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THE SALUTARY SIMPLIFICATIONS IN ASTRONOMICAL PROCEDURES BY GAṆEŚA DAIVAJÑA

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Abstract: Gaṇeśa Daivajña is one of the great luminaries of post–Bhāskara period who is responsible for upholding the great tradition of Indian astronomy and Mathematics through his very creative and popular works. Gaṇeśa has simplified the procedures of computations of planetary positions and eclipses by completely dispensing with trigonometric functions, without sacrificing the correctness of the results. These procedures are otherwise laborious by the traditional methods. For this reason, even today, his works are famous among *pañcāṅga* makers in most parts of India.

This paper presents the special features of his popular text *Grahalāghava* and its computational effectiveness in the calculations of planetary positions and eclipses.

Keywords: Classical Indian astronomy; Gaṇeśa Daivajña; *Grahalāghava*; *pañcāṅga*; astronomical tables; almanac.

1 INTRODUCTION

There is a common belief that after Bhāskara II there was a decline in the growth of mathematics and astronomy in India. While there may be some truth in this belief, thanks to historical reasons, it is also true that the post-Bhāskara period saw rigorously innovative activity in mathematics in the regions south of the Vindhyas. In fact, Kerala became the base of enormous and rich mathematical output—often anticipating developments in European mathematics. In other parts of India too there were luminaries, like Gaṇeśa Daivajña, upholding the grand tradition of Indian astronomy and mathematics.

In fact, Gaṇeśa Daivajña's astronomical works are widely used among the makers of traditional *pañcāṅga* (astronomical almanacs) in most parts of India today.

2 DATES AND THE PLACE OF GAṆEŚA

Gaṇeśa Daivajña's father was the famous astronomer Keśava Daivajña and his mother's name was Laksmi. He was born in 1507 CE (śaka 1429) at Nandigrāma, a village on the shore of the Indian Ocean.

According to S.B. Dikshit (1981), *Nandigrāma* is currently called Nandagāon, and is in the Janjeera State on coast in the Konkan region of State of Maharashtra. It lies about 40 miles to the south of Mumbai (Bombay), at latitude 18° 22′ 60″ N., and longitude 72° 55′ 00″ E. This identification is supported by Ohashi (1998) in his commentary on Verse 7 of the translation work of the *Pratodayantra*. The *Pradayantra* is a small text consisting of 13 verses on the construction of the column dial.

The family (Figure 1) belonged to the gotra

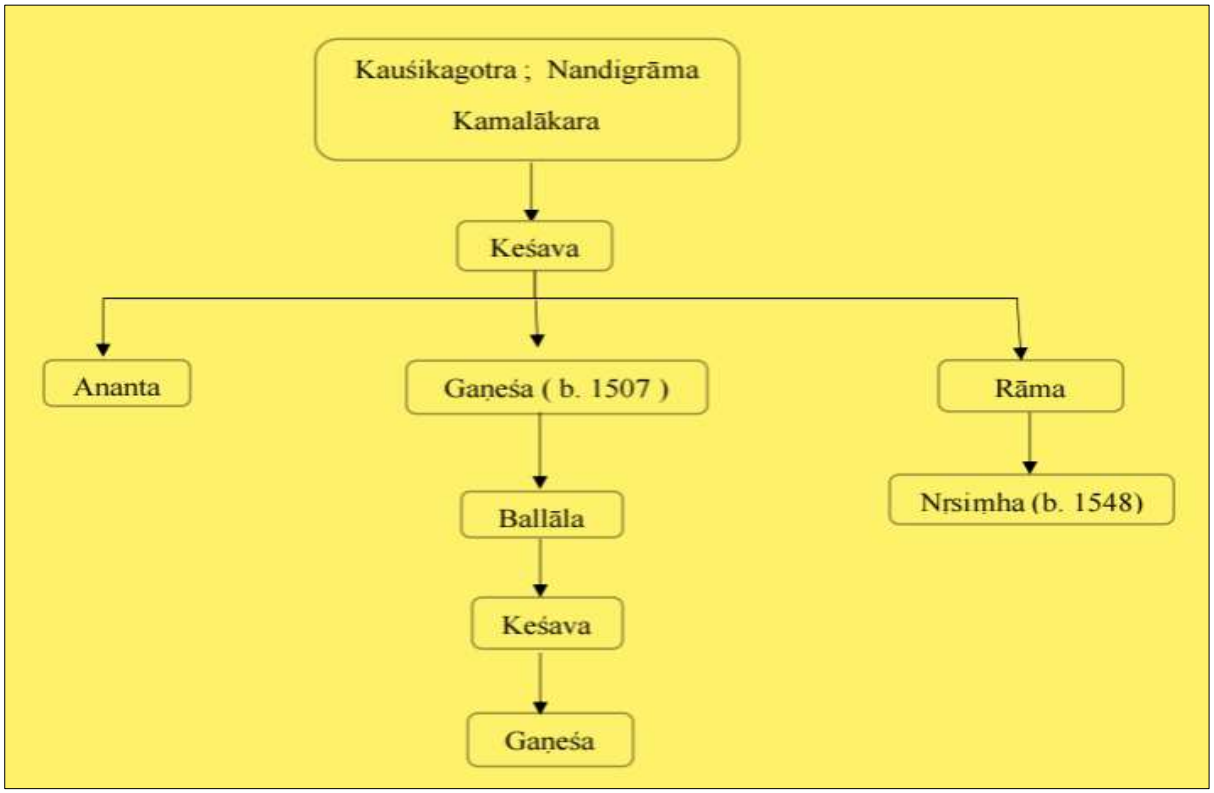


Figure 1: The Family Tree of Gaṇeśa Daivajña (genealogical chart: the author.

(patrilineal ancestry) of *Kuśika*. Gaṇeśa's grandfather (Keśava's father) was Kamalākara, also an eminent astronomer. Gaṇeśa's teacher in astronomy was his own father, while Keśava's teacher was Vaijanātha. These details are mentioned in the work called *Muhūrtatatva* by Keśava Daivajña (Dikshit, 1981).

3 THE WORKS OF GAṆEŚA DAIVAJÑA

Gaṇeśa composed several important works in the fields of mathematics, astronomy, astrology, etc. among which his astronomical treatise *Grahalāghava* (GL) is the most famous. In fact, the remarkable popularity of the *Grahalāghava* surpassed that of his father's *Graha Kautuka* which was an important text in its own right.

Ganeśa's other works were:

(a) The *Laghu* and *Bṛhat–Tithicintāmaṇi*;
(b) A commentary on Bhāskara's *Siddhānta Śiromaṇi*;
(c) A commentary on Bhāskara's *Līlāvatī* called *Buddhivilāsinī*;
(d) The *Vivāha vṛndāvanatīkā*; and
(e) The *Srāddhanirṇaya* .

Gaṇeśa also mentions another work of his, namely the *Parva nirṇaya*. Ganeśa's commentary on the *Līlāvatī*, called *Buddhivilāsinī*, is an extremely useful text to understand the rationale for formulae and methods.

Among Ganeśa's astronomical works, the *Grahalāghava* appears to have been composed first, when he was young. The epoch of the *Grahalāghava* is Monday 19 March 1520 CE. The *Tithichintamaṇi* (1525 CE) and the *Grahalāghavasārinī* (epoch 18 March 1520 CE) are the tables based on the *Grahalāghava*.

The *Tithicintāmaṇi* is a set of astronomical tables composed by Gaṇeśa Daivajña in 1525 CE. The *Tithicintāmaṇi* is a popular text of Gaṇeśa *pakṣa* and is widely used even to today by *pañcāṅga* makers. The *Tithicintāmaṇi* consists of astronomical tables to determine the five calendrical components—the five elements of *pañcāṅga*, namely the *tithi* (lunar day), *nakṣatra* (asterism), *yoga* (conjunction), *karaṇa* (half a *tithi*) and *vāra* (the week day), and also the *rāśi* (zodiac sign) and *saṅkrāntis* (solar ingress).

The text *Tithicintāmaṇi* comprises two versions viz, the *Bṛhat tithicintāmaṇi* and the *Laghu tithicintāmaṇi*, with the former a larger version of the latter. The main version of the text *Tithicintāmaṇi* is popular throughout the country, and its extended version, the *Bṛhat tithicintāmaṇi*, is used mainly in the Western part of India. The *Bṛhat tithicintāmaṇi* comprises 27 verses, while the other version has 18 verses and related tables. They are used in determining the five *pañcāṅga* elements.

The first four elements of *pañcāṅga* namely the *tithi, nakṣatra, yoga* and *karaṇa*, require the true positions of the Sun and the Moon. The fifth

element, the *vāra* (weekday), can be determined without the needing the positions of heavenly bodies. Gaṇeśa has given a simple procedure for computing these elements. Once a *pañcāṅga*-maker obtains the required annual constants (the *dhruvas*) for the *tithi, nakṣatra* etc. for the beginning of a solar year the rest of the work is simple and rather mechanical. By adding or subtracting the related elements and using the tables in the text, these can be obtained easily.

There are commentaries on the *Tithicintāmaṇi* wriitten by

- Ganeśa's nephew, Nṛsimha (b. 1586)
- Viśvanātha (1634) of Benares
- Bāpū and Yajñeśvara
- Viṣṇu, on the *Bṛhat tithicintāmaṇi* (*subodhiṇī,*1575).

The commentaries by Viśvanātha and Viṣṇu (*subodhiṇī ṭīkā*) have been published.

The *Buddhivilāsinī* commentary on Bhāskara's *Līlavatī* dates to the year 1545 CE. Also, the other work, the *Pāta sāraṇī,* was composed sometime after 1538 CE.

Ganeśa's *Vṛndāvanatīkā* gives the date of its composition in a very fascinating manner:

> Take 12 as the number for the *samvatsara*(*hāyan*). Add one (*lava*) for *ayana* to it. Add 6 to the sum of these two numbers (i.e., to 12 + 1) so as to obtain (19) as the number of the *yoga*. Add 4 to the sum which would give 23 as the number of the *nakṣatra* and one (*lava* 0 for the *pakṣa*. If the *pakṣa* is added to one more *pakṣa* (i.e., 1+2), it would give 3 as the number denoting the week day. Take 1 as the *tithi* number and 11 as the month number. Multiply the sum of all these numbers by 21 and increase the product by 9 (*nanda*). The result is the *śaka* year number.

This gives us the following:

Samvat Ayana Yoga Nakṣ Pakṣa Vāra Tithi Māsa Śaka

(12 + 1 + 19 + 23 + 1 + 3 + 1 + 11) × 21 + 9 = 1500

i.e., (71 × 21) + 9 = 1491 + 9 = 1500.

This means that the text, the *Vivāha vṛndāvanatīkā*, was composed on *Māgha* (11th lunar month) *Śukla* (1st *pakṣa*) *Pratipat* (1st *tithi*) of *śaka* 1500 (1578 CE), *Bahudhānya* (12th) *samvatsara*, during *Uttarāyaṇa* (1st *ayana*), *Dhaniṣṭhā* (23rd) *nakṣatra* and *parigha* (19th) *yoga*.

The date of the said text indicates that Gaṇeśa was about 71 years old when he composed it. However, Dikshit (1981) refers to a manuscript copy of the *Vṛndāvanatīkā* which gives the year of its composition as *śaka* 1476 (1554 CE), *Ananda samvatsara*, which is 24 years earlier than the above-cited date (1578 CE). In the *Grahalāghava*, the positions of the planets have been given for the moment of sunrise on Monday, the New Moon Day of *Phālguṇa* of *śaka* 1441, which corresponds to 19 March 1520 CE (Julian).

4 THE *GRAHALĀGHAVA*: ITS POPULARITY AND COMMENTARIES

Among Ganeśa's works, the *Grahalāghava* is the most popular astronomical text. The text is considered as the most comprehensive, exhaustive and easy-to-use text among the *karaṇa* works (handbooks) on classical Indian astronomy. Ganeśa has made the computation of planetary positions and eclipses very easy by simplifying the procedures, that otherwise are laborious, by the methods explained in texts like the *Sūryasiddhānta*, Brahmagupta's *Brahmasputasiddhānta* and *Khaṇḍakhādyaka,* Bhāskara's *Siddhāntaśiromaṇi*, etc. Currently, the *Grahalāghava* is extensively used by Indian *pañcāṅga*-makers, particularly those in Maharashtra, Gujarat, and Northern part of Karnataka, the Hyderabad Deccan region of Andhra Pradesh and by the Deccanis of Vāraṇāsi, Gwalior and Indore. It is also significant that even the Government almanacs published in Indore and Gwalior use the *Grahalāghava* and the *Tithicintāmaṇi* of Gaṇeśa Daivajña.

The *Grahalāghava* carries with it very useful and authoritative commentaries by reputed astronomers like Gangādhara (1586 CE), Mallāri (1602 CE) and Viśvanātha (c. 1612 CE). Viśvanātha has given a large number of examples in his commentary, to illustrate the methods of the *Grahalāghava.* Viśvanātha was the brother of the highly skilled astronomer Viṣṇu who composed a *karaṇa* with the epochal year 1608 CE. The *karaṇa* text is based on the *Sūryasiddhānta*. Viṣṇu also wrote a commentary on Gaṇeśa Daivajña's *Bṛhat tithicintāmaṇi* in which he explains the theory. Viśvanātha has written an *udāharaṇa* on his brother's *karaṇa*.

Gangādhara's commentary on the *Grahalāghava* is called the *Manoramā*. Gangādhara's father was Nārāyaṇa, who authored the *Muhūrta mārtāṇḍa*. Gangādhara was a Vājasaneyī Brāmhmin, and belonged to the *Kauśikagotra*, the same as Gaṇeśa Daivajña. He lived in a village called Tapar, to the North of the Ghṛṣṇeśvara (Lord Shiva) temple, which is North of Devagiri (Daulatabad).

The two popular commentators of the *Grahalāghava* are Viśvanātha and Mallāri. They were born in illustrious Mahārāstrian Brāhmin families of astronomers. Mallāri's father was Div-

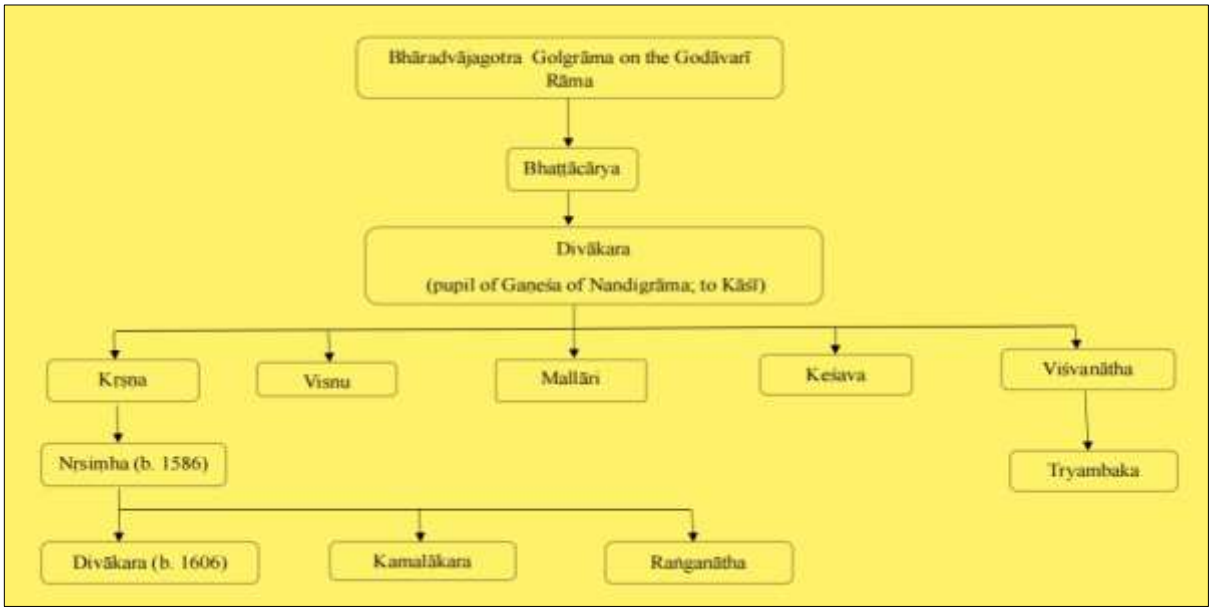


Figure 2: The lineage of Gaṇeśa's pupil, Divākara (genealogical chart: the authors).

ākara (Figure 2), a pupil of Gaṇeśa. Kamalākara, author of the *Siddhānta tattva viveka*, and Ranganātha, who wrote a commentary on the *Sūryasiddhānta*, were descendents of Viśvanātha and Mallāri. Nṛsiṃha, a nephew and pupil of Gaṇeśa, wrote his commentary, the *Harṣakaumudī*, in 1548 CE. The other commentators were Nārāyaṇa (*Kaśī*, before 1635 CE) and Kamalākara (before 1662 CE).

Apart from these commentaries there are other commentaries and translations of the *Grahalāghava*. Some of these are

(1) Edited with the commentary of Mallāri by L. Wilkinson, Calcutta 1843;
(2) Edited with the commentaries of Mallari and Visvanatha by Bhālacandra, Kasi 1865;
(3) Edited with the commentary of Viśvanātha and a Marathi translation by K. S. Goḍabole and V.K.J. Gadre, 2nd edition, Bombay 1873, 5th edition, Poona 1914, and 6th edition, Poona 1926;
(4) Edited with the commentary of Mallāri, Mumbal 1875, Benares 1877, Dilli 1877, Bombay 1882, Mumbal 1883, Calcutta 1886;
(5) Edited with the commentary of Viśvanātha and a Bengali translation by Raisikamohana Caṭṭo-pādhyāya, Calcutta 1887;
(6) Edited with the Hindi translation of Jiyārāma Śāstri by Rāmeśvara Bhaṭṭa, Kalyaṇa-Bombay 1889;
(7) Edited with the commentary of Mallāri by Hariprasāda Śarman, Bombay 1901;
(8) Edited with the commentaries of Mallāri and Viśvanātha by Sudhākara Dvivedi, Benares 1904, reprinted Bombay 1925;
(9) Edited with the commentary of Mallāri and the Telugu commentary of Maṅgipūḍi Vīrayya Siddhāntigār, Musalipatam 1915;
(10) Edited with his own Sanskrit commentary and Hindi translation by Sītārām Jhā MM 142, Benares 1932, reprinted Benares 1941 and Dillī 1975;
(11) Edited with the commentaries of Viśvanātha and Yogeśvara Jhā and a Hindi commentary by Kapilesvara Sastrin, KSS 142, Benares 1946, and edited with the commentary of Mallari and his own Hindi *ṭīka* by Rāmacandra Pāṇḍeya, Jambu 1976.

All these shows how popular the text is.

5 SPECIAL FEATURES OF THE *GRAHALĀGHAVA*

The *Grahalāghava* was originally a palm-leaf manuscript. The title of the text, '*Grahalāghava*', means 'planetary computations made simpler'. Its most extraordinary feature is its systematic substitution of algebraic approximations and tabular interpolation for trigonometric procedures that require tabulated sine values.

Some of the special features of this simplification in the *Grahalāghava* are given below, along with relevant tables.

(1) To avoid handling large number for the *ahargaṇa* (the number of civil days elapsed since a chosen epoch), Gaṇeśa Daivajña introduced a cycle (*cakra*) of 4016 days, which is approximately 11 solar years. This modified *ahargaṇa* never exceeds 4016 and hence is handy. So, numerical error in multiplication etc. can be avoided. For each planet, a number call-

ed the *dhruva* is given (Figure 3). The *dhruva* or *dhruvaka* represents the residual motion of a body in a *cakra* after removing the completed revolutions. Depending on the number of elapsed *cakras* since epoch, the appropriate multiple of the *dhruva* is subtracted from the planet's epochal mean longitude or *kṣepaka*. Then the number of elapsed days in the current *cakra* multiplied by the daily mean longitudinal increment is added to the result to obtain the mean longitude at the desired time. Figure 4 shows examples of *kṣepakas*.

(2) In order to make the procedures simpler for *pañcāṅga*-makers and benefit beginners in astronomy who may be ignorant of trigonometry, Gaṇeśa completely avoided *sine* (*jyā*) and *co-sine* (*koṭijyā*) functions by adopting good approximations for these trigonometric functions.

(3) The dropping of trigonometric ratios did not seriously affect the accuracy of the results, but it greatly simplified the procedures. Gaṇeśa used very good, and mathematically justifiable, approximations for trigonometric functions. He was a very accurate calculator. Even the other *karaṇa* texts that used trigonometric functions had given values of sines of angles in the intervals of 15° or 3¾°.

(4) The text of the *Grahalāghava* has 187 verses spread across 16 chapters. The title of the 16 chapters are given below:

(i) *Madhaymādhikārah* (Mean Positions of the Planets)
(ii) *Ravi Candra Sphuṭādgikārah* (True Positions of the Sun and the Moon)
(iii) *Pañctārāspaṣṭādhikārah* (True Positions of the Five Star-Planets)
(iv) *Triparaśnadhukārah* (Three Problems of Direction, Place and Time)
(v) *Candragrahaṇādhikārah* (Lunar Eclipses)
(vi) *Sūryagrahaṇādhikārah* (Solar Eclipses)
(vii) *Udayāstādhikārah* (Rising and Setting of the Planets)
(viii) *Grahacchāyādhikārah* (Planetary Shadows)



Figure 3: Examples of *dhruvas.*

(ix) *Nakṣatrachāyādhikārah* (Shadows of the Stars)
(x) *śṛṅgonnatyadhikārah* (Rising of the eMoon's Hhorns)
(xi) *Grahayutyadhikārah* (Conjunctions of the Planets)
(xii) *Māsagaṇādhikārah* (Accumulated Lunar Months)
(xiii) *Grahaṇadvaya Sādhanādhikārah* (Eclipses Using the Almanac Short-Cut Method)
(xiv) *Pātādhikārah* (Declinations of the Sun and the Moon)
(xv) *Pañcāṅgagrahaṇādhikārah* (Obtaining a Lunar Eclipse using Accumulated Lunar Months)
(xvi) *Upasamhārādhikārah* (The Concluding Chapter)

(5) While highlighting the special features of his text, Gaṇeśa declares in Verse 4 of Chapter 16:

पूर्वे प्रौढ़तराः क्वचित्किमपि यच्चक्रुर्धनुर्ज्यें विना
ते तेनैव महातिगर्वकुभृदुच्छृङ्गेऽधिरोहन्ति हि ।
सिद्धान्तोक्तमिहाखिलं लघु कृतं हित्वा धनुर्ज्यें मया
तद्गर्वो मयि मास्तु किञ्च यदहं तच्छास्त्रतो वृद्धधीः ॥ ४ ॥

An English translation is:

> The earlier preceptors very proudly rose to the peaks of their fame even though in only a few places they did calculations without using *sines* etc. But I have made calculations for the entire *siddhānta* (astronomy) simple (*lāghava*) by omitting *sine*s etc. throughout. My intelligence (and knowledge) is enriched by their works and hence I do not exhibit pride (in my achievements).

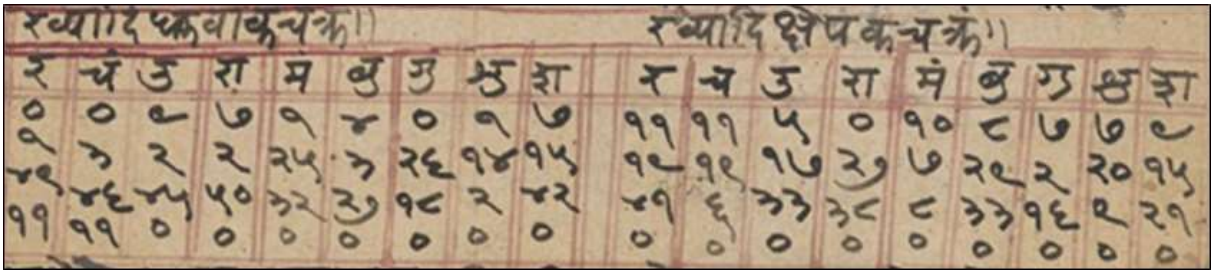


Figure 4: Examples of *kṣepakas*.

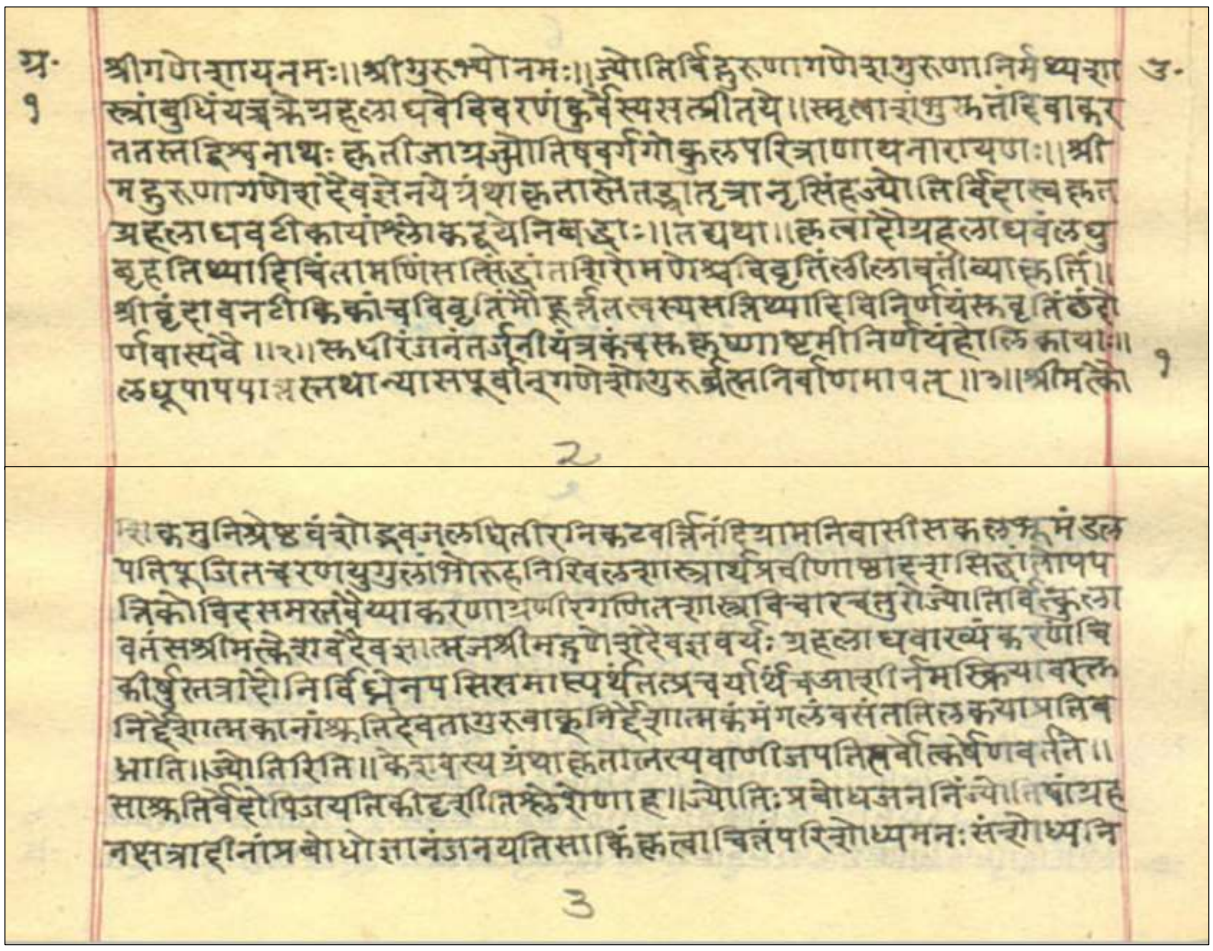


Figure 5: A numerical version of the versified scaled *manda*-equation table in a manuscript of the *Grahalāghava* (University of Pennsylvania: 390 1822 f. 8r).

The *Grahalāghava*'s practical usefulness is further enhanced in the chapters on simplified eclipse predictions by employing almanac (*pañcāṅga*) data and a method for extending their use to specific dates preceding the epoch of Saka 1520. Overall, the work reflects Gaṇeśa Daivajña's ingenious computational techniques, combining simplicity and efficiency for the user with sophisticated and often innovative construction on the part of the compiler. Examples of parts of two folios taken from the *Grahalāghava* are shown in Figure 5.

6 COMPUTATION OF THE *AHARGAṆA* ACCORDING TO THE *GRAHALĀGHAVA*

The *Ahargaṇa* is a basic parameter used for calculating mean positions of the planets and other elements. The number of civil days elapsed since a chosen fixed epoch is called the *ahargaṇa*, literally meaning 'heap of days'. The intercalary months (*adhikamāsa*) play an important role in calculating the *ahargaṇa*.

To avoid a large number for the *ahargaṇa*, Gaṇeśa Daivajña adopted a cycle that is handy and also avoids huge numbers for the *ahargaṇa* by the conventional method, for multiplication. For example, the *Ahargaṇa* according to the GL for the date of Thursday 15 February 2024 is given by *cakra* = 45 and *ahargaṇa* = 3320. For the same date, *Kaliahargaṇa* = 18,71,890.

7 COMPUTATION OF TRUE POSITIONS

In Indian traditional methods to find the true positions of the celestial bodies two important corrections are applied to the mean positions. They are called as *mandaphala*, the equation of centre and *śīghraphala*, the equation of conjunction. The equation of the centre is applied if the celestial body's orbit is eccentric and thus the motion is not uniform. The usual formula for equation of centre is given by $a/R \sin(m)$ where $R = 360°$, a = periphery of the *manda* epicycle (in degrees) and m is the anomaly (from the apogee) called the *mandakendra.*

The *Grahalāghava* explains the method of finding the *mandaphala* of the Sun, as given in the following steps:

(1) Find the *mandakendra* (m = MK) of the Sun. MK = Mean planet – *mandocca* where the *mandocca* is the mean apogee.
(2) Find the *bhuja* of the MK (hereafter denoted by BMK).

(3) Subtract *bhuja*/9 from 20 i.e., (20 – BMK/9).
(4) Multiply the results of step (3) and BMK/9.
(5) Divide the result of step (4) by 9.
(6) Subtract the result of step (5) from 57.
(7) Express the results of step (6) and step (4) as *vikalas* (seconds of arc) and divide the result of step (4) by that of step (6).

The result is the *mandaphala* of the Sun. If these steps are put in the form of a mathematical expression, then,

the *mandaphala* of Sun = $\frac{\left(20-\frac{BMK}{9}\right)\frac{BMK}{9}}{57-\left\{\frac{\left(20-\frac{BMK}{9}\right)\frac{BMK}{9}}{9}\right\}}$

Note that

(1) If the *mandakendra* is within 6 *raśis* from the *meṣa* (i.e., 0° < MK < 180°) then the *mandaphala* is additive.
(2) If the *mandakendra* is within 6 *raśis* from the *tula* (i.e., 180° < MK < 360°) then the *mandaphala* is subtractive.
(3) The *bhuja* of MK is considered as follows:
   a) If 0° < MK < 90° then BMK = MK
   b) If 90° < MK < 180° then BMK = 180° – MK
   c) If 180° < MK < 270° then BMK = MK – 180°
   d) If 270° < MK < 360° then BMK = 360° – MK

Table 1: A comparison of Baskara I's values and the real values for sin *θ*.

Angle *θ*(°)	Values of sin *θ*	
	Bhāskara	Actual
00	0.000	0.000
10	0.175	0.174
20	0.343	0.342
30	0.500	0.500
40	0.642	0.643
50	0.765	0.766
60	0.865	0.866
70	0.939	0.940
80	0.985	0.985
90	1.000	1.000

7.1 Rationale for the *Mandaphala* of the Sun

It is interesting to note that Bhāskara I (629 CE), even as early as in the seventh century gave a very good rational approximation for the *sine* of an angle. In fact, it is important to note that the angle need not be small. His formula is

$\sin\theta \approx \frac{4(180-\theta)\theta}{40500-(180-\theta)\theta}$ where the angle *θ* is in degrees.

The *sine* values according to Bhāskara I are compared with the actual values (obtained from calculators or tables) correct to 3 decimal places for angles from 0° to 90° at intervals of 10° in Table 1.

Sripati Bhatta's expression for the *jyā* (sine) of the *mandakendra* is given in the verse

दोः कोटिभागरहिताऽभिहताः खनागचन्द्रास्तदीयचरणोनशरार्कदिग्भिः ।
ते व्यासखण्डगुणिता विहृताः फलन्तु ज्यार्भिर्विनाऽपि भवतो भुजकोटिजीवे ॥

According to the above verse,

The *mandakendra jyā* = $\frac{(180-BMK)BMK\times120}{10125-\frac{(180-BMK)}{4}BMK}$

i.e., *Jyā* (MK) = $\frac{(180-BMK)BMK\times480}{40500-(180-BMK)BMK}$

$= \frac{\left(\frac{180-BMK}{9}\right)\frac{BMK}{9}\times480}{\frac{40500}{9\times9}-\left(\frac{180-BMK}{9}\right)\frac{BMK}{9}}$ (dividing by 9 × 9)

$= \frac{\left(20-\frac{BMK}{9}\right)\frac{BMK}{9}\times480}{500-\left\{\left(20-\frac{BMK}{9}\right)\frac{BMK}{9}\right\}}$ (1)

The above derivation is based on the significant and unique formula of Bhāskara I (629 CE).

Now, according to the *Grahalāghavam*, the *parama mandaphala* (i.e., maximun *mandaphala*) of the Sun = 125°/57° ≈ 2° 11′ 34″.

Thus, the *mandaphala* of the Sun

$= \frac{125}{57}\times\frac{mandakendra\ \ jyā}{120}$

$= \frac{125}{57\times120}\times\left[\frac{\left(20-\frac{BMK}{9}\right)\frac{BMK}{9}\times480}{500-\left\{\left(20-\frac{BMK}{9}\right)\frac{BMK}{9}\right\}}\right]$

$= \frac{\frac{125}{57}\left[\left(20-\frac{BMK}{9}\right)\frac{BMK}{9}\times4\right]}{500-\left\{\left(20-\frac{BMK}{9}\right)\frac{BMK}{9}\right\}}$

$= \frac{\frac{500}{57}\left[\left(20-\frac{BMK}{9}\right)\frac{BMK}{9}\right]}{500-\left\{\left(20-\frac{BMK}{9}\right)\frac{BMK}{9}\right\}}$

$= \frac{\left[\left(20-\frac{BMK}{9}\right)\frac{BMK}{9}\right]}{\frac{500}{\frac{500}{57}}-\frac{\left(20-\frac{BMK}{9}\right)\frac{BMK}{9}}{\frac{500}{57}}}$

$= \frac{\left[\left(20-\frac{BMK}{9}\right)\frac{BMK}{9}\right]}{57-\frac{\left(20-\frac{BMK}{9}\right)\frac{BMK}{9}}{8.7719298}}$

i.e., *Mandaphala* of the Sun

$\approx \frac{\left[\left(20-\frac{BMK}{9}\right)\frac{BMK}{9}\right]}{57-\left\{\frac{\left(20-\frac{BMK}{9}\right)\frac{BMK}{9}}{9}\right\}}$

Note that the exact formula for the *mandaphala* of the Sun is a/R sin m, with R = 360°. When M = 90°, sin m = 1. Thus, the maximum value of the equation of centre (*parama mandaphala*) = a/R radians.

According to the *Grahalāghava*,

$\frac{a}{R}\times\frac{180}{\pi} = \frac{125}{57}$ degrees

$\frac{a}{2}\times\frac{1}{\pi} = \frac{125}{57}$ (taking R = 360°)

$a = \frac{2\pi\times125}{57}$ degrees

$= 13°.778915$

The value of a for the Sun is between 11°.80781 and 12°.31284, based on the eccentricity of the Earth's orbit.

7.1.1 Example 1: Finding the *Mandaphala* of the Sun

The m*andocca* of the Sun = 78° (fixed). Suppose, the Mean Sun = 34° 13′ 42″. Then the *mandakendra* of the Sun (MK)
= *mandocca* – Mean Sun = 78° – 34° 13′ 42″
= 43° 46′ 18″

Since 0 < MK = 43° 46′ 18″ < 90^0, the BMK = 43° 46′ 18″. Therefore, the *mandaphala* of the Sun

$$= \frac{\left[\left(20-\frac{BMK}{9}\right)\frac{BMK}{9}\right]}{57-\left\{\left(20-\frac{BMK}{9}\right)\frac{BMK}{9}\right\}} = \frac{73.616558}{48.820382}$$

$$= 1°.5079062 = 1°\ 30'\ 28''.$$

The *mandaphala* is additive because 0 < MK = 43° 46′ 18″ < 180°.

Note that according to the formula, using the *sine* function, we have the *mandaphala* of the Sun = $\frac{a}{R}\sin m$ (where $a = 14°, R = 360°, m = 43°46'18''$)

$$= \frac{14}{360}\sin(43°46'18'') = 0.0269027\ radians = 1°\ 32'\ 29''.$$

We note that the *mandaphala* of the Sun, without the *sine*-function, is very close to that obtained using the trigonometric function. The error is just 2′.

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7.1.2 Example 2: : Finding the True Daily Motion of the Moon

To find true daily motion of the Moon, a correction called the *gatiphalam* is applied to the mean daily motion. According GL, the expression for the *gatiphalam*(Δn) is given by

$\left[\left(11-\frac{koti}{20}\right)\frac{koti}{20}\right]\left(2+\frac{2}{6}\right)$ where

koti = 90° – BMK.

The modern formula reads:

$\Delta n = -\frac{b}{R}cosM\left[\frac{dM}{dt}-\frac{dA}{dt}\right]$, where $b =$ 31° circumference of Moon′sepicycle, $R = 360°$
$\Delta n = 28'44''46'''$

Suppose for a given day, the MK of the Moon = $115°\ 12'\ 17''$
The BMK = 180° – MK = $64°\ 47'43''$
Koti of MK = 90° – BMK = $25°\ 12'17''$

The *Gatiphalam*, using the GL formula = $28'38''2'''$. Using the modern formula the value is $28'44''46'''$.

We note that the case of the Moon, the difference between the values of Δn according to the GL and the formula based on trigonometry is only about $7''$.

7.1.3 Rationale for the *Gatiphala* of the Moon

The radius of the bigger circle is considered as 3438 whereas the radius of smaller circle is taken as 120.

The ratio of these $\frac{3438}{120} = \frac{382\times9}{30\times4} = \frac{191\times9}{15\times2\times2} \approx \frac{13\times9}{2\times2} = \frac{13}{2}\times\frac{9}{2} = \left(\frac{22-9}{2}\right)\times\frac{9}{2} = \left(11-\frac{9}{2}\right)\times\frac{9}{2}$

Since the maximum value of the centre (*parama kendra kotyāmśa*) = 90, we write the above ratio as

$$\left(11-\frac{9}{2}\right)\times\frac{9}{2} = \left(11-\frac{90}{20}\right)\times\frac{90}{20}$$
$$= \left(11-\frac{koti}{20}\right)\times\frac{koti}{20}$$

According to the GL *paramagatiphala* (Maximum motion) of the Moon across the radius = $65\ |15 = \frac{273}{4}$

Therefore, by proportion, the required *gatiphalam* of the Moon $= \frac{\left(11-\frac{koti}{20}\right)\times\frac{koti}{20}}{13\times\frac{9}{4}}\times\frac{273}{4}$

$$= \left(11-\frac{koti}{20}\right)\times\frac{koti}{20}\times\frac{21}{9}$$
$$= \left[\left(11-\frac{koti}{20}\right)\times\frac{koti}{20}\right]\left(2+\frac{2}{6}\right)$$

8 THE APPROXIMATION FOR THE MOON'S LATITUDE IN LUNAR ECLIPSE CALCULATIONS

In spite of the approximations introduced in the GL, doing away with trigonometric ratios, the predictions of eclipses are fairly reliable on account of improved and updated values of the related parameters. One of the important parameters required is the latitude of the Moon, which is called the *candra śara* in Classical Indian texts.

According to the GL, the formula for the latitude of the Moon is $\frac{11}{7}(M-R)$ where *M* and *R* are respectively the true longitudes of the Moon and Rāhu (the ascending node of the Moon) and (M – R) considered here is the *bhuja* (not exceeding 90°) of the difference.

Note that the approximate formula for *śara*

follows from 120 $\sin\theta \approx \frac{72}{35}\theta$ (θ in degree) when θ is small, as given by Gaṇeśa (see GL, Chapter 4, verse 22). In this chapter while finding *bhuja* using different method he takes *trijyā*(R) = 573 and 573 $sin1^0$ = 10 (approximately). Therefore, $120 \sin 1^0 = \frac{10\times120}{573} \approx \frac{72}{35}$. This leads to $120sin\theta \approx \frac{72}{35}\theta$, when θ is small.

According to the traditional texts the Moon's latitude (*śara*) is given by

$\beta = 270'$ $\sin(M-R) \approx 270 \times \frac{72}{120\times35}(M-R)$ $kalās\ (minutes\ of\ arc)$

$\beta \approx \frac{162}{35}(M-R)\ kalās.$

Dividing the above result by 3, we get

$\beta \approx \frac{54}{35}(M-R) \approx \frac{11}{7}(M-R)\ aṅgulas.$

(1 $aṅgula$ = 3 minutes of arc).

The approximations in this case are justified since under the possible circumstances of an eclipse the argument (M – R) is indeed small. The text GL takes the maximum latitude of the Moon as 270′ whereas the modern value is 308′.

9 CONCLUDING REMARKS

In this paper we have highlighted the special features of the *Grahalāghava* and presented the computational procedures and accuracy of methods in calculating planetary positions and eclipses after simplifying the procedures, according to the GL by explaining them with illustrative examples. We observe that the simplifications made in the mathematical procedures by dropping the trigonometric functions did not seriously affect the accuracy of the results. This is because Gaṇeśa used fairly good and mathematically justifiable algebraic approximation, which is apt in the different circumstances. The sine values of intermediate angles were also approximated by linear interpolation. Therefore, results were comparable to those obtained from other astronomical texts and also modern ones.

Finally, we should not that apart from his astronomical works, Ganeśa Daivajña also contributed significantly to the field of mathematics. To mention just a few areas of interest:

(1) The value of π by revising the dimensions of the polygon inscribed in a circle.
(2) Construction of rational quadrilaterals.
(3) Rationale for four sides to form a trapezium.
(4) Geometrical proof of Pythagoras' theorem.
(5) Gaṇeśa's commentary on the *Līlāvatī* called *Buddhivilāsnī* is also an extremely useful text to help us understand the rationale for the formulae and methods used by Bhāskara II and his predecessors.

However, these mathematical aspects are not discussed here since they lie outside the scope of the present paper.

10 BIBLIOGRAPHY

Dikshit, S.B., 1969. *English Translation of "Bharatiya Jyotish Sastra", Part 1*. Delhi, Controller of Publications.

Dikshit, S B., 1981. *English Translation of "Bharatiya Jyotish Sastra", Part 2*. Delhi. Controller of Publications.

Ganeśa Daivajña, 1946. *Grahalāghavam – with Sanskrit commentary of Viśvanātha Daivajña and Madhuri, Sanskrit–Hindi commentary by Yugeśvara Śāstri*. Banaras, Chowkhamba Sanskrit Series Office.

Ganeśa Daivajña, 1946. *Grahalāghavam – with Sanskrit commentary of Mallāri and Viśvanātha and Hindi commentary by Kedaranāth Joshi*. Varanasi, Chowkhamba Sanskrit Series Office.

Montelle, B., and Plofker, K., 2018. *Sanskrit Astronomical Tables*. Cham (Switzerland), Springer (Sources and Studies in the History of Mathematics and Physical Sciences).

Ôhashi, Y., 1998. The cylindrical sundial in India. *Indian Journal of History of Science*, 33(4), S147–S205.

Pingree, D., 1981. *Jyotiḥśāstra – Astral and Mathematical Literature. Volume 4*. Wiesbaden, Otto Harrassowitz.

Rao, S. Balachandra, Uma, S.K., and Venugopal, P., 2003. Lunar eclipse computation in Indian astronomy – with special reference to Grahalāghavam. *Indian Journal of History of Science*, 38(3), 255–271.

Rao, S. Balachandra, and Uma, S.K., 2006. Grahalāghavam of Ganeśa Daivajña. English exposition, mathematical notes etc. *Indian Journal of History of Science*, 41(1) to 41(4), S1–S411.

Rao, S. Balachandra, Uma, S.K., and Venugopal, P., 2009. Grahalāghavam – special features and efficacy of procedures. In Narliker, J.N. (ed.), *Science in India*. New Delhi, PHISPC-Viva Books. Pp. 197–220.

Rao, S. Balachandra, 2016. *Indian Astronomy – Concepts and Procedures*. Bengaluru, M.P. Birla Institute of Management.

Rupa, K., Venugopal, P. and Balachandra Rao, S., 2013. An analysis of the Mandaphala table of Makaranda and revision of parameters. *Ganita Bharatī*, 35(1-2), 221–240.

Sharma, S.R., 2010, Historical Note: Nadigrama of Ganeśa Daivajña. *Indian Journal of History of Science*, 45(4), 569–574.

Uma, S.K., and Rao, S. Balachandra, 2018. Ahargana in Makarandasarini and other Indian astronomical texts. *Indian Journal of History of Science*, 53(1), 16–32.



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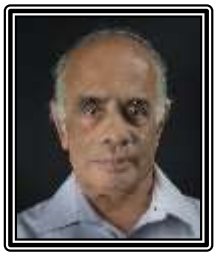
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