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A SURVEY OF ABU ʾL-WAFĀʾ'S SOLAR AND STELLAR OBSERVATIONS

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Abstract: The observational legacy of Abu ʾl-Wafāʾ al-Būzjānī, one of the tenth-century Islamic precursors in forming and developing trigonometry and spherical astronomy in the same form as we know today, is investigated in this paper. The relevant data have been, partly, presented in his *Majisṭī* and, partly, preserved by his younger contemporary and correspondent, Abū al-Rayḥan al-Bīrūnī. Of five surviving stellar observations, his ecliptical or polar longitudes for Capella, Deneb, Mizar, and Regulus are accurate to within –⅙°, but there is an error of –½° in the case of Altair. It is shown that his value of 1°/75ʸ for the rate of precession is the result of his comparison of his own value for the longitude of Capella in the 970s with the corresponding value that can be considered as the star's longitude in the *Almagest* for Timocharis' time.

He established an accurate solar theory on the basis of a series of observations made about the equinoxes and Summer solstice in 974 C.E. as well as at mid-Winter, mid-Spring and mid-Summer in 976 C.E. His times of the first three events were accurate to within –1⅙ hours, and the errors in the successive intervals of time between the last three events amounted to +1½ hours. As a result of this remarkable degree of accuracy, the errors in the longitude of the Sun never exceeded the narrow range of –1′ to +4′ at the time. A systematic negative error is detected in the solar noon-altitudes with a mean of about –5′, which, in combination with a positive error of the same size in the meridian altitude of Capella, supports the hypothesis that either there was a northward tilt in the set-up of his meridional ring, or the observing site had a northward slope.

Keywords: Islamic astronomy, medieval astronomy, Abu ʾl-Wafāʾ, Capella, precession

1 INTRODUCTION

This paper deals with a survey and assessment of the observational activities performed by Abu ʾl-Wafāʾ al-Būzjānī (10 June 940–after 25 May 997; Dold-Samplonius, 2016; Youschke-vitch, 1960; *BEA*, pp. 188–189; Suter's entry in *EI*$_2$, Vol. 1, p. 159), a Persian mathematician and astronomer active in Baghdad (Kennedy, 1984; Kennedy and Mawaldi, 1979; Saidan, 1974; Sesiano, 1998; Raynaud, 2012). He was one of the precursors in laying down the corner-stone of trigonometry and spherical astronomy in the same form as we know it today. The main bulk of his astronomy has come down to us, though incompletely, in his work titled *Majisṭī* (without the Arabic definite article 'al-'). The work is extant in a unique and incomplete manuscript in the Bibliothèque Nationale in Paris (Arabe 2494).

The *Majisṭī* is divided into seven 'books' (*maqāla*), each into sections called 'ype/ sort/ kind' (*nawʿ*), some of which are, in turn, divided into subsections called 'chapters' (*faṣl*). The names of the first two sections are borrowed from the Arabic *Almagest* in both Ḥajjāj and Isḥāq-Thābit's translations. Besides, our author introduces the three main Parts of his work in the end of I.1 and I.2, each called 'general class/sort' (*jins*):

> … each main Part includes a number of books and chapters. In the first Part, we speak of the topics which must be explained before dealing with the motions of the heavenly bodies, which is in five books. The second Part is on the motions of the celestial objects, which are called the motion in longitude and the motion in anomaly. The third part is on the issues which are closely related to the motions of the heavenly objects. (Abu ʾl-Wafāʾ, *Majisṭī*, f. 2r).

He does not specify how many books the last two parts are comprised of; however, it seems that each one consists of a single book; actually, the subject matter of the second and third Parts agree, respectively, with the contents of books VI and VII.

Book I is on planar trigonometry, and books II–V are, generally, on spherical trigonometry and spherical astronomy; in particular, he gives us some information on the instruments he used in his observations and his measurement of the obliquity of the ecliptic, the latitude of Baghdad (II.2.1; see below, Sect. 2.1) and the ecliptical and celestial coordinates of some fixed stars (see Sect. 3.1). Book VI is, on the whole, devoted to the basic hypotheses employed for the construction of the Ptolemaic planetary models, as the list of its contents given in detail on ff. 81v–82r denotes. The first five sections (VI.1 to VI.5) deal with the general topics and the discussion on the various hypotheses deployed in the Ptolemaic cinematic models in order to account for the planets' motions in longitude and in anomaly (i.e., the eccentric deferent and the epicycle). VI.6.2, which

is the last chapter on the Sun, has left unfinished on f. 93v, and the next leaf starts from somewhere towards the end of VI.9.2. So, VI.7 (on the Moon, including nine chapters), VI.8 (on Venus, consisting of three chapters), the entire of VI.9.1, and apparently a good deal of the initial part of VI.9.2 are absent from the only surviving Paris MS (the first three chapters of VI.9 are on Mercury, but its fourth chapter, which is the last chapter of Book VI, is a brief summary of the types of the planetary orbs, motions, and inequalities, and the sizes of their eccentricities, epicycles' radii, and orbital inclinations). Book VII discusses at length on the ways through which the inequalities in the tropical and anomalistic motions of the Sun, Moon, and five planets were discovered and on how they are accounted for by means of the hypotheses explained in book VI.

It comes as a matter of surprise that our author's famous contemporary biographer, al-Nadīm, does not mention the *Majisṭī* among his works. Rather, he refers to two other works of his, titled the *Kitāb al-kāmil* (*Complete/Perfect book*) and the *Wāḍiḥ zīj* (*Clarifying zīj*), and, surprisingly, gives the same list of contents for both, which happen to coincide with our au-thor's three main Parts of the *Majisṭī* mentioned above (al-Nadīm, ed. Flügel, Volume 1: 283, ed. Tajaddod, page 341, Eng. translation, Vol-ume 2: 667–668). Accordingly, it seems that all three titles refer to one and the same work: seemingly, called a *zīj* for his astronomical tables, named a complete/perfect work for its comprehensibility, and titled a *Majisṭī*, a simple literal designation indicating that the work in question resembles Ptolemy's *al-Majisṭī* (*Great Syntaxis*) in genre, topics and volume, among other aspects.

Apart from the incomplete copy at our disposal, Abu ʾl-Wafāʾ's empirical findings have been preserved in the works of his outstanding contemporary junior correspondent, Abū al-Rayḥān al-Bīrūnī's (973–1048 C.E.) *al-Qānūn al-masʿūdī* and *Taḥdīd*. He tells us that Abu ʾl-Wafāʾ carried out his observations at *Bāb al-Tibn* (the Straw Gate) in Baghdad. He prearranged with Abu ʾl-Wafāʾ to measure the time of the maximum totality of a lunar eclipse, as the first was based in Khwārizm and the latter was living in Baghdad at the time, and then to derive the difference in terrestrial longitude between the two locations. The lunar eclipse in question was the one that occurred in 387 H. (≈ 997 C.E., NB. the year 387 H. started on 13 January 997, according to the Islamic astronomical calendar, or on the day after it, according to the Islamic civil calendar (Bīrūnī, *Taḥdīd*: 250, English translation, pages 214–215, E.S. Kennedy's commentary, page 164). Of the two lunar eclipses that occurred in that year, on 25 May and 17 November, only the first (a partial eclipse of magnitude of 0.55) was visible at both localities. Of course, in the region of Khwārizm, the Moon set before the end of the eclipse. A minor part of the trigonometrical and spherical astronomy tables of the *Majisṭī* has been preserved in a *zīj* written by a certain al-Baghdādī towards the end of the thirteenth century (van Dalen, 1993: 70–76, 144, 158, 170–172).[1]

In the modern scholarship on the historiography of medieval astronomy, Abu ʾl-Wafāʾ's *Majisṭī* is very often remembered in association with the misinterpretation of VII.2.10 (Abu ʾl-Wafāʾ, *Majisṭī*, ff. 99v–100r) by Louis Pierre Eugéne Amélie Sédillot (1808–1875, the son of the French orientalist Jean Jacques Emmanuel Sédillot, 1777–1832) (Sédillot, 1845–1849(1): 42ff), which gave birth to the blatantly erroneous idea about the discovery of the lunar variation by Abu ʾl-Wafāʾ. The controversy took four decades to finally be resolved by Carra de Vaux (1892).[2]

In the following sections, we aim to specifically address two main issues. Firstly, how accurate were Abu ʾl-Wafāʾ's solar and stellar observations? Although the available data are limited, when scrutinized technically they can throw some light on the degree of competence of the observer. Secondly, how did Abu ʾl-Wafāʾ attain his value for the rate of precession? Furthermore, we will pinpoint the dates of his stellar observations, and present a working hypothesis about a structural defect in his meridional instrument.

Recently an edition of the *Majisṭī* has been published (Moussa, 2010), and a part of it on spherical astronomy has been studied (e.g., Delambre, 1819: 156–170; King, 2004–2005(1): 113; Moussa, 2011), but the subject matter of this paper has remained unexplored to date. Thanks to the marvellous digital era, the unique copy of the *Majisṭī* is accessible online, so references are made to the manuscript rather than to a printed transcript.

2 THE SUN AND THE LATITUDE OF BAGHDAD

2.1 Instruments and Observations

In II.2.1&2, Abu ʾl-Wafāʾ (*Majisṭī*, ff. 19r–20r) mentions three instruments that were used to measure the meridian-altitude of the Sun and stars. The first two are basically the classic meridional instruments described in the *Almagest* I.12 (Toomer, [1984] 1998: 61–63): a mural quadrant drawn on a plaque installed facing the southern side of the local horizon, and a solid

ring; the only difference is that both, we are told, could be equipped with a sighting device, consisting of a ruler with two perforated pinnulas, known as an alidade. The third is a simple conic-shaped gnomon. Since his preserved quantitative data are related to heavenly objects passing through both the northern and southern regions of the meridian of Baghdad, he should have used the meridional ring and/or had 'two' mural quadrants (the second facing the north); a systematic error committed by our author, which will be discussed later, gives the strong impression that the former was the case. Judging from his extant observational records, the scale of the instrument(s) appears to have been graduated to every minute of arc. Accordingly, its radius should have been about 3.5 m (around the same size as the southern solid mural quadrant built at Maragha Observatory some 300 years later—Mozaffari, 2018a: 616–617), if not larger (e.g., the Northern and Southern meridional quadrants drawn on a wall at Istanbul Observatory in the 1570s were of about 7.5-m radius; Mozaffari and Steele, 2015: 353; Mozaffari, 2018b: 235).

Abu ʾl-Wafāʾ remarks that, 'for successive years', he measured the solar noon-altitudes about the solstices in order to determine the basic parameters, the obliquity of the ecliptic ε and the latitude φ of Baghdad. The extremal annual noon altitudes of the Sun at the Summer and Winter solstice were equal, respectively, to 80;10° and 33;0°, from which he derived ε = 23;35° and φ = 33;25° (in II.2.3; Abu ʾl-Wafāʾ *Majisṭī*, f. 20r). Note that throughout this paper, besides the ordinary decimal numbers we express the historical values in the standard sexagesimal notation, in which a semicolon is used for the separation of the integer and fractional parts, and a colon for separating the digits; e.g., *a*,*b*;*c*,*d* stands for $a \times 60^2 + b \times 60^1 + c \times 60^{-1} + d \times 60^{-2}$.

The data obtained from his observations for the purpose of determining the Sun's/Earth's orbital elements have been preserved by Bīrūnī (see below). The only reference to this in the extant copy of the *Majisṭī* is found in VII.1:

> … by the aforesaid instruments, we observed the Sun when it was in the vicinity of the two-third of [the zodiacal sign] Gemini [viz. at a longitude of about 80°, near the apogee] and of Virgo and Pisces [i.e., at the orbital quadratures] … and found that its true motion in longitude differs from its mean longitudinal motion by approximately 2° in the time intervals between [every] two [consecutive] observations (Abu ʾl-Wafāʾ, *Majisṭī*, f. 97r).

2.2 Assessment of Observations and Solar Theory

The geocentric altitudes of the Sun (i.e., measured by a fictitious observer locating at the Earth's centre) at a specific location in the Northern Hemisphere at the Summer and Winter solstices are, respectively, $h_{max} = 90° - \varphi + \varepsilon$ and $h_{min} = 90° - \varphi - \varepsilon$. The change in the solar declination, and therefore in its noon-altitude is practically zero on the days around the two Solstices. As observed from the Earth's surface, the apparent altitudes, h', are affected by atmospheric refraction, R, and the solar parallax, π: $h' = h + R - \pi$. The solar parallax is, in actuality, so small as to be negligible, but in Ptolemaic astronomy and the Medieval traditions dependent upon it, it was quite large, because the solar distance from the Earth was taken as about 1210 terrestrial radii, or thereabouts, which made the Sun's horizontal parallax amount to about 0;2,51°. We can assume that a typical Medieval astronomer took such erroneous values for the solar parallax (shown as π'') into account in order to neutralize the parallax's effect. His measurement might also contain an error dh. Thus, $h'' = h' + \pi'' + dh$. Then

$$\varepsilon'' = \varepsilon + \tfrac{1}{2}(R_{ss} - R_{ws}) + \tfrac{1}{2}(\pi''_{ss} - \pi''_{ws}) + \tfrac{1}{2}(dh_{ss} - dh_{ws}) \quad (1a)$$

and

$$\varphi'' = \varphi - \tfrac{1}{2}(R_{ss} + R_{ws}) - \tfrac{1}{2}(\pi''_{ss} + \pi''_{ws}) - \tfrac{1}{2}(dh_{ss} + dh_{ws}) \quad (1b)$$

The subscript 'ss' and 'ws' indicate, respectively, the Summer and Winter solstices. Note that the refraction and parallax terms in both equations are negative; R_1 is so small as to be negligible (~10″) in Baghdad (and, generally, for many places in the Islamic domain), but $R_2 \approx 1.5'$. This obviously shows that even though the noon-altitudes measured by a Medieval astronomer were completely error-free, and also that he did not consider the erroneous parallax theory at his disposal, the values measured for the obliquity of the ecliptic and the latitude of place should have been below the real values by about 1′ for most of localities in the Middle East

If the latitude of a place is precisely known (as at Maragha or Samarqand Observatories), the errors dh can be computed directly, but, under the condition that it is not known with a good degree of certitude. As was the case with Baghdad Observatory of the 970s, only the limits of the errors could be demarcated. Nonetheless, the round-shape city of *The 1001 Nights* appears to have been centred about φ = 33;20°, and was not spread beyond the range of 33;17° – 33;23°, so that a tolerance of

±3′ should be considered (note that every arc-minute of difference in terrestrial latitude corresponds to a distance of about 1.85 km on the Earth's surface).

There is no evidence that our author took into account the parallax, as he just presents his observed extremal noon-altitude values, and immediately derives ε and φ. At the time

$\varepsilon = \varepsilon_0$ (= the mean obliquity of the ecliptic) + nutation = 23;34,19° – 8″ = 23;34,11° ≈ 23;34°,

while our author has ε'' = 23;35°. As a result, his solar noon-altitudes certainly contain some errors, as $dh_1 - dh_2 \approx 3'$. The errors could not be less than $dh_1 \approx -1⅓'$ and $dh_2 \approx -4⅓'$ if φ = 33;23°, and could not exceed –7⅓′ and –10⅓′ if φ = 33;17°; for φ = 33;20°: –4⅓′ and –7⅓′.

An initial set of observations for the purpose of determining the basic parameters ε and φ seems to have been done at a primary stage of his observing program in 343 Y. (363–364 H./974 C.E.).[3] At the time, he carried out his first round of solar observations[4] for the purpose of determining the solar orbital elements by the seasons method (see Mozaffari, 2013(1): 317–320), of which we have information only about the values he measured for the lengths of the seasons and the time of occurrence of the Autumnal equinox through the medium of Bīrūnī (*Taḥdīd*: 301, English translation, p. 270, E.S. Kennedy's commentary, p. 228; *al-Qānūn*, VI.6, 1954–1956(2): 640):

3 hours after the beginning of sunrise on Friday, 30 Pachon [9th month] 1722 Nabonassar [= 18 September 974, modern: 9:18 MLT].

In order to determine the moment that the Sun arrived at a specific longitude (e.g., λ = 0° for the vernal equinox), a Medieval astronomer ought to have continuously measured its meridian altitude on successive days around the event, and thus, the wanted time could be determined by means of interpolating (usually, linearly) between the two closest values intercepting the critical noon altitude $h_c = 90° - \varphi + \delta(\varepsilon, \lambda)$. Let the two observed altitudes be $h_c - \chi_1$ and $h_c + \chi_2$ if the altitude on the first day is less than that on the second day (let us call this as the 'ascending situation', which is the case, e.g., with vernal equinoxes), or $h_c + \chi_1$ and $h_c - \chi_2$ if the opposite is correct (the 'descending situation', which is the case, e.g., with Autumnal equinoxes). Here, $\chi_1 + \chi_2 = \Delta\delta(\varepsilon, \lambda)$ is the difference in the solar declination, or in noon-altitude, between the two consecutive days

$T = \chi_1 / \Delta\delta$ of a day after true noon on the first day, and
$\chi_2 / \Delta\delta$ of a day before true noon on the second day.

Thus, the probable errors will amount to

$$dT = (d\delta - d\varphi - dh - R) / \Delta\delta \quad (2a)$$

of a day in an ascending situation, and

$$dT = (d\varphi + dh + R - d\delta) / \Delta\delta \quad (2b)$$

of a day in an ascending situation of a day in a descending situation, in which $d\delta \approx d\varepsilon \sin\lambda$. Note that since any Medieval astronomer was very well aware of the fact that $\Delta\delta$ cannot surpass an already known limit ($\Delta\delta$ = 24′ for the equinoxes), the errors inevitably became of a systematic nature. At the equinoxes ($d\delta$ = 0) in Baghdad: $R_{eq} \approx 40''$. This simply means that the refraction pushes forth the time of the Spring equinox by about ⅔ of an hour, but makes a delay in the time of the Autumnal equinox by the same amount. However, under ideal circumstances (i.e., dh = 0), its effect would have been considerably neutralized by $d\varphi = -\frac{1}{2}(R_{ss} + R_{ws}) \approx -50''$.

It is rather obvious that, on 18 September 974, our author measured the noon-altitude of the Sun by χ_2 = –3′ below his critical altitude h_c = 90° – φ'' = 56;35°, i.e., at h = 56;32°, whose error is $dh_{ae} \approx -6'$. The surprising interaction of errors causes the measured time of the Autumnal equinox to have a small error $d\varphi + dh + R \approx +5 - 6 + ⅔ \approx -⅓$ of an hour.

He measured the lengths of the Spring and Summer seasons in 343 Y. (363 H./974 C.E.), respectively, as

93;30,8 and 93;7,10 days (3)

(Bīrūnī, *al-Qānūn* VI.7: 1954–1956(2): 654–655; Mozaffari, 2013(1): 322; note that that Persian year started on 23 March 974, and thus, the Spring equinox, occurring on 15 March, was in advance of it by a week), which are about

+57 and –9 minutes of time

in error, since the times of the two vernal equinoxes and the Summer solstice in between were as listed in Table 1. Consequently, it is evident that his time of the Summer Solstice of 974 C.E. had a trivial error of about –⅙ of an hour, which is remarkably accurate, for example

Table 1: Dates of the Equinoxes and the Summer Solstice in Baghdad.

	Baghdad MLT		JDN
Spring equinox	15 March	19:10	2076885.1755
Summer solstice	17 July	06:17	2076978.6382
Fall equinox	18 September	09:18	2077071.7640

Table 2: Dates of the mid-Winter, mid-Spring and mid-Summer in Baghdad.

	Baghdad MLT		JDN
Mid-Winter	30 January	02:34	2077570.4836
Mid-Spring	30 April	14:53	2077661.9968
Mid-Summer	02 August	18:21	2077756.1413

in comparison with the strikingly erroneous time his colleague, Abū Sahl al-Kūhī, measured in Baghdad for the Summer solstice of six years earlier: the first hour of the night whose morning was Saturday, 16 Ḥazīrān (the 9th month) 1299 Alexander (= 16 June 988; JDN 2082092) (see *al-Qānūn* VI.6 (1954–1956(2): 642–643; F*: f. 96r, B*: ff. 111r–v, C*: f. 111r, I*: f. 173r, K: p. 280, L*: f. 115r, R: p. 247). Bīrūnī correctly detected, with the aid of his solar theory, that this figure should have been more than –19 hours in error, and discussed it at some length, in connection with an assessment of Mumtahan/Ḥabash's solar theory.

In a similar fashion, it is a simple matter to deduce that the error in Abu ʾl-Wafāʾ's time of the Spring equinox of 974 C.E. nearly amounts to $-1^{1}/_{6}$ hours. From (2), we find that he committed an error of $dh_{ve} \approx -4½'$. According to Bīrūnī (*al-Qānūn* VI.7: 1954–1956(2): 658; Mozaffari, 2013(1): 326), he established his final solar theory two years later, in 345 Y. (365 H./976 C.E.), by means of the mid-seasons method (see Mozaffari, 2013(1): 320–323). He measured the interval of time between the mid-Winter and mid-Spring, and from the latter to mid-Summer, which respectively yielded

91;34,25 and 94;9,7,30 days (4)

(Note, again, that 345 Y. began on 22 March 976, and hence, it will be more proper to say that the observations about the mid-Winter were performed towards the end of 344 Y.) These values are about

+87 and + 11 minutes of time

in error—the occurrence times of the mid-seasons are listed in Table 2.

At mid-seasons, $\Delta\delta = \pm 17.5'$ and $d\delta = \pm ½'$. Refraction in Baghdad at mid-Springs and mid-Summers amounts to about ⅓′ and at mid-Autumns and mid-Winters reaches ~$1^{1}/_{6}'$. From these, according to (7) and (8), it resulted in $dh_{m\text{-}w} - dh_{m\text{-}sp} \approx -1'$ and $dh_{m\text{-}su} + dh_{m\text{-}sp} \approx -9.5'$. The conclusive results are as follows. The negative systematic error we have found earlier in the noon-altitudes at the solstices and equinoxes is also, indeed, the case with the mid-seasons. Furthermore, the sum of the errors at mid-Spring and mid-Summer has a lesser size than the sum of the errors at the equinoxes ($dh_{ve} + dh_{ae} \approx -4.5' - 6' = -10.5'$), and also, the error at the mid-Winter is greater than the error pression that the negative error smoothly and gradually increases from the greatest annual noon-altitude at the Summer solstice to its minimum value at the Winter solstice. This, in combination with the error Abu ʾl-Wafāʾ committed in the meridian altitude of Capella, fuels speculation about probable discrepancies in the set-up of his meridional instrument(s). A working hypothesis in this respect will be presented later (see below, Section 3.4).

Our author derives two sets of values for the solar orbital elements, the first set results in the eccentricity $e \approx 2;4½^{p}$ and the longitude of the apogee $\lambda_A \approx 84½°$, and from the second set, $e = 2;5^{p}$ and $\lambda_A \approx 84^{2}/_{3}°$. As shown elsewhere, the second value for e shows an improvement over the first one as compared to the corresponding theoretical values expected to derive by the application of the two methods deployed, whereas the second value for λ_A can be considered an improvement over the first one in comparison with the corresponding modern value at the time (Mozaffari, 2013(2): Tables 4 and 5).

We do not know about the value he derived for the Sun's mean motion/the length of the solar year, and thus, it is not known whether he established a solar theory of Type I or of Type II. As discussed in depth elsewhere (Mozaffari, 2018b), the medieval solar theories can be categorized distinctly in two groups, depending on whether an astronomer employed Ptolemy's egregiously erroneous equinox times (with errors $\gtrsim$ +1 day) (Type I) or not (Type II). Under the first condition, the length of the solar year will be inevitably lesser, and so the Sun's mean motion will become larger, which, in turn, causes striking positive systematic errors in the solar longitude after passing a good span of time.

Abu ʾl-Wafāʾ's junior correspondent, Bīrūnī, constructed a solar theory of Type I, whereas his elder contemporary, Ibn al-Aʿlam (d. 985 C.E.), appears to be the first Medieval astronomer who had success in laying down a stable solar theory of Type II (note that this, also, means he believed or recognized that Ptolemy's solar theory could not be correct in Ptolemy's own time). Nevertheless, on account of Abu ʾl-Wafāʾ's remarkably accurate derivation of the time of the Autumnal equinox of 974 C.E., his solar theory should not have been affected too much by any probable errors in the mean motion/epoch position in the 970s. The errors in the solar longitude during a solar year resulting from the errors in both sets of the orbital el-

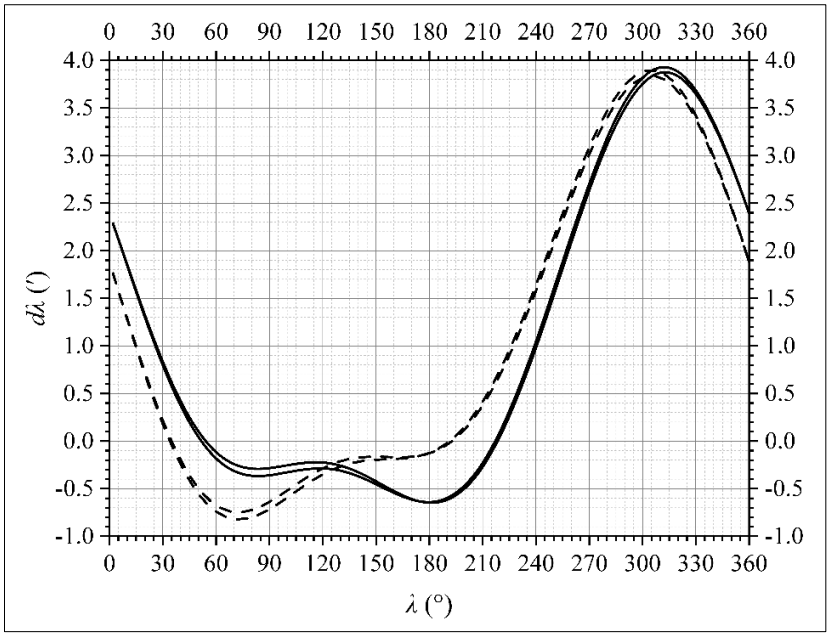


Figure 1: The errors in Abu ʾl-Wafāʾ's solar theory, as plotted against the true longitude for the beginning and end of the 970s. The dash graphs are related his first theory, and the continuous graphs pertain to his finalized theory (plot: Mohammad Mozaffari).

ements mentioned above, together with making allowance for a small upward shift of 1′ due to the trivial error of –⅓ of an hour in the time of the autumnal equinox of 974 C.E., are presented in pictorial form in Figure 1. They do not surpass a small range of about –1′ to +4′. The attained degree of accuracy was a crucial, decisive contributing factor in his success of measuring remarkably accurate values for stellar longitudes.

3 STARS

3.1 Observations

The most important aspect of Abu ʾl-Wafāʾ' *Majisṭī* is the preservation of his measured values for the stellar coordinates in book V, which chiefly deals with the transformation of coordinates between various celestial systems (horizontal, equatorial, and ecliptical) and time-keeping. The information about his stellar observations can be found among his worked examples. They are few in number. We have been provided with data for four stars: Capella (Ar. *al-ʿAyyūq*, α Aur), Deneb (Ar. *al-Ridf*, α Cyg), Altair (Ar. *al-Nasr al-ṭāʾir*, α Aql), and Mizar (Ar. *Rafīq al-suhāʾ*, ζ UMa, i.e., ''the *Suhāʾ*''s companion', where *Suhāʾ* is the Arabic name of Alcor, 80 Uma—on the star names, names, see Mozaffari, 2016–2017: 95).

Capella appears to have been our author's favourite star, as most of the worked examples throughout book V are related to it. In V.1.2 (Abu ʾl-Wafāʾ, *Majisṭī*, f. 68v), we are told that the measurement of Capella's ecliptic coordinates was performed in 365–366 H. (9/10 September 975–17/18 August 977): β = +22;50° and λ = 67;35°. In V.5 (Abu ʾl-Wafāʾ, *Majisṭī*, f. 75v), he says he measured Capella's declination δ = +44;9° and the transit (or mid-heaven) degree (i.e., the polar longitude; the longitude of the point on the ecliptic which transits the local meridian at the same time as a given celestial object does) λ_M = (62 + ½ + ¼)° (= 62;45°); in V.3.4 (Abu ʾl-Wafāʾ, *Majisṭī*, f. 75r), he derives *RA* = 60;40,22…° and λ_M = 62;45,25…° from the ecliptical longitudes) in 366 H. (28/29 August 976–17/18 August 977).

In V.4 (Abu ʾl-Wafāʾ, *Majisṭī*, ff. 75r–v), we have been supplied with something of Deneb and Altair. On Deneb, we are told: when the Sun was at the beginning of the zodiacal sign Scorpius (i.e., λ = 210°), we found the difference in time between the sunset and the culmination of Deneb to correspond to 11;30° of the revolution of the celestial sphere. From this, our author derives the mid-heaven degree as λ_M = 299;**3**7° (recalculated: 299;27°; maybe, a scribal error has occurred because of the similarity between the Arabic alphanumerics for 20/ ٢ and 30/ ٣). On Altair, we are told: when the Sun was at 6° in the zodiacal sign Taurus (i.e., λ = 36°), we found the difference in time between the meridian passage of Altair and sunrise as equal to (9 + ½ + ⅓)° of the revolution of the celestial sphere, from which he derives λ_M = (283 + ¼ + ⅕)° (= 283;2**7**°; recalculated: 283;28°).

Table 3: The longitude of Capella as measured in the Medieval Islamic period.

	Pt*	Mt	AW	IY	IK	ISh	UB
λ	55;00°	65;05°	67;35°	68;41°	71;10°		74;43°
	55;57	**65;32**	**67;36**	**68;22**	**71;09**		**74;00**
$d\lambda$	–57′	–27′	–1′	+19′	+1′		+43′
β	+22;30	+22;50	+22;50	+22;33	+22;40		+22;42
	+22;50	**+22;51**	**+22;51**	**+22;51**	**+22;51**		**+22;51**
$d\beta$	–20′	+1′	+1′	–18	–11′		–11′
RA			60;40			67;40°	
			60;41			**67;34**	
dRA			–1′			+6′	
δ			+44;09			+44;40	
			+44;09			**+45;03**	
$d\delta$			0			–23′	

* Abbreviations and epochs: Pt = Ptolemy (137), Mt = *Mumtaḥan zīj* (829), AW = Abu ʾl-Wafāʾ (976/7), Ibn Yūnus (1032), IK = *Īlkhazī zīj* (1231), ISh = Ibn al-Shāṭir (1363), UB = Ulugh Beg's *Sulṭānī zīj* (1437).

In V.14 (Abu ʾl-Wafāʾ, *Majisṭī*, f. 80v), the ecliptic coordinates of Mizar are given as β = +55;40° (which is Ptolemaic) and λ = 151;10°, from which he measured δ = 59;57,1***3***,***55***° (re-calculated: …1<u>4</u>,<u>21</u>), and, in turn, the extremal altitudes of the star in the upper and lower transits, respectively, about 63;28° and 3;22°.

In addition, we have information about a fifth star: Regulus. There is no mention of it in the unique surviving copy of the *Majisṭī*, but it has been preserved in Bīrūnī's *al-Qānūn al-mas'ūdī* VI.8 (Bīrūnī, 1954–1956(2): 677). The latter says that Abu ʾl-Wafāʾ measured the longitude of Regulus as λ = 135;30° in the year 343 Y. (23 March 974–22 March 975 C.E.), which coincides with the period of his first round of solar observations.

3.2 Accuracy and Comparison

In Table 3, Abu ʾl-Wafāʾ's values for the coordinates of Capella are compared with other important measurements, made by Ptolemy and in the medieval Islamic period, as arranged in a chronological order. All measured coordinates are ecliptical, except for Ibn al-Shāṭir, whose are equatorial.[5] As can be seen obviously, exclusively, in the case of Abu ʾl-Wafāʾ, both latitude and longitude values are excellently accurate to a single minute of arc, and his declination value is error-free. Of course, his latitude value was not unprecedented; it was obtained, apparently for the first time, in the Mumtaḥan observational program. In the case of the longitude, his degree of accuracy was yielded only once again in the observations conducted by the main staff of Maragha Observatory (except for Muḥyī al-Dīn al-Maghribī, d. 1283 C.E., who performed his massive observational program independently), whose results have come down to us through the non-Ptolemaic star table incorporated into the *Īlkhānī zīj*.

In the case of Mizar, Abu ʾl-Wafāʾ's value for the longitude is highly accurate, with a trivial error of –2′, which can be taken as the best degree of accuracy achieved in the Islamic period in measuring this star's longitude, notwithstanding the fact that we only know of two other corresponding values from the Islamic period: Ibn al-Shāṭir's value of 193;20° for the right ascension of the star is about –1;7° in error, and the error in Ulugh Beg's value of 158;4° for its longitude amounts to about +21′. In contrast, our author adopted the Ptolemaic value for the star's latitude, which is about –40′ in error; as a result, his calculated declination is about –28′ off at his time, which, as combined with his error of +5′ in the latitude of Baghdad and taking the atmospheric refraction into account, makes the errors of, respectively, about +33′ and –35′ in his values for the extremal altitudes of the star in the upper and lower culminations. For unknown reason(s), the orthogonal components of the coordinates (i.e., altitude, declination, and latitude) of the circumpolar stars in the Medieval Islamic period suffered from inexplicable large errors. The Banū Mūsā's extremal altitude values of γ, ε, and ζ UMa 862–863 C.E., as reported by Bīrūnī (*Taḥdīd*: 66–67, English translation, pp. 37–38, E.S. Kennedy's commentary, pp. 16–18), have the errors of up to ±⅓° (exclusively for ζ UMa: ±¼°). Ibn al-Shāṭir's values for the declinations of ε and ζ UMa also have the large errors of more than –1° (of course, for the other star in the neighbourhood, i.e., η UMa. his error is quite small: $d\delta$ = +1′). Among the data that have come down to us from the period in question, solely in the star catalogue accommodated in Ulugh Beg's *Sulṭānī zīj*, we can find the coordinates of the five stars in this region of the sky, i.e., γ, δ, ε, ζ, and η UMa, to have been congruently precise ($d\lambda$ < ±1/2° and $d\beta$ ≤ ±1/4°).

Table 4: The derivation of the polar longitudes of Deneb and Altair.

	$\lambda_\odot$	$A\varphi_\odot$		$\Delta t°$	*RA*	λ_M
Deneb	210;0°	200;09°	90° +	11;30°	301;39°	299;27°(1)
		200;11		**11;29**	**301;40**	**299;29**
d		–2′		+1′	–1′	–2′
Altair	36;0	24;28	270 –	9;50	284;38	283;28(2)
		24;31		**9;20**	**285;11**	**283;58**
d		–3′		+30′	–33′	–30′

(1) Text: …,37; (2) Text: 27

For Regulus, Bīrūnī only provides us with Abu ʾl-Wafāʾ's longitude value, which is, again, remarkably accurate, with a small error of about –7′. The other measurements of this star in the Medieval Islamic period, also, reached the same order of error, except for the observations carried out by the Mumtaḥan team, Ibn al-Aʿlam, and Muḥammad b. Aḥmad b. Yūsuf al-Samarqandī (the latter two's measurements are reported by Ibn Yūnus, d. 1009 C.E.), which suffer from the large errors of about –½°.

For the determination of the polar longitudes of Deneb and Altair, the two objects were seemingly observed at the times of their meridian passage, which were measured in relation to the moments of sunrise or sunset. The procedure is straightforward. The right ascension of a point on the celestial equator simultaneously rising or sitting with a heavenly object in a specific geographical location is called its oblique ascension $A\varphi$. The separation between the oblique ascension of the Sun at sunrise or sunset and the right ascension of a star transiting the local meridian comprises a quadrant plus, or minus, the time interval between the two moments, as expressed in the amount of the rotation of the celestial sphere, $\Delta t°$. Knowing the solar longitude $\lambda_\odot$, its oblique ascension $A\varphi_\odot$ can be computed; thus, $RA = A\varphi_\odot + (90° + /270° –) \Delta t°$, from which the polar longitude λ_M is derived (cf. Table 4).

We are not told about the two stars' longitudes, but it is expected our author's value for the longitude of Deneb to be as accurate as those of Capella, Regulus, and Mizar, but his value for Altair's longitude should have been about –½° in error.

3.3 Demarcation of Abu ʾl-Wafāʾ's Times of Observations

Although Abu ʾl-Wafāʾ gives no exact date in association with his stellar observations, it is possible to ascertain some as well as pinpoint others by narrowing the periods in the 970s he declares.

Thus, if the data of Deneb and Altair were related to real observations, throughout the 970s, it was on 18 October in the years 971, 975, and 979 that the Sun was near the longitude of 210° at sunset in the horizon of Baghdad; the middle date falls in the period of our author's observations of Capella. When passing across the meridian at 18:1 MLT, Deneb's altitude was over 81°, and the Sun's depression about –9°. The difference in azimuth between the Sun and Deneb amounted to 96°. Under these circumstances, Deneb, with a visual magnitude of 1.25, could be observed conveniently.

The Sun was located at the longitude of about 36° on 21 April when rising on the local horizon in the years 972, 976, and 980; again, the middle date falls within the period Abu ʾl-Wafāʾ gives for his observations of Capella. When transiting the meridian at 4:43 MLT, Altair's altitude was over 63°, the Sun's depression was about –7°, and their azimuthal difference around 111°. Under these conditions, Altair—with an apparent visual magnitude of 0.77—was completely observable. The visibility conditions have been assessed with the aid of charts and analyses presented by Belokrylov et al. (2011) and Tousey and Koomen (1953).

Note that, on either date, our author had not yet established his second solar theory, and so, it is reasonable to assume that he deployed his first one at the time.

With the Sun taken as the reference body, the most suitable dates for the observation of the meridian transit of Regulus were in May (after sunset) and/or in October (before sunrise) in 974. In May, our author was still in the middle of his first round of solar observations, and his first solar theory was probably ready for use before October.

The proper periods of observations in the case of Capella were March (after sunset) and/or August (before sunrise) in the years 976 and/or 977.

The errors in the solar theory on the aforesaid dates and periods are so small as to be negligible (remaining below ±1′), except for March when they could amount, at the most, to about +3′ (cf. Figure 1).

3.4 Analysis

What was behind the significant accuracy achieved by Abu ʾl-Wafāʾ in measuring the stellar

longitudes and the latitude of Capella should have had a story on its own.

Both of Abu ʾl-Wafāʾ's dated pieces of information on Capella, mentioned in 3.1, may be taken as 'observation reports'. Thus, it seems *prima facie* suspicious what he did in reality: whether he first measured the ecliptical coordinates with the armillary spheres, and then transformed them into equatorial ones, or the reverse was true. Since he describes only the meridional instruments, and there is no mention of an armillary instrument of any kind in his work it is most reasonable to think that the latter was the case—in fact, the only justified option. The entire contents of his *Majisṭī* can be testimony to his use of the method of spherical astronomy, originated from the *Almagest* (esp., VIII.5; see, e.g., Marx, 2020: 24–30, for an elucidation) and developed in his *magnum opus* for the same purpose. However, he did not attempt to establish a straightforward link between his observational data and measurements, and they are scattered throughout his worked examples.

The method was in practical use in the Medieval Middle East long before Abu ʾl-Wafāʾ (a notable example is al-Battānī's explicit mention of it in his stellar observations), but he significantly expanded upon it and substantially contributed to establishing it firmly through his comprehensive explanations and clear numerical examples. From the late Islamic period, Muḥyī al-Dīn al-Maghribī's (d. 1283 C.E.) observations and measurements of the coordinates of the Sun, Moon, planets and eight bright stars at Maragha Observatory in the 1260s–1270s C.E. are among the most illustrative surviving examples of employing it (see Mozaffari, 2018a: 601–602, 630–631). Ibn al-Shātir's star table arranged in the equatorial coordinates contains the preserved results of other examples, which in itself provided later Near Eastern astronomers and religious time-keepers (Egypt and Syria) with a suitable ground to prepare the star tables in equatorial coordinates (Mozaffari, 2022a: 190).

Any measurement by either method needs a reference celestial object whose coordinates have been already determined. The Sun was the best candidate during the initial steps of an observational program, and our author evidently deployed it in the cases of Deneb and Altair. As we have seen earlier, his solar theory seems, in all likelihood, to have been accurate enough to fulfil essential requirements for tolerably precise and accurate measurements of the stellar longitudes, at least, in the 970s. As time passed, bright stars (e.g., Regulus or Capella, his favourite star) could serve for the same purpose.

The method of spherical astronomy basically consists of the following steps, a part of which has been already portrayed, in practice, in the derivation of the polar longitudes of Deneb and Altair. One measures the meridian altitude and the interval of time between that moment and the time of observation of a reference object in a specific situation (e.g., the Sun rising or sitting), from which he calculates the declination, right ascension, and polar longitude of the star. Then, by computing the acute angle between the meridian and the ecliptic and the difference in declination between the star and its mid-heaven degree, the ecliptical coordinates can be determined (for more explanation and practical examples, see Mozaffari, 2018–2019: 182–189).

Let us briefly compare the two methods with each other. The errors are in

(1) the basic parameters, φ and ε, and in
(2) the longitude/right or ascension of the reference body would have the same consequences in both methods.
(3) Working with a fixed meridional instrument is expected to be significantly less prone to sizable errors (either due to the observer or owing to instrumental discrepancy) than such complicated instrument as the armillary spheres, in which random errors of any source and sort were quite probable to commit, and in which construction various kinds of structural defects were possible to occur (for a review, see Evans, 1987: Part 2); only one example of a deficient structure, according to Bīrūnī's *al-Qānūn* VI.6 (Bīrūnī 1954–1956, Volume 2: 636–637), was that "… the rings are stretched, because of their weight, so they elongate as they are made excessively big …", a difficulty Taqī al-Dīn Muḥammad b. Maʿrūf (1526–1585 C.E.) actually experienced in his gigantic armillary spheres (whose meridian ring was over 4.5-m in diameter) constructed at Istanbul Observatory (Taqī al-Dīn, *Sidra*, K: f. 15r–v, N: f. 18v–19v, V: f. 22v–23v).[6] Generally speaking, there was a tendency in Medieval Middle-Eastern astronomy to replace it with altazimuth instruments (see Mozaffari, 2018a: 632–633).
(4) The accuracy of the spherical astronomy method is substantially dependent upon the computational adequacy of the calculator and/or of the necessary auxiliary astronomical or trigonometric tables of various kinds at his disposal. However, no serious difficulty was expected to happen in this regard, because generally Islamic calculators and tables, and particularly Abu ʾl-Wafāʾ and his numerical tables, were sufficiently accurate, as can be seen in those already inspected cases pertinent to him.

(5) The 'Achilles' Heel' of the method is that it needs an accurate timekeeping method or device, simply because an error of a single minute in measuring the time-spans can potentially cause errors of 15′ in right ascension and, nearly, of the same order in the transit degree and longitude. As we have seen at the end of 3.2, the errors in the basic parameters ε and φ are small enough to exert a quite small negative impact in the transformations $\lambda\odot \rightarrow A\varphi_{\odot}$ and $RA \rightarrow \lambda_{M}$; rather, it is $\Delta t°$ that has profound implications for RA and λ.

A peculiar characteristic of the method is that there might be a considerable disparity in the degree of accuracy achieved in the determination of longitude, which mainly arises from the diverse range of errors in the time-intervals, as seen in the case of Deneb and Altair, but if even they are at the same level of accuracy, the errors in the time-intervals might quite probably be compensated for or aggregated by the occasional mistakes committed by an astronomer in calculations and the errors in the auxiliary astronomical and trigonometric tables at his disposal, and the longitude/right ascension of the reference body (which, in turn, is dependent upon the solar theory). The best example in this regard is Muḥyī al-Dīn's measurements (see Mozaffari, 2018a: 630–631).

Abu ʾl-Wafāʾ seems almost certainly to have used the method of timekeeping by the Sun and stars, which he explains in detail in his work (see below). It is worth noting that the early Islamic astronomers did not intend at all to use water- or sand-clocks. Our author's junior contemporary, Bīrūnī, has left for us remarks against their reliability for astronomical purposes (Sheynin, 1973: 116). This situation is, indeed, in stark contrast to what can be found in the post-classical Islamic period (after 1050 C.E.), when clocks of every sort are mentioned in treatises (e.g., in Khāzinī's *On Experimental Astronomy* I.4; *Zīj*, V: f. 7r, L: f. 57r), and were employed, in practice, for timekeeping at the observatories established in Maragha (at least, two water-clocks, probably, a mechanical version of the Chinese origin) and Istanbul (the 1570s, a mechanical wheel-and-gear clock),[7] and probably at Samarqand Observatory, since both sand- and water-clocks are mentioned in Kāshī's *Khāqānī zīj* (see Mozaffari, 2020–2021: 83–84).

The definite advantage of timekeeping by stars over the medieval clocks is that the technological limitation of the day has a less undesirable influence in the former than in the latter. Timekeeping with stars could potentially enable astronomers to measure narrower intervals of times as well as the times to a finer and better degree of accuracy than what could be achieved with a clock of any type. We have seen that in both cases of Deneb and Altair, the errors in $\Delta t°$, as converted to time, correspond to the deviations of 4 seconds and 2 minutes. A Medieval clock could scarcely measure time to such a degree of accuracy, as we recall that the mean absolute value of the errors of the durations measured by the water-clocks of Maragha Observatory was over 2 minutes, and the mechanical clock utilized at Istanbul Observatory some 300 years later suffered from notorious systematic errors of between –½ and –1 hour, the size of which changed variably with respect to intervals of time.

Returning to Abu ʾl-Wafā, we are not given sufficient details about his observations of Capella, Regulus, and Mizar. On the orthogonal coordinates, first and foremost, we should consider that the error of +5′ in his value for the latitude of the place of observations makes an error of the same size and sign in the declination of the Northern stars and an error of –5′ in the case of the Southern ones. Capella transits the meridian of Baghdad in the northern half of the horizon. Hence, since his measured declination is correct, there should have been an error of about +5′ in his value for the meridian altitude to compensate the error in the latitude of locality. This positive error in combination with the errors in the solar noon-altitude of –4′, <–4.5′, <–5′, <–5.5′, and –7′ at, respectively, the Summer solstice, mid-Spring/ Summer, equinoxes, mid-Winter, and Winter solstice (see above, Section 2.2) may prompt a variety of speculations about the set-up of his meridional ring. A plausible working hypothesis may be that the vertical axis/diameter of the meridional instrument deployed might have had a trivial northward tilt of about 5′, or so, arising either from the slope of the ground towards the north, or simply because of a wrong orientation of the cardinal directions of the ring's graduations (up, down, north, south) on the wall (see Figure 2). A plumb-line mechanism utilized at the time[8] could hardly ascertain to such a degree of accuracy that the ground was precisely level, the wall was plumb, the vertical direction was correctly marked on the wall, and/or the ring's vertical diameter is soundly placed aligned with it. However, the scanty nature of the evidence available does not allow us to go further with such a conjectural hypotheses. Moreover, this range of errors in altitude, all below ⅙°, seems an optimistic estimate of the errors committed by Abu ʾl-Wafāʾ, not least because of our realization of the large errors of ±½° in his values for the extremal altitudes of Mizar, as if our author never observed this star with his meridional in-

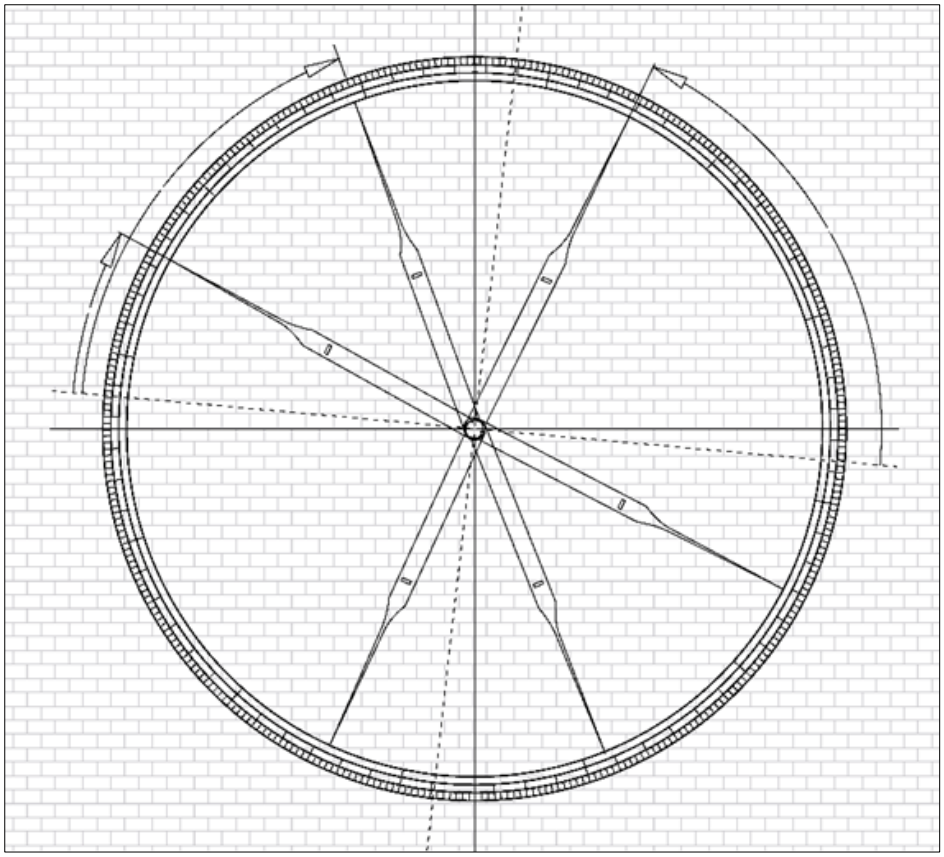


Figure 2: A probable structural defect as a northward tilt in the orientation a meridional ring or a northward slope of the ground causes the measured altitudes of the celestial bodies passing across the meridian due south to become less than true values, while the reverse occurs for the northern objects (diagram: Mohammad Mozaffari).

strument(s). On the one hand, such a narrow accuracy is expected of a simple small portable instrument; on the other hand, it should not have been impossible to detect errors of such size even by a not-too-large, but sound, portable device (e.g., an astrolabe or handy quadrant of 12-cm radius, or thereabout). The errors in Mizar's altitudes turn out to be crucial, since our author suggests it as a clock-star for timekeeping at night (see below).

We now turn our focus to evaluate how he could measure the time intervals with such fairly good degree of accuracy as his longitude values conspicuously indicate. This problem is crucial not only in the present case study, but scrutinizing the issue in some depth may, also, dissolve the doubts usually surrounding the efficiency of Medieval practical astronomy. We know that the altitude measurements by the use of simple portable devices could not be accurate enough out of the plane of meridian—say, to ±½°, or so. The observational records of the solar and lunar eclipses from the Medieval Islamic period contain the altitudes of clock stars at various phases (which was apparently for the purpose of determining the time of each phase), most of which are given in integer degrees, and so, are accurate to ±1° (see Said and Stephenson, 1996; 1997; Steele, 2000, Chapter 4; Stephenson, 1997: Chapter 13

An instance of Abu ʾl-Wafā's timekeeping strategy is that, in V.14 (Abu ʾl-Wafāʾ, *Majisṭī*, ff. 80v–81r), he presents the formula

$$\operatorname{versin} HA = \frac{\sin h_{\max} - \sin h}{\cos\varphi \cos\delta},$$

which is an equivalent of

$$\cos HA = \frac{\sin h - \sin\varphi \sin\delta}{\cos\varphi \cos\delta},$$

in order to instruct the reader how to measure time at night from the altitude of a circumpolar, permanently-over-horizon star. In a worked example, the hour angle of Mizar, when h = 30°, is computed as HA = 56;48…°. Albeit, when a star is near crossing the local meridian (in the highest and lowest altitudes, for a circumpolar star), the variation in altitude is quite small with respect to time; so, the use of stars for timekeeping at those times was, in practice, impossible, but they could be used conveniently when located at their intermediate altitudes.

The errors in Abu ʾl-Wafāʾ's values for the declination of Mizar and the latitude of place cause the errors of, at most, ±1° in *HA* of the

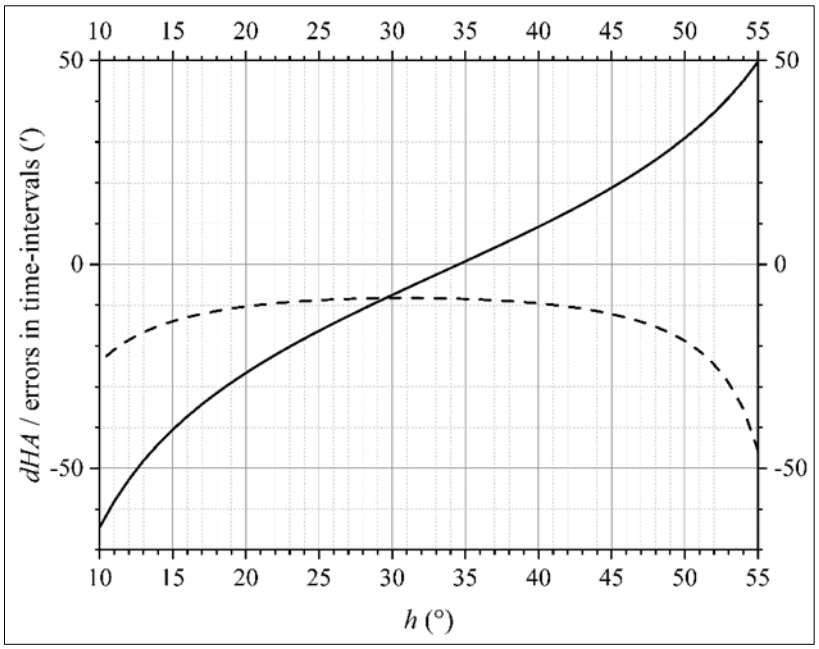


Figure 3: The continuous curve displays the errors in *HA* of Mizar arising from the errors in φ of Baghdad and δ of the star, as plotted against its altitude *h*. The dashed curve shows the errors in time-intervals corresponding to $\Delta h = \pm 5°$.

star at the endpoints of the altitude range of 11°–56°; the errors reduce to below ±10′ in the intermediate altitudes of 29°–40° (in the aforesaid worked example, *dHA* = –8′), and completely vanish/disappear at *h* = 34.55°; cf. the continuous curve in Figure 3).

Of course, for the purpose of determining *RA*, λ_M, and λ of another star, we essentially need to know 'time-intervals', i.e., differences in *HA* ('time') between two moments, and hence the aggregate errors ($dHA_2 - dHA_1$) can be either more or less than the individual ones.

Since the errors in *HA* are of the same sign and order for a long part of the intermediate altitudes, the errors in time intervals would naturally decrease. Assume that the wanted time-intervals correspond to a change of 5° in altitude, their errors would be considerably less than the errors in the times: they vary between –8′ and –24′ at *h* = 10°–52°, and remain ≤ –10′ for *h* = 20°–42° (see the dashed curve in Figure 3).

Nevertheless, we have already seen that our author committed the large errors of ±½° in measuring the extremal altitudes of the star. The errors in altitude of this size can, in itself, cause egregious errors in *HA*, whose minimum value would never be less than about 0;40°.

In the case of *dh* = –½°, *dHA* = +40′ occurs at *h* = 10°–11°; the errors increase too smoothly to reach a value of +2° at *h* = 55° (see the upper continuous curve in Figure 3). Nevertheless, for the time intervals corresponding to a difference in altitude of 5°, the errors remain below –10′ for a good portion of the intermediate altitudes, specifically, *h* = 10°–40° (cf. the upper dashed curve in Figure 4).

In the case of *dh* = +½°, *dHA* > –40′ for *h* < 47°; since then, the errors decrease, and approach to zero in the vicinity of the upper culmination, at *h* = 61° (cf. the lower continuous curve in Figure 3). But, once again, for the time intervals corresponding to a difference in altitude of 5°, the errors remain below –10′ for a half of the intermediate altitudes, now in the range *h* = 29°–50° (see the lower dashed curve in Figure 3).

Therefore, these, and many other possible similar cases, may explain how, notwithstanding the discrepancies in the basic parameters, and committing the errors of ±½° in altitude, a Medieval astronomer could succeed in arriving at a degree of accuracy in the calculated values for *HA* and time-intervals for a specific range of the intermediate altitudes to be congruent with an accuracy of about ±10′ in the longitude measurements. Needless to say, there are equally many other situations in which the altitude-errors lead to gross discrepancies in the time-intervals. For example, when *dh* does not remain at the same sign during the time interval corresponding to $\Delta h = \pm 5°$, but changes, e.g., from +½° at its origin and –½° at its end, or *vice versa*. In the first situation, the size of error in the time-spans would not be less than

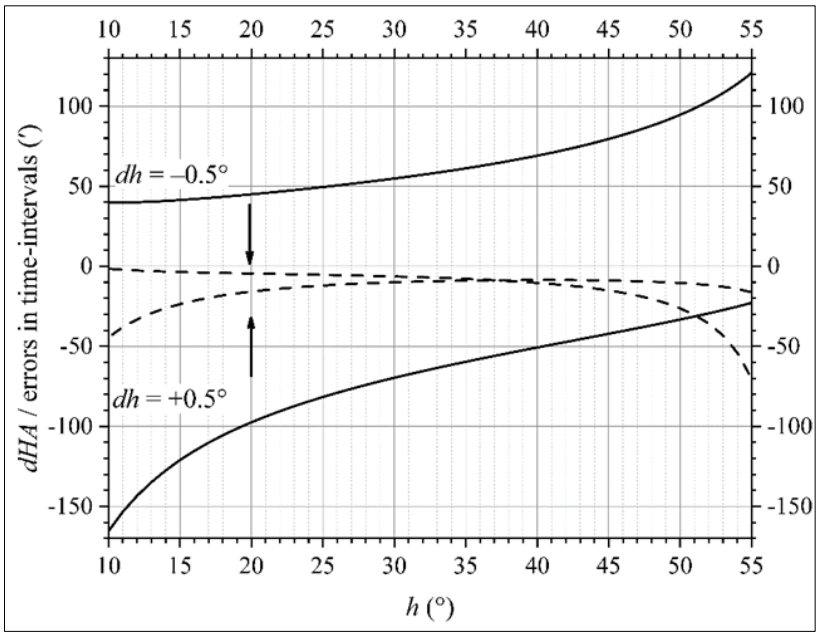


Figure 4: The continuous curve illustrates the errors in *HA* of Mizar arising from the errors in φ and δ together with dh = ±½°, as plotted against its altitude h. The dashed curve exhibits the errors in time-intervals corresponding to Δh = ±5°.

−2.1° in the Eastern half of the horizon, and +1.8° in its Western half (see Figure 5). In the latter case, the errors are, respectively, not less than +1.8° and −2.1° (cf. Figure 6). Nevertheless, such big errors were quite probably controllable with the aid of the *Almagest* star catalogue or any previous star table, so that, as soon as an astronomer established a rate of precession, he could define a range of acceptable differences in longitude between the *Almagest* and his own values, and then, no strange value surpassing too much the limits could be considered valid and trustworthy (e.g., for Abu ʾl-Wafāʾ, the differences in longitude for Mizar, Regulus, and Capella is about 13°: 12;53° ± 17′). This strategy was apparently an effective way of preventing any blatant errors from oc-curring and holding newly measured and previ-ously known stellar longitude values, to a great extent, consistent with each other. The com-parative star table in the *Īlkhānī zīj* can attest to its functional use (see Mozaffari, 2016).

3.5 The Rate of Precession

Bīrūnī compares Abu ʾl-Wafāʾ's value λ = 135;30° for the longitude of Regulus, measured in 974/975 C.E., with Hipparchus' corresponding value of 119;50°, measured in 129/128 B.C.E. (an error of about −34′), as mentioned in the *Almagest* VII.2 (Toomer, [1984] 1998: 328), and derives the value of 1° in 70 Persian (= Egyptian) years (of 365 days, unvarying) plus 4 months (of 30 days). However, this is not Abu ʾl-Wafāʾ's formal value: in the very last sen-tence of his *Majisṭī* VI.9.2 (which is the second of the three chapters that comprise his lengthy discussions on the planet Mercury), he explicitly states that the motion of its apogee is "… approximately equal to 1° in 75 years." (Abu ʾl-Wafāʾ, *Majisṭī*, f. 94r).[9] We can be almost certain that Abu ʾl-Wafāʾ maintained the same figure for the rate of precession.

In the early Islamic period, following Ptolemy, it was accepted that the planetary apsidal and nodal lines move progressively along the ecliptic at the rate of precession, which was only extended to include the apsidal line of the Sun/Earth, after the discovery of its motion in the ninth century. It was not until towards the end of the thirteenth century that Jamāl al-Dīn Muḥammad b. Ṭāhir b. Muḥammad al-Zaydī, a Persian astronomer in the service of the Yuan Dynasty of China (1260–1368 C.E.), put a distinct difference between the apogeal and precession motions, assigning the rate of 1° in 66 Persian years to the first and taking 1°/73^{y} for the latter (see Mozaffari, 2019: 80a, n. 85), a result that Ibn al-Shāṭir independently reached, with the same motion for the apogees/nodes, but adopting the rate of 1°/70^{y} for the precession, within a century later (Mozaffari, 2017: 21–24).

Let us speculate on how Abu ʾl-Wafāʾ could arrive at his specific value for the rate of precession, which is the second smallest estimation hitherto brought to light from the Medieval Islamic period (for a review of other known values see Mozaffari, 2017: 6–7); I could find, however,

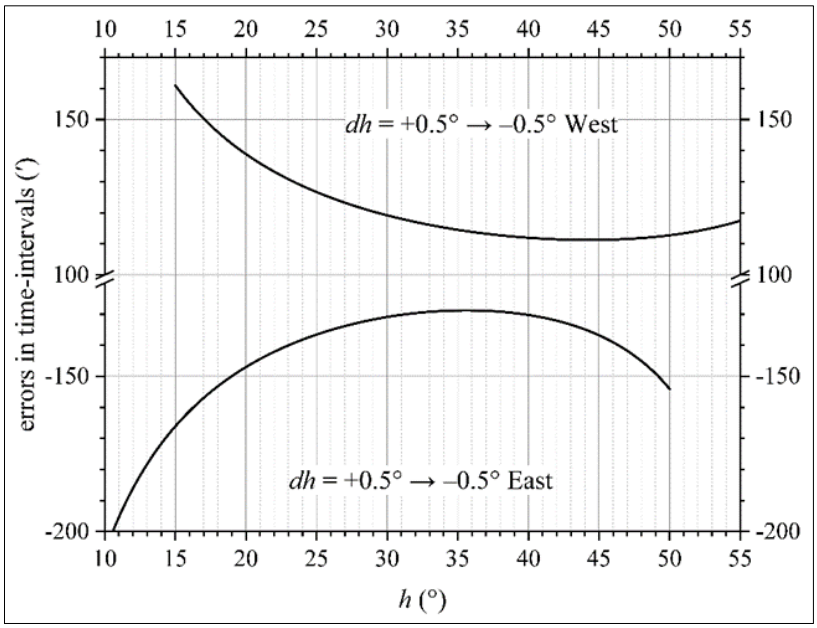


Figure 5: The errors in time-intervals corresponding to Δ*h* = ±5° arising from the errors in φ and δ together with *dh* = +½° (at the beginning) → –½° (at the end), as plotted against its altitude *h*, for the eastern and western halves of the local horizon (note that there is a break in the ordinate).

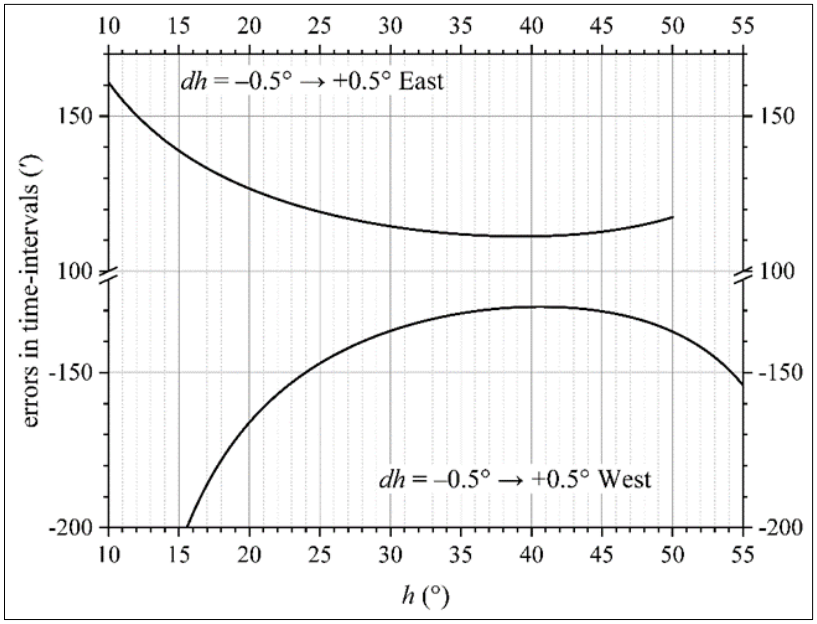


Figure 6: The errors in time-intervals corresponding to Δ*h* = ±5° arising from the errors in φ and δ together with *dh* = –½° (at the beginning) → +½° (at the end), as plotted against its altitude *h*, for the eastern and western parts of the local horizon (note that there is a break in the ordinate).

no trace of its later use in the Medieval Middle Eastern astronomical corpus. His stellar longitudes, as we have seen earlier, are accurate to a good degree; hence, his very low estimate of the rate of precession appears, in all probability, to have arisen from his use of an initial longitude value for making comparison, which suffered from a large positive error (not, like Hipparchus' value for the longitude of Regulus, whose negative error inevitably results in a larger value for the rate of precession) and/or belonged to a time long ago from him. But which one? Besides Hipparchus' value for the longitude of Regulus, Ptolemy puts forward, in *Almagest* VII.3 (Toomer, [1984] 1998: 334–338), the ancient observational records of the occultations of Pleiades, Spica, β Sco by the Moon from his predecessors, Timocharis, Agrippa,

and Menelaus, and derives the three stars' longitudes on the basis of his own lunar theory for the dates and times reported, falling in 295, 294, and 283 B.C.E. and 92 and 98 C.E. Ptolemy nicely tries to convince the reader that all differences in longitude are in excellent agreement with his rate of precession of 1°/100ʸ, so the longitudes calculated for Timocharis' time are by either 4;10° or 4;20° less than the corresponding values tabulated in the *Almagest* star catalogue (epoch: 20 July 137 C.E.). We do not know whether Abu ʾl-Wafāʾ measured the longitudes of the three stars mentioned here, and if he, in actuality, did it, we do not have the results. Moreover, we do not have more ancient values for the longitude of any of the stars observed by him than Hipparchus' value for the longitude of Regulus.

Nevertheless, there might have been a way for a medieval astronomer to think that he has already given access to a more ancient catalogue than (in fact, the original version and archetype of) the one included in the *Almagest*, as explained in the following. For unknown reasons, none of the Medieval Islamic astronomers believed that the *Almagest* star catalogue was a work performed by Ptolemy himself. Although his claim on the authenticity of his stellar observations in *Almagest* VII.4 ("… using the same instrument [i.e., the armillary spheres], because the astrolabe rings in it are constructed to rotate about the poles of the ecliptic, we observed as many stars as we could sight down to the sixth magnitude.") has already been correctly and clearly rendered into Arabic in the ninth century (see Toomer, [1984] 1998: 339; Arabic *Almagest*: Ḥajjāj: L: f. 110v; Isḥāq–Thābit: S: f. 98r; TN: f. 116v), nevertheless, it was apparently understood in the way that he measured the stars' magnitudes down to the 6th magnitude! The consensus was that Ptolemy borrowed the catalogue from Menelaus. Bīrūnī was of the opinion that the catalogue was taken from Timocharis, although he was not sure about the magnitudes. Even a late Islamic commentator, al-Bīrjandī (d. 1525/6 C.E.), presented the profound hypothesis that Ptolemy had Hipparchus' values for the longitudes and declinations of the stars at his disposal, measured only their declinations, and then derived the longitudes for his own time from the differences in declination between Hipparchus' and his times (Mozaffari, 2016: 303–307).

If someone believed that Ptolemy actually borrowed his catalogue from Timocharis, then he could restore Timocharis' alleged original values for the longitudes for 295/294 or 283 B.C.E by simply subtracting, respectively, 4;20° or 4;10° from the corresponding values tabulated in the *Almagest*. By doing this conversion, the negative errors in the longitudes at Ptolemy's time turn out to be positive for Timocharis' day. Let us examine whether this solution works in the case of Regulus: in the *Almagest*: 122;30° (with an error of about –1.5°); thus, for Timocharis' time: 118;10° (with an error of around +4′); comparing the latter with Abu ʾl-Wafāʾ's value by taking the time interval of 1270 years results in the rate of precession of 1° in 73.3 years, very close, but not quite identical, to his value.

Let us consider Capella, our author's favourite star. Note that although the stars located near the ecliptic (notably Regulus and Spica) were used more for measuring the precession of the equinoxes during the Medieval period, the use of Capella for fulfilling this purpose was not unprecedented. Notably it was used by al-Farghānī in 856–857 C.E. (Mozaffari, 2016–2017: 74), and the determination of the longitudes of Capella and Vega was at the top of Muḥyī al-Dīn al-Maghribī's (d. 1283 C.E.) 'wish list', in addition to the longitudes of eight other stars that he measured at Maragha Observatory in 1267–1268 C.E. in deriving the rate of precession (even though he was unable to do it, because there was no northern mural meridional quadrant at the Observatory; see Mozaffari, 2018a: 634). For Capella, in the *Almagest*: 55;0° (with an error of about –1°); hence, for Timocharis' time it was 50;40° (with an error of around +40′); comparing the latter with Abu ʾl-Wafāʾ's value yields the rate of 1° in 75.1 years, completely in agreement with his formal value. Therefore, it seems most likely that he derived his value for the rate of precession and apogeal motions in such a way. I think this is the most likely scenario that led our author to his specific value for the rate of precession. If this coincides with what really happened, and if he had explained it clearly somewhere in his work, even though it is not included in the copy at our disposal, then he appears quite probably to have planted in Bīrūnī's mind the seed of the idea that Ptolemy compiled his catalogue on the basis of an alleged early prototype that had come down to him from Timocharis.

It is worth noting that Bīrūnī also compares his value of 189;24° for the longitude of Spica, measured in 1009 C.E. (with an error of about –39′), with the value of 172;20° that Ptolemy computed from his lunar theory for the date and time provided in Timocharis' observational record of the star's occultation by the Moon in 294 B.C.E. (with an error of about +20′). This had resulted in a rate of precession of 16 cycles (of 360°) in 160696125 days (or about 1° in 76.4 years), close to Abu ʾl-Wafāʾ's formal value.

(Bīrūnī's presentation of his determination of the rate of precession is confused (see Mozaffari, 2013(2): 405, 411, n. 127), and his formal value (1°/69^{y}) seems, quite probably, to have its basis in the comparison between Ptolemy's and his values for the longitude of Spica.)

4 PLANETARY THEORY

Nothing of Abu ʾl-Wafāʾ's planetary observations and measurements has been preserved in the extant copy of the *Majisṭī* or by Bīrūnī. Although during the last five years of his life he appears to have been quite busy writing his *Essentials on Practical Geometry for Craftsmen* (Raynaud, 2012), after his latest recorded stellar observation (discussed earlier) he still had two decades ahead of him to continue his astronomical observations and extend them to the planets. Yet for a variety of reasons, one gets the strong impression that he did not deal with the planets from a practical aspect.

The planetary eccentricity and epicycle radii values mentioned in VI.9.4 are all Ptolemaic, except those for Venus, for whose double eccentricity (i.e., the distance between the equant point and the terrestrial centre in the Ptolemaic model), he gives 2;5^{p} (Abu ʾl-Wafāʾ, *Majisṭī*, f. 95r), which is equal to his value for the solar eccentricity. However, it is not known whether, as with other early Islamic astronomers, our author followed the Indian Midnight System in keeping, too, the apsidal lines of the Sun and Venus coinciding with each other.[10]

For the orbital inclinations, the spaces for writing down the values for Mars, Venus, and Mercury have left blank, but for Saturn and Jupiter, we are given, respectively, 2;26° and 1;24°, which are also the values recorded by Ptolemy in *Almagest* XIII.3; however, "… for the sake of what is more appropriate …", he preferred to work with the round numbers 2;30° and 1;30°.[11]

Further, we have seen that Bīrūnī deployed Abu ʾl-Wafāʾ's few stellar observations together with his own in the derivation of the rate of precession, and considered his solar observations among others from his Islamic predecessors. But in the case of planetary astronomy he had to modify Ptolemy's theories in a purely artificial way in order to reach a new set of parameter values (see Mozaffari, 2017: 13). Bīrūnī, meanwhile, was suffering from a serious eye disease, as he clearly remarks in the *Taḥdīd*, due to looking at solar eclipses during his youth (Bīrūnī, *Taḥdīd*: 168, English translation, pp. 131). This may have been a justified excuse for him not to get seriously involved in planetary observations. If any planetary theory from Abu ʾl-Wafāʾ was at his disposal, he was no doubt the first to employ it in his *opus magnum*.

In the thirteenth century the anonymous *Shāmil zīj* was compiled on the basis of the *ʿAlāʾī zīj* of Ibn al-Fahhād of Shīrwān (a practical astronomer of twelfth century), which enjoyed a wide dissemination in India, Yemen and Byzantium (Mozaffari, 2017: 16–18; 2019b). At the beginning of the *Shāmil zīj* we are told that the underlying planetary mean motions and positions in the *ʿAlāʾī zīj* were taken from Abu ʾl-Wafāʾ. The controversary surrounding this completely false attribution to Abu ʾl-Wafāʾ has remained unclarified during the 66 years following the publication of Kennedy's study of Islamic *zījes* (see Kennedy, 1956: no. 29 on page 129 and the diagram of *zījes* related to the *ʿAlāʾī zīj* on page 169). A close inspection of the *ʿAlāʾī zīj* shows that this cannot be true. Ibn al-Fahhād used the solar and lunar theories of Mumtaḥan/Ḥabash's ninth century *zij*, together with a rate of the apogeal/processional motion of 1°/66^{y}, which distinctly differs from Abu ʾl-Wafāʾ's. The error in the Mumtaḥan/Ḥabash's solar theory was well-known in the end of the tenth century, and Bīrūnī includes many examples of it throughout his works. At the Autumnal equinox of 974, the Mumtaḥan/Ḥabash's Sun was at a longitude of 180;12°, which corresponds to a sizable error of about –4½ hours in the event's timing. It is extremely improbable that such striking a deficiency went unnoticed, and Abu ʾl-Wafāʾ adopted this inaccurate solar theory. In the case of the planetary theories, which constitute the core of the puzzle and, indeed, the most difficult part of it from a technical aspect, a close analysis of Ibn al-Fahhād's planetary parameters evidently demonstrates that they were:

(1) derived from his observations, independent of, or bearing no significant relation to, any other available theory, which is the case with Mars;

(2) the results of his modifications/deteriorations of the parameters of other theories on the basis of the results he achieved from his observations, which is the case with Saturn and Jupiter: he changed ʿAbd al-Raḥmān al-Khāzinī's (fl. 1100–1130 C.E.) mean motions in longitude and/or radix mean longitudes on the basis of his observation of the conjunction between the two planets in 1166 C.E. (Mozaffari, 2022b); or

(3) were taken directly and faithfully from other sources known and available at the time, which was the case with Venus, and his source theory was, again, Khāzinī's.

This topic requires separate, detailed treatment, and will be the subject of a future paper (now, see Mozaffari 2023).

5 CONCLUDING REMARKS

The results of both sets of Abu ʾl-Wafāʾ's observations have been scrutinized for the first time in this study, as straightforwardly reported in his *Majisī* or by Bīrūnī.

Concerning the stellar longitudes, his ecliptical or polar longitudes for Capella, Deneb, Mizar, and Regulus were accurate to within –⅙°, but there was an error of –½° in the case of Altair. According to our reconstruction of the way in which he reached his specific value of 1°/75^{y} for the rate of precession, he selected Capella. His value for its longitude (and, also, its latitude) was the most accurate among the other values known during Medieval Islam. Then, his desire to make astronomical observations over the longest time interval possible led him to compare his longitude with what was considered in the *Almagest* to be the star's longitude in Timocharis' time (i.e., the value given in the *Almagest* catalogue minus 4;10° or 4;20°).

Of his extensive solar observations, only the extremal annual noon-altitudes are available in the *Majisṭī* (with errors of about –4′ and –7′). The rest have come down to us through Bīrūnī: the time of the autumnal equinox of 974 C.E. (the error is about –⅓^{h}) in addition to his values for the lengths of Spring and Summer in the same year (which enabled us to determine the errors in the times of the summer solstice of about –⅙^{h} and the vernal equinox of about –1⅙^{h}), and the successive intervals of time between the mid-Winter, mid-Spring, and mid-Summer as measured in 976 C.E. (with errors of about +1½^{h} and +⅙^{h}). The resulting solar theories were accurate to within –1′ to +4′.

We have shown that his meridian-altitudes suffered from a systematic error of about ±5′ (negative for the six noon-altitudes of the Sun in the Southern region of the local horizon and positive for Capella in the Northern horizontal zone). We have hypothesized that this might have been because of a northward disorientation in the set-up of the meridional ring or because the ground at his observing site had a northward slope of the same size.

To sum up, the available data, although limited, reflect a complete mastery of Abu ʾl-Wafāʾ's operations and techniques and demonstrates that he was fully competent in conducting a purposed observational program and adept at establishing a solar theory and achieving highly or tolerably accurate longitudes for reference stars.

We have seriously considered the bewildering dilemma over the planetary theories of Abu ʾl-Wafāʾ in relation to the ones presented in the twelfth-century *ʿAlāʾī zīj* composed by Ibn al-Fahhād. According to a wild claim by the anonymous author of thirteenth century *Shāmil zīj*, the latter were not original, but were taken from the former. This unverified claim—crucial not the least because the *ʿAlāʾī zīj* was a highly influential *zīj* in the late Islamic period—remained unchallenged for 66 years after its formulation by the late E.S. Kennedy in 1956. In our study, we have cleaned up the situation and rejected the fraud, but due to page constraints only the results have been summarized here. Details will be systematically presented in another paper, in association with an analysis of the origins of the planetary theories from the classical period of Islamic astronomy until Ibn al-Fahhād's time.

6 NOTES

1. The date of the completion of al-Baghdādī's *zīj* in the unique extant copy is given as *Muḥarram* 684 H. (= March–April 1285) on the colophon (f. 255r), but it seems to be the date of an earlier version or of the composition of canons (explanatory section), because the calculations related to the solar eclipse of Monday, 28 *Dhi ʾl-qaʿda* 699 H. (= 15 August 1300) is given ff. 132v–133r, and of the lunar counterpart of Sun-day, 14 Ramadan 685 H. (= 3 November 1286) on ff. 135r–136r.
2. A brief discussion of the controversy is given in Dreyer (1906: 252–257) and Neugebauer ([1957] 1969: 206–207). It is worthwhile that a less-known anachronistic idea asserted by Sédillot (1845–1849(1): 282–287) is maintained by Dreyer, surprisingly, just before his discussion on Abu ʾl-Wafāʾ (precisely, on page 252): drawing a false analogy between the difference between the greatest and least values measured in the Islamic period for the inclination of the lunar orbit, 4;45° and 5;3°, and its true variation, between about 4;57° and 5;18°, misled him to remark that "[Islamic astronomers] miss[ed] a remarkable discovery, that of the variation of the lunar inclination"!
3. The three calendars mentioned throughout this paper are: the Y. = Yazdgirdī (epoch = 16 July 632, length of year = 365 days, fixed); the H. = Hijra (epoch = 15/16 July 622, lunar); and the C.E. = Common Era.
4. It is not known why Bīrūnī (*Taḥdīd*: 100, English translation, page 69, E.S. Kennedy's commentary, page 43) gives the years 365–366 H., which is the only period mentioned by Abu ʾl-Wafāʾ's in his *Majisṭī* in connection with his observations of Capella (cf. Section 3.1).
5. The data from other Islamic stellar measure-

ments used here for making comparison with Abu ʾl-Wafāʾ's values have already been presented by Mozaffari (2016: 296–297; 2016–2017: 92–100; 2018a: 601–602; 2022a: 168–171; see, also, Knobel (1917). For their epochs, see Mozaffari (2022a: 164–165). See, also, Ibn Yūnus' report of the observations of Regulus by the early Islamic astronomers in his *Ḥākimī zīj* (L: 106–108) and in Caussin (1804: 143–155).

6. After setting up the rings, Taqī al-Dīn examined the instrument in order to find whether any deviation occurred in the rings' diameters because of their weight. He found that, in the meridian ring, the horizontal diameter became larger than the diameter perpendicular to the horizon by about 4 mm. He suspended the ring, and saw that the opposite situation occurred, namely, the perpendicular diameter now became larger than the horizontal one by the same amount. In order to fix the deficiency caused by the weight pressure applied by the upper segment of the meridian ring to its lower part, he installed two large inclined pillars (*zand*) at the two [Northern and Southern] sides of the perpendicular (*ʿamūd*) base of the instrument to bear the meridian ring's weight from its Northern and Southern sides, in such a manner that there were some space between either and the perpendicular base. He also fixed three other defects in his armillary spheres, which are among the best examples of the troubleshooting of Medieval observational apparatus.
7. For Taqī al-Dīn Muḥammad b. Maʿrūf's solar and lunar observations made at Istanbul Observatory see Mozaffari and Steele, 2015.
8. For two sketch drawings in Medieval Islamic treatises, see, e.g., King, 1979: 36–37.
9. It deserves mentioning that it does not come as a surprise at all to find a Medieval astronomer to have computed a value for a specific parameter by the use of observational data reported by another astronomer, which may be different, as in this case, from the value derived by the latter from his or her own data. Bīrūnī did so frequently throughout his *al-Qānūn al-masʿūdī*. A notable example is Bīrūnī's derivation of a non-Ptolemaic value for the lunar eccentricity from Hipparchian-Ptolemaic data in his *al-Qānūn* VII.7.1 (1954–1956, Vol. 2, pp. 789–795, F: ff. 116r–v, B: ff. 246v–248r, C: ff. 142r–143r, I: ff. 213v–215r, K: pp. 341–344, L: ff. 142r–143r, R: pp. 305–307). Another note is that although Abu ʾl-Wafāʾ does not specify the *type* of 'year' for his value, but even taking the length of the lunar year used in the Islamic calendar, from Bīrūnī's derivation of $1°$ in $70\frac{1}{3}^{y}$, a value of about $1°$ in $72\frac{1}{2}$ lunar years results, so that Bīrūnī's value can by no means be reconciled with Abu ʾl-Wafāʾ's one in this way.
10. His contemporary, Ibn al-Aʿlam, was seemingly the first Islamic astronomer who rejected this erroneous hypothesis and kept Venus' apogee behind the solar one in longitude, as it is in the *Almagest* (Mozaffari, 2019a).
11. Toomer ([1984] 1998: 605), and the latitude tables in *Almagest* (XIII.5: 632), opposite the arguments of 90°. I owe the correct translation of the quoted expression to Alan Bowen (pers. comm., 2022). G.J. Toomer (ibid.) translates it as "… to achieve greater symmetry." The two extant ninth-century Arabic translations are, also, not so accurate as regards this expression: Ḥajjāj has *min ajl ḥusn al-taqdīr*, "… for the sake of good estimation" (LE: f. 202v), and Thābit-Isḥaq: *li-l-tashīl fī al-ʿamal*, "… in order to facilitate the procedure/operation." (S: f. 195r, PN: f. 163r, TN: f. 220v, LO1: f. 207r). The translators actually tried to guess the reason behind Ptolemy's rounding here.

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