the astronomer Zu Chongzhi (祖沖之, 429–500). He was a talented engineer who not only invented the Zhinan-che (an instrument with a magnetic compass), but he also compiled the Daming Calendar between AD 457 and 464 that included the effect of precession for the first time. From the latter fact, it is possible to suppose that for his calendar reform he made observations of the 28-Xiu constellations again or produced a new star map in which the star positions were calculated by Chinese precession theory (1° per 50 years) at that time, then one of its copies was transmitted to Japan in a much later date. In any case, this Japanese star map will be an interesting target for further studies.

6.5 The Observation Epoch of Ulugh Beg's Star Catalogue

Finally let us change our view on star maps and catalogues from East Asia to those by Arabic astronomers in the Middle East. It is well known that the Greek astronomy that culminated in Alexandria was not inherited by the Romans but by the Islamic people after the ninth century. Thabit ibn Qurra (836–901), and Muhammad al-Battani (ca. 850–929) both made efforts to develop further the astronomy in Ptolemy's *Almagest*. In parallel with such activities pre-modern astronomical observatories were constructed in Malaga, Samarkand and Istanbul (Hoskin and Gingerich, 1997).

On the basis of the star catalogue in the *Almagest*, Islamic astronomers tried to improve it by observations and gave Arabic proper names to bright stars, some of which are now familiar to us, such as Rigel (β Ori), Aldebaran (α Tau), Altair (α Aql), Vega (α Vir), Deneb (α Cyg) and others. The representative work on Islamic constellations is the *Book of the Fixed Stars* authored by the Persian astronomer Abd al-Rahman al Sufi (903–986) in AD 964. In his book al-Sufi describes each of the Islamic 48 constellations in comparison to the corresponding Greek ones, illustrating star charts with constellation shapes and their member stars (e.g., see Hafez et al., 2011).

At Samarkand, the old capital of Uzbekistan and an important oasis city on the Silk Road, the remains of an ancient astronomical facility were unearthed by the Russian archaeologist Vassily Vyatkin in 1908. From Islamic historical records it was soon identified as the huge astronomical observatory constructed by Ulugh Beg (1394–1449), the fourth Timurid Sultan. Prior to that position, he was the long-serving Governor of Samarkand, and was highly interested in intellectual matters. He was not only an excellent astronomer and mathematician but also a patron of science and culture, inviting many talented scholars to make the city an intellectual center.

The underground remains of the observatory were found to be the graduated arc of a gigantic sextant with a radius of 36m, the size intended for increasing the measuring accuracy (Figure 15). Ulugh Beg completed it in 1429 and by commanding subordinate astronomers he made precise observations of the transit altitudes of the Sun and more than a thousand stars. It is said that this resulted in the Ulugh Beg star catalogue of 1437 (the *Zij-i-Sultani*) which included the data of 993 stars. This catalogue was a substantial revision of al-Sufi's star table. Furthermore, using other astronomical instruments Ulugh Beg could improve the precession constant in the ecliptic longitude to be 51.4″ per year (the correct value is 50.2″ per year), and the obliquity of the ecliptic (ε) to be 23° 30′ 17″ (the correct value at that time was 23° 30′ 48″).

In order to confirm the observation epoch of the Ulugh Beg star catalogue, we analyzed his measurements by the simulation method with the quadratic precession theory. The data were taken from the Appendix in Yabuuchi's translation of *Hevelius Star Atlas* (1993) and given in Table A7. Through the Bayer desig-

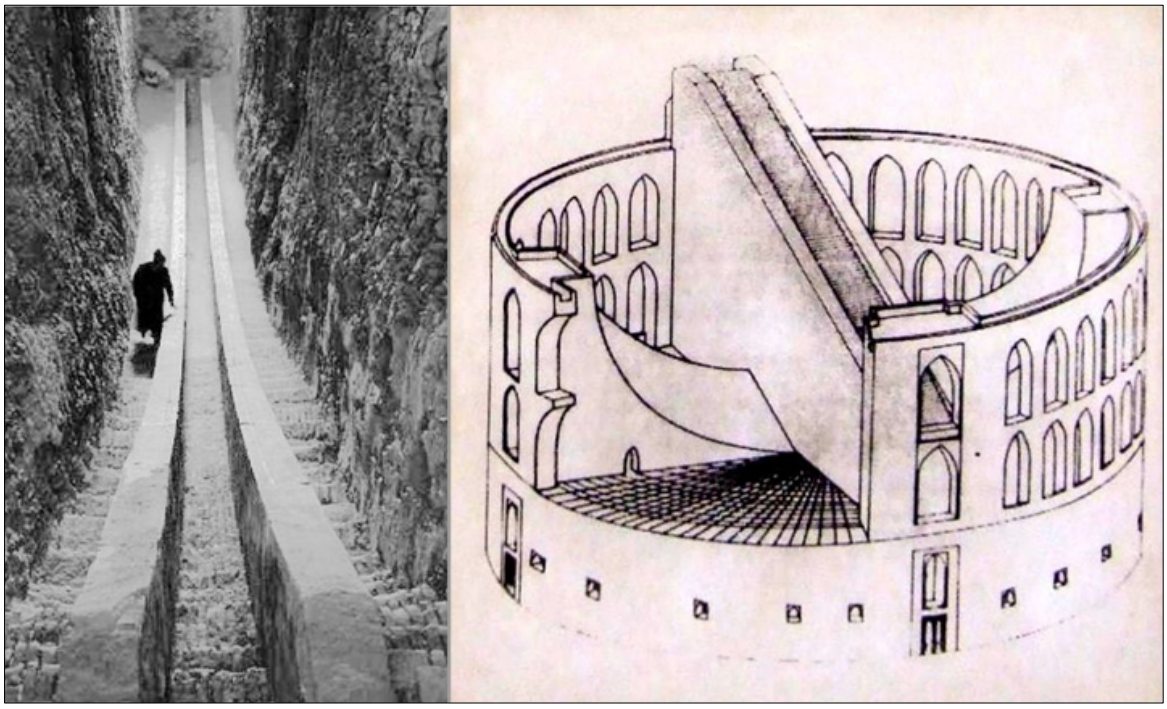


Figure 15: The graduated underground arc of the Ulugh Beg's gigantic sextant at Samarkand (left) and a restoration image of the observatory (right) (courtesy: Ulugh Beg Observatory Museum).

nations and the Bright Star Catalog (Hoffleit and Jascheck, 1982), we identified the corresponding 28-Xiu Juxing stars except for the No.18 Mao constellation (the Pleiades), and calculated their right ascensions and declinations in Table A7 from the Newcomb's formula (Mueller, 1969) assuming $\varepsilon = 23.513°$ at AD 1430.

Once the equatorial coordinates of the Juxing stars were obtained, we applied the same dating method to them as that used for the *Almagest* stars (Section 5). Here are the resulting estimates of the observation dates: from right ascension analysis: 1450 ± 10 years (the mean residual: 0.4°), and from declination analysis: 1425 ± 25 years (the mean residual: 0.3°), so both results are consistent with each other. Although the lower bound of the former estimate is different by a few years from the completion year of the Ulugh Beg catalogue (1437), this seems partially due to the fact that the simulation method frequently used random numbers. So, the above outcomes show that our statistical analysis adopting the 28 Juxing stars can provide a correct date within an error of 10 years or so.

Regarding the measuring accuracy of the Ulugh Beg star catalogue, both Shevchenko (1990) and Krisciunas (1993) have made detailed error analyses of it. They found that the mean error of the catalogue was about 16′. Hence, we see that our residuals of 0.3–0.4° are nearly compatible with their values as well.

7 CONCLUDING REMARKS

From the above-mentioned results, it can be seen that our 28-Xiu BS method provides more precise dating of old star maps and catalogues than the conventional approaches used in the past, with a sample number of just a few tens to be analyzed. So, what were the main reasons for our successful estimations?

Observed positions of stars suffer generally from the instrumental and setting errors of the armillary sphere used for measurements, sometimes leading to a bias in dating. Unless analyzed stars are scattered all over the sky, even when numbering more than a hundred, the influence from the above biased errors on the estimated observation date was inevitable. To cope with the problem, some researchers introduced setting and instrumental errors in their analysis parameters as well (e.g., Maeyama 1977; Sun and Kistemaker 1997).

On the other hand, the 28-Xiu constellation system used in our analysis consists of only 28 Juxing stars, but all of them are fairly bright; the faintest one is No. 17 Wei 胃 (35 Ari) with the visual magnitude of 4.6. First, this situation allowed us to apply our method successfully to many historical star maps and catalogues.

Another advantage of using the 28-Xiu constellations for the purpose of dating analysis is that they are distributed over a wide declination range in the sky (from –35° to +25°) irregularly all along the equator and the ecliptic.

This situation canceled out an excessive appearance of positive or negative (O−C) values in measurements caused by instrumental and setting errors of the armillary spheres used, thus resulting in less biased dating as a whole.

The third merit of adopting the 28-Xiu constellations is due to smallness of the data size. As the 28 Juxing stars can be measured easily within a year, this ensures that all of their observations were conducted usually in the same year. By contrast, if we try to use for analysis too many stars recorded in star maps and catalogues in an attempt to increase the dating accuracy, it may conversely endanger the assurance that all the data are from the same observation epoch—note that there is no way of knowing the observation date of each star, unless it is clearly written in the historical records.

In conclusion we emphasize that the most essential point in using the 28-Xiu constellations method for dating analysis is to allow us to apply it to many historical star maps and catalogues in a unified way, and to compare them from a common viewpoint.

8 ACKNOWLEDGEMENTS

The author expresses his gratitude to three anonymous referees, whose comments and advice were helpful in improving the original manuscript.

9 NOTES

1 This paper is a concisely reorganized version of the Japanese book published by the same author in 2018 (Nakamura, 2018), correcting a few mistakes and adding subsequent new findings and insights.

2 Confidence level is sometimes called 'confidence coefficient' as well. The meaning of a confidence level 95%, for instance, is as follows: if measurements of a statistical quantity are made a hundred times independently, the statistical estimates of the true value of the quantity fall 95 times into the corresponding confidence interval. Naturally, the larger the confidence level, the wider the width of the confidence interval; namely if we set up higher reliability of estimation, the error range inevitably becomes wider.

3 If we compare Figure 6 or Figure 7 with Figure 12 (*Suzhou tianwen tu*), one can easily notice that the ecliptic of the Kitora star map is quite wrong in terms of its orientation. Although the true reason of the failure is unknown, one possibility is that when the Kitora painter drew the ecliptic referring to the original star map designed on a sheet of paper, presumably he put it in a wrong direction by mistake (Miyajima, 1999).

4 The inner and outer circles (規 Gui) express the sky regions in which stars never sink below or never rise above the horizon and their sizes are calculated from an assumed value of the geographic latitude. From this relationship some researchers have claimed that the two circles allow us to determine the location where the stars in a star map were observed. However, Nakamura (2015; 2018) showed that, through a total surveying of the Chinese literature contained in the *Siku Quanshu* (四庫全書) completed in 1782 and consisting of 79,000 books and documents, the two circles were drawn mainly for the convenience of users of a star map, and in general had nothing to do with the place where the stars were observed.

If the above view is justified, the geographic latitude obtained from the inner Gui circle of the Kitora star map of 37.6–38.1° (close to the location of the capital of the Koguryeo Kingdom in ancient Korea, ca. first century BC to seventh century AD), may indicate the possibility that the original of the Kitora star map was transferred to Japan not directly from China but via the Korean Peninsula. That is to say, it is likely that the inner circle was added anew to the original Chinese star map to meet the need to use the star map in Korea.

5 At present, the Cartesian coordinates (x, y), or the polar coordinates (r, θ) like those adopted in the Kitora star map to express positions of stars are one of the most elementary ways for planar graphical representation. Interestingly, however, such an approach was surprisingly recent in the Western world. According to Friendly et al. (2010), the earliest known use of the one-dimensional graph, to show the route from Toledo, Spain to Rome, was invented in 1644 by Michael F. van Langren (ca. 1600–1675), who is known to have published a broadsheet lunar map for the first time in 1645 (Whitaker, 1989).

6 In China from ancient times, astrological astronomy had politically been of top-priority importance, because it was believed that the Chinese Emperor was chosen by Heaven to rule over his country. Accordingly, from the Late Han dynasty on, the Government passed a law to prohibit ordinary people from being involved in astronomy. For instance, the *Tang liutien* (*The Six Institutional Classification of the Tang Dynasty*) of AD 738 stated that if a person

who was not a Court Astronomer studied astrological books or used astronomical instruments (such as an armillary sphere) for observations, he would be punished (Nakayama, 1969: 15). And this regulation also applied to the *Da Qing Huidien shili* (*Collected Institutions of the Qing Dynasty*), which was written in 1690. Given this situation, it would have been almost impossible for a non-Governmental citizen to observe stellar positions using an armillary sphere or produce a star map. Hence there was very little possibility that the oservations other than those of the times shown in the table in Figure 3 were actually performed, including the one of AD 300 obtained by Sôma (2016).

7 Following Yoshihara (2009), we clearly explain calculation process of the 28-Xiu BS method using the Kitora data (Table 1). First, we regarded 24 dates in the last column given in Table 1 as the original population whose mean value is expressed here by Θ, and on the basis of the population we made up n sets of artificial data groups by random resampling (permitting multiple sampling of the same data value). Then the boot strap theory teaches us that the confidence interval of the mean for the original population can be obtained by $[\Theta - \sigma_n Z_n(\beta)$ and $\Theta + \sigma_n Z_n(1-\beta)]$, where σ_n, $Z_n(\beta)$ and $Z_n(1-\beta)$ are statistical quantities calculated from all the above n sets of pseudo-data groups generated in a computer for β = 90%. Note that Θ is not always located at the center of the confidence interval depending upon the values of $Z_n(\beta)$ and $Z_n(1-\beta)$. Although there are several procedures to estimate σ_n and Z_n, it is common to select the one giving the minimum width of the confidence interval as the final result (Chernick, 1999; Yoshihara, 2009).

8 During actual observations in modern astronomy, we sometimes encounter situations where we are forced to handle the data with meager sample numbers and/or of low quality caused by noise due to the instrumental detection limit, poor weather, and so on. In the case of astronomical observations, the observer cannot generally set up the measuring conditions himself, or try again to make the same measurements—this is an important difference between astronomy and physics experiments carried out in a laboratory. To compensate for such inconvenience, we often perform a model analysis instead.

9 For the Juxing star No. 22 (井, Jing) the minimum of $\Sigma_i(O-C)_i^2/n$ did not exist during AD 200–1700.

10 Although scientific Mercator-type star maps like the one in the book *Xin yixiang fayao* (新儀象法要) published in 1094 by Su Song (see Section 6.1) must have needed some sort of lattices for the purpose of an exact design, to the author's knowledge, there seem to be no such products of Chinese origin drawn on latticed paper. In the Chinese history of geography, however, one can find a very famous example of a latticed map, called *Yuji tu* (禹跡圖). This map of continental China inscribed in 1137 on a stone monument is now preserved in the Museum of Historical Monuments in Xian city; the unit of the lattice measures a hundred Chinese lis (li: about 400 meters). Meanwhile, the astronomer Shen Kuo (沈括, 1030–1094) of the Song Dynasty and Huan Chang (黃裳), the author of *Suzhou tianwen tu* inscribed in the twelfth century, were deeply involved in the production of such scientific geographical maps as well (Needham, 1959b). Thus, it is reasonable to consider that techniques of latticed mapmaking were applied by them also to star maps at around that time.

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11 APPENDIX

Table A1: Linear approximation of precession.

					Rate (°/100y)		
No.	28-Xiu	Juxing	RA1 (°)	DC1 (°)	dRA/dt	dDC/dt	dΔRA/dt
1	角	α Vir	175.95	0.49	1.252	−0.592	0.039
2	亢	κ Vir	187.36	0.13	1.291	−0.510	−0.008
3	氐	α Lib	196.43	−5.68	1.283	−0.548	0.093
4	房	π Sco	211.36	−17.66	1.376	−0.472	0.014
5	心	σ Sco	216.69	−17.93	1.39	−0.437	0.121
6	尾	μ Sco	221.36	−31.35	1.511	−0.345	0.012
7	箕	γ Sgr	240.45	−26.97	1.523	−0.245	−0.003
8	斗	φ Sgr	250.77	−25.57	1.52	−0.145	−0.097
9	牛	β Cap	277.00	−18.13	1.423	0.104	−0.051
10	女	ε Aqr	284.70	−14.08	1.372	0.172	−0.033
11	虚	β Aqr	296.38	−11.98	1.339	0.272	−0.042
12	危	α Aqr	305.73	−7.97	1.297	0.343	−0.086
13	室	α Peg	321.88	5.93	1.211	0.439	0.023
14	壁	γ Peg	338.41	4.92	1.234	0.505	0.000
15	奎	ζ And	346.66	13.87	1.234	0.520	0.045
16	婁	β Ari	2.51	10.88	1.279	0.514	0.052
17	胃	35 Ari	13.43	18.53	1.331	0.489	0.036
18	昴	17 Tau	28.16	16.61	1.367	0.421	0.002
19	畢	ε Tau	39.22	13.24	1.369	0.351	−0.040
20	觜	λ Ori	56.94	6.90	1.329	0.213	−0.090
21	参	δ Ori	58.04	−3.30	1.239	0.207	0.240
22	井	μ Gem	65.96	21.50	1.479	0.122	−0.001
23	鬼	θ Cnc	98.76	23.19	1.478	−0.190	−0.125
24	柳	δ Hya	102.63	11.28	1.353	−0.221	−0.117
25	星	α Hya	117.34	−0.90	1.236	−0.343	−0.040
26	張	υ Hya	124.02	−6.18	1.196	−0.394	0.005
27	翼	α Crt	140.85	−7.74	1.201	−0.502	0.028
28	軫	γ Crv	159.13	−5.91	1.229	−0.574	0.023

RA1 and DC1 are the right ascension and declination in degrees for AD 1, and dRA/dt and dDC/dt are their variation rates per century. dΔRA/dt is the Xiudu rate per century. This approximation is valid with errors of less than 0.5–0.8° during the period from 200 BC to AD 1300.

Table A2: Quadratic approximation of precession.

			1900.0		RA			DC		
No.	28-Xiu	Juxing	RA (°)	DC (°)	A	B	C	A	B	C
1	角	α Vir	199.981	−10.639		0.0128	175.570	0	−0.0055	−0.211
2	亢	κ Vir	211.890	−9.808		0.0129	187.310	0	−0.0053	0.099
3	氐	α Lib	221.288	−15.581		0.0132	196.030	0	−0.0050	−6.359
4	房	π Sco	238.200	−25.826	5.0E-07	0.0142	210.910	4.0E-07	−0.0049	−18.300
5	心	σ Sco	243.777	−25.353		0.0144	216.230	5.0E-07	−0.0046	−18.562
6	尾	μ Sco	251.274	−37.876		0.0158	220.850	6.0E-07	−0.0043	−31.970
7	箕	γ Sgr	269.658	−29.584		0.0156	239.930	7.0E-07	−0.0028	26.810

8	斗	φ Sgr	279.852	−27.094		0.0155	250.320	7.0E-07	−0.002	26.184
9	牛	β Cap	303.848	−15.097		0.0144	276.600	7.0E-07	0.0007	−18.737
10	女	ε Aqr	310.566	−9.862		0.0139	284.280	6.0E-07	0.0014	−14.714
11	虛	β Aqr	321.574	−6.011		0.0135	295.980	5.0E-07	0.0025	−12.609
12	危	α Aqr	330.162	−0.806		0.0131	305.330	4.0E-07	0.0033	−8.603
13	室	α Peg	344.945	14.667		0.0123	321.520	0	0.0049	5.240
14	壁	γ Peg	2.021	14.628		0.0126	338.060	0	0.0054	4.248
15	奎	ζ And	10.509	23.723		0.0126	346.320	0	0.0055	13.195
16	婁	β Ari	27.278	20.319		0.0131	2.260	0	0.0054	1.015
17	胃	35 Ari	39.395	27.282		0.0137	13.157	0	0.0051	17.858
18	昴	17 Tau	54.734	23.799		0.0140	27.929	−4.0E-07	0.0050	15.871
19	畢	ε Tau	65.694	18.959		0.0139	39.058	−5.0E-07	0.0044	12.519
20	觜	λ Ori	82.407	9.867		0.0134	56.749	−6.0E-07	0.0031	6.180
21	參	δ Ori	81.724	−0.358		0.0125	57.842	−6.0E-07	0.0030	−4.010
22	井	μ Gem	94.228	22.565		0.0149	65.842	−7.0E-07	0.0023	20.733
23	鬼	θ Cnc	126.473	18.433		0.0147	98.600	−7.0E-07	−0.0009	22.470
24	柳	δ Hya	128.090	6.053		0.0135	102.450	−6.0E-07	−0.0012	10.551
25	星	α Hya	140.668	−8.225		0.0124	117.130	−5.0E-07	−0.0026	−1.607
26	張	υ Hya	146.667	−14.378		0.0120	123.810	−4.0E-07	−0.0032	−6.894
27	翼	α Crt	163.725	−17.766		0.0122	140.610	0	−0.0048	−8.427
28	軫	γ Crv	182.665	−16.987		0.0125	158.750	0	−0.0054	−6.586

RA and DC values for the year AD 1900.0 in the 4th and 5th columns are taken from *The Bright Star Catalogue* (Hoffleit and Jaschek, 1981). RA and DC of an arbitrary year can be calculated by $At^2 + Bt + C$, where t = the Christian era. This approximation is valid with errors of less than 0.1° between 200 BC and AD 1800.

Table A3: An example of model analysis for the assumed date of AD 300.0.

AD300.0			SD (1.5)	SD (1.0)	SD (0.7)
No.	28-Xiu	Juxing	Year	Year	Year
1	角	α Vir	289	365	285
2	亢	κ Vir	225	330	289
3	氐	α Lib	406	379	255
4	房	π Sco	290	307	284
5	心	σ Sco	347	347	300
6	尾	μ Sco	186	329	323
7	箕	γ Sgr	543	131	283
8	斗	φ Sgr	104	369	198
9	牛	β Cap	142	267	289
10	女	ε Aqr	309	315	294
11	虛	β Aqr	302	329	281
12	危	α Aqr	242	351	299
13	室	α Peg	426	344	273
14	壁	γ Peg	428	270	280
15	奎	ζ And	276	262	306
16	婁	β Ari	320	251	333
17	胃	35 Ari	276	303	280
18	昴	17 Tau	316	281	306
19	畢	ε Tau	334	317	315
20	觜	λ Ori	304	304	285
21	參	δ Ori	127	305	293
22	井	μ Gem	368	282	273
23	鬼	θ Cnc	389	371	390
24	柳	δ Hya	315	316	2.84
25	星	α Hya	351	248	290
26	張	υ Hya	319	122	251
27	翼	α Crt	310	261	266
28	軫	γ Crv	438	361	303
Mean			310	301	280
S.D.			97	62	63
28-Xiu-BS			[282, 337]	[282, 319]	[261, 298]

As for SD(1.5), SD(1.0) and SD(0.7), refer to the text (Section 4.5).

Table A4: Xiudu and Qujidu data of *Shishi xingjing*.

No.	28-Xiu	Juxing	Xiudu	Qujidu
1	角	α Vir	12	91
2	亢	κ Vir	9	
3	氐	α Lib	16	94
4	房	π Sco	5	108
5	心	σ Sco	5	109
6	尾	μ Sco	18	124
7	箕	γ Sgr	11	118
8	南斗	φ Sgr	26.25	116
9	牽牛	β Cap	8	110
10	須女	ε Aqr	12	106
11	虛	β Aqr	10	104
12	危	α Aqr	17	99
13	室	α Peg	16	85
14	東壁	γ Peg	9	86
15	奎	ζ And	16	70
16	婁	β Ari	12	80
17	胃	35 Ari	14	72
18	昴	17 Tau	11	74
19	畢	ε Tau	16	78
20	觜觿	λ Ori	2	84
21	参	δ Ori	9	94.4
22	東井	μ Gem	33	70
23	輿鬼	θ Cnc	4	68
24	柳	δ Hya	15	77
25	星	α Hya	7	90
26	張	υ Hya	18	97
27	翼	α Crt	18	99
28	軫	γ Crv	17	99
		Sum	366.25	

Note that the sum of all the Xiudus is not 365.25° but 366.25°, probably due to a copying mistake in the past literature.

Table A5: Dating analysis of the *Almagest* data.

No.	28-Xiu	Juxing	RA_{pt}	DC_{pt}	Xiudu (°)	Rel-RA	Year
1	角	α Vir	176.2	−0.5	11.6	0.0	130.4
2	亢	κ Vir	187.8	−0.5	9.0	11.6	114.6
3	氐	α Lib	196.8	−6.6	14.7	20.7	117.0
4	房	π Sco	211.5	−18.3	5.4	35.4	115.3
5	心	σ Sco	216.9	−18.8	4.5	40.8	112.0
6	尾	μ Sco	221.4	−32.1	19.7	45.3	17.2
7	箕	γ Sgr	241.1	−27.8	9.9	64.9	82.3
8	斗	φ Sgr	251.0	−26.2	26.7	74.9	42.8
9	牛	β Cap	277.7	−18.4	7.3	101.6	83.3
10	女	ε Aqr	285.0	−14.4	11.9	108.8	61.2
11	虛	β Aqr	296.9	−12.5	9.1	120.7	66.3
12	危	α Aqr	306.0	−8.4	16.3	129.9	61.0
13	室	α Peg	322.3	5.7	16.5	146.1	64.1
14	壁	γ Peg	338.8	4.4	8.7	162.6	66.5
15	奎	ζ And	347.5	13.6	16.1	171.4	25.8
16	婁	β Ari	3.6	10.7	10.0	187.5	28.0
17	胃	35 Ari	13.6	18.1	26.7	197.5	70.4
18	昴	17 Tau					
19	畢	ε Tau	40.3	12.8	17.5	224.1	77.0
20	觜	λ Ori	57.8	6.7	0.9	241.6	58.3
21	参	δ Ori	58.7	−4.1	8.3	242.5	48.4
22	井	μ Gem	66.9	20.9	31.8	250.8	106.2
23	鬼	θ Cnc	98.7	22.4	4.5	282.5	20.3
24	柳	δ Hya	103.2	10.1	14.2	287.0	66.8
25	星	α Hya	117.4	−2.0	7.0	301.3	95.5
26	張	υ Hya	124.4	−6.7	16.4	308.3	59.9

27	翼	α Crt	140.8	−8.7	18.3	324.7	106.2
28	軫	γ Crv	159.1	−7.1	17.1	342.9	126.2
						mean	74.9
						SD	33.1

* RA_{pt} and DC_{pt} are converted values from the ecliptic longitudes and latitudes given in the *Almagest* (Grasshoff, 1990). Since we could not identify No. 18 (昴) Juxing star in the *Almagest* catalogue, the actually analyzed number of stars are 27.

Table A6: Dating analysis of the star map of King Qian Yuanguan's tumulus.

No.	28-Xiu	Juxing	Xiudu (°)	DC (°)	Year
1	角	α Vir	9.5	76.1	1090
2	亢	κ Vir	8.0	78.3	1187
3	氐	α Lib	14.0	69.5	1124
4	房	π Sco	5.5	82.0	929
5	心	σ Sco	11.5	73.2	847
6	尾	μ Sco	15.0	68.1	812
7	箕	γ Sgr	12.5	71.7	519
8	斗	φ Sgr	27.0	50.6	734
9	牛	β Cap	12.5	71.7	934
10	女	ε Aqr	11.0	73.9	1048
11	虚	β Aqr	9.0	76.9	1035
12	危	α Aqr	14.5	68.8	924
13	室	α Peg	54.5	10.4	912
14	壁	γ Peg			
15	奎	ζ And			
16	婁	β Ari			
17	胃	35 Ari	14.5	68.8	1069
18	昴	17 Tau	15.5	67.4	948
19	畢	ε Tau	15.0	68.1	1141
20	觜	λ Ori	0.0	90.0	978
21	参	δ Ori	13.5	70.3	854
22	井	μ Gem	25.5	52.7	
23	鬼	θ Cnc	2.0	87.1	1008
24	柳	δ Hya	17.5	64.4	990
25	星	α Hya	24.0	54.9	905
26	張	υ Hya			
27	翼	α Crt	15.5	67.4	721
28	軫	γ Crv	12.5	71.7	956
				mean	942
				SD	156

Table A7: Ulugh Beg's 28-Xiu observations.

			Ecl.-long.			Ecl.-lat.		RA	Dec.
No.	28-Xiu	Juxing	Sec.*	°	′	°	′	α (deg)	δ (deg)
1	角	α Vir	6	16	10	−2	9	194.05	−8.36
2	亢	κ Vir	6	26	52	3	0	206.01	−7.59
3	氐	α Lib	7	7	52	0	45	215.74	−13.47
4	房	π Sco	7	24	40	−5	27	230.84	−24.27
5	心	σ Sco	8	0	28	−3	45	237.43	−23.98
6	尾	μ Sco	8	7	55	−15	15	243.10	−36.72
7	箕	γ Sgr	8	23	49	−7	12	262.87	−30.56
8	斗	φ Sgr	9	2	19	−3	54	272.60	−27.39
9	牛	β Cap	9	26	10	4	45	297.25	−16.31
10	女	ε Aqr	10	3	49	8	9	304.20	−11.43
11	虚	β Aqr	10	15	43	8	48	315.42	−7.80
12	危	α Aqr	10	25	31	10	9	324.38	−3.48
13	室	α Peg	11	15	55	19	0	339.32	11.81
14	壁	γ Peg	0	1	22	12	24	356.23	11.90
15	奎	ζ And	0	13	25	17	18	5.20	21.17
16	婁	β Ari	0	27	7	7	51	22.18	17.78
17	胃	35 Ari	1	9	40	10	54	33.45	25.05
18	昴	17 Tau							
19	畢	ε Tau	2	1	10	−2	54	59.65	17.62
20	觜	λ Ori	2	16	31	−13	30	76.71	9.39

21	参	δ Ori	2	14	34	−23	57	75.92	−1.19
22	井	μ Gem	2	27	31	−1	15	87.32	22.24
23	鬼	θ Cnc	3	27	40	−1	15	119.50	19.47
24	柳	δ Hya	4	2	25	−12	30	121.86	7.49
25	星	α Hya	4	19	31	−22	30	134.69	−6.31
26	張	υ Hya	4	28	10	−26	0	141.40	−12.29
27	翼	α Crt	5	15	55	−22	42	158.10	−15.33
28	軫	γ Crv	6	2	46	−14	18	176.71	−14.19

* 'Sec.' in the ecliptic longitude column means the sequential number of the zodiacal constellations (Grasshof, 1990).

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His recent books, for the past ten years, are: *Deciphering the Ancient Starry Sky from the Kitora Tumulus Star Map* (2018, University of Tokyo Press, in Japanese); *The Emergence of Astrophysics in Asia: Opening a New Window on the Universe* (2017, Springer, co-edited by Wayne Orchiston); *A History of Oriental Astronomy* (2014, Maruzen Publ., in Japanese); *Five Thousand Years of Cosmic Visions* (2011, University of Tokyo Press, co-authored by Sadanori Okamura, in Japanese); *Highlighting the History of Astronomy in the Asia-Pacific Region: Proceedings of the ICOA-6 Conference* (2011, co-edited by Wayne Orchiston and Richard G. Strom); and *Mapping the Oriental Sky: Proceedings of the ICOA-7 Conference* (2011, National Astronomical Observatory of Japan, co-edited by Wayne Orchiston, Mitsuru Sôma and Richard G. Strom).

Tsuko is on the Editorial Board of the *Journal of Astronomical History and Heritage*. Asteroid Tsuko (6599), a member of Flora family, was named after him in 1991. One of his favorite things is to visit domestic and overseas art museums and in particular to look for historical paintings relating to astronomy.