



低频宽带吸振技术研究综述与展望

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收稿日期: 2025-12-26; 接受日期: 2026-02-11; 网络版发表日期: 2026-03-10

国家自然科学基金项目(批准号: 52422504, U2541231)和浙江省自然科学基金项目(编号: LZ26E050004)资助

摘要 随着航空航天等复杂装备逐步大型化、轻质化和高精度化, 导致结构模态密集, 低频振动问题凸显, 严重影响装备的安全性、可靠性与运行稳定性等指标. 吸振技术是将吸振器附着于振动主结构, 通过调谐有效抑制目标频率的结构振动. 因其不改变主结构刚度特点, 吸振技术在工程中得到了广泛应用. 本文围绕低频宽带振动抑制需求, 梳理了国内外近年来吸振技术的研究进展与发展脉络, 重点综述了被动式吸振器、可调谐式吸振器以及非线性能量汇的研究现状, 总结归纳了吸振器参数优化方法与准则, 并对低频宽带吸振技术的发展趋势进行了展望, 为高端装备的低频宽带振动抑制提供理论参考与技术支持.

关键词 吸振, 动力吸振器, 低频振动, 非线性能量汇, 振动抑制

1 引言

航空航天、先进集成电路制造装备、高精密机床等高端装备是保证国家安全、支撑战略性新兴产业发展、提升综合国力的基石, 随着以上装备逐步向轻量化、大型化和高精度方向不断发展, 系统低频模态愈加密集, 低频振动问题愈发突出^[1]. 尤其是在复杂和多源激励条件下, 低频振动因其具有能量大、传播距离远、衰减速度慢、抑制难度大等特点, 成为制约高端装备动态性能提升的关键因素之一^[2]. 其不仅会导致设备内部精密部件指标漂移、精度下降、结构损伤, 严重时还可能引发安全事故, 大幅缩短装备的服役寿命并降低其运行可靠性^[3]. 因此, 对装备有害低频振动进行有效抑制, 确保其在复杂动力学环境下的长期、

稳定、可靠运行, 是实现高端装备“高精、超稳、超静”的重要保证, 对国民经济和社会发展具有重要的意义.

吸振技术因其结构简单, 易于实现, 同时不改变主系统整体刚度等优点, 被广泛应用于多种装备与机械结构的振动抑制^[4]. 吸振技术的基本原理是在主结构上附着一个附加质量-弹簧单元, 即动力吸振器(dynamic vibration absorber, DVA), 通过调节吸振器与主结构之间的频率匹配关系, 即调谐, 使主结构在目标频率附近实现反共振, 主结构振动响应得到极大抑制, 而吸振器振动增大, 即实现吸振. 然而, 在实际工况下装备所受激励形式复杂、频带范围宽, 传统线性吸振器通常仅对单一频率有效, 当吸振器频率偏离最优频率时不仅将无法吸振, 甚至可能还会加剧系统振动^[5].

引用格式: 潘侠圭, 严博. 低频宽带吸振技术研究综述与展望. 中国科学: 技术科学, 2026, 56: 952–970

Pan X G, Yan B. A state-of-the-art review and research progress on low-frequency and broadband vibration absorption technology (in Chinese). Sci Sin Tech, 2026, 56: 952–970, doi: 10.1360/SST-2025-0443

此外, 吸振器的低频振动抑制能力主要依赖于增大附加质量或降低刚度来实现. 附加质量过大会增加结构负载, 对工程可实施性产生不利影响; 而刚度过低则会削弱系统抗变形能力, 使得系统在共振处产生过大位移, 影响系统稳定性和安全性. 这使得传统吸振器难以满足装备对低频宽带振动抑制的需求.

综上所述, 本文围绕高端装备的低频宽带振动抑制需求, 系统梳理了近年来的国内外低频宽带吸振技术研究进展, 主要涉及被动式吸振器、可调谐式吸振器、非线性能量汇(nonlinear energy sink, NES)等的基本原理、发展脉络及应用, 此外, 对吸振器的参数优化方法也进行归纳与整理, 以期对复杂装备的低频吸振提供理论参考与技术支持.

2 被动式吸振器

被动式吸振器发明最早、应用最为广泛, 其结构形式的演化与性能优化始终是吸振技术领域的研究热点^[6]. 动力吸振器最早由Frahm于1911年提出^[7], 其基本原理如图1(a)所示. 随后, Ormondroyd^[8]在吸振器中引入了阻尼元件, 形成了调谐质量阻尼器(tuned mass damper, TMD), 也称Voigt式吸振器, 如图1(b)所示. 研究发现, 在不同阻尼下, 安装吸振器后主系统幅频响应曲线始终经过两个固定点, 这一规律即为定点理论, 为吸振器的参数优化奠定了理论基础. 随着装备性能的提升, 传统Voigt式吸振器的振动抑制效果逐渐难以满足需求. Ren^[9]在此基础上提出将与主系统连接的阻尼元件直接接地, 构建了接地式动力吸振器, 如图1(c)所示. 理论研究表明, 接地式动力吸振器可获得比Voigt式动力吸振器更好的振动抑制效果, 但由于阻尼元件与地基相连, 其安装条件也受到了一定限制. 此外, 三要素式吸振器也是一种Voigt式吸振器的改进形式^[10], 如图1(d)所示. Asami等人^[11,12]的研究表明, 相同质量比下, 三要素式吸振器具有比Voigt式吸振器更好的振动抑制效果, 同时他们结合 H_∞ 优化和 H_2 优化方法对三要素式吸振器的参数进行了优化.

随着装备向大型、轻质方向发展, 固有频率不断降低, 传统被动式吸振器在低频段的适用性逐渐受到限制. 因此, 研究人员相继设计了多种新型吸振结构, 以实现吸振器与主系统在低频范围内的有效匹配. Wang等人^[13]提出了一种具有负刚度的动力吸振器, 并

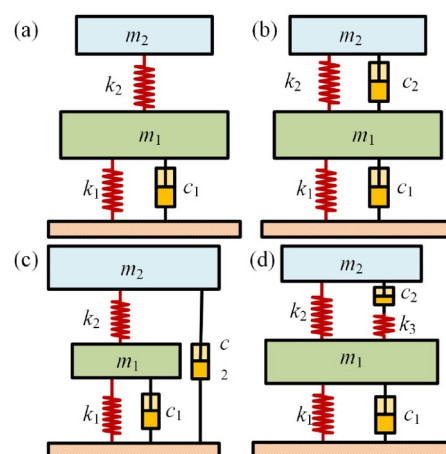


图1 (网络版彩图)动力吸振器. (a) 无阻尼; (b) Voigt型; (c) 接地式; (d) 三要素式吸振器

Figure 1 (Color online) Dynamic vibration absorber. (a) Undamped DVA; (b) Voigt DVA; (c) grounded DVA; (d) three-element DVA.

在此结构中增加了一个与地基相连的弹性元件. Shen等人^[14]对该结构的响应特性进行了分析, 其幅频响应曲线依然通过两个不动点, 因此可以采用 H_∞ 优化方法进行最优参数设计. 随后, 相关研究进一步探讨了该吸振器在不同激励下的减振效果与适用范围^[15,16]. 此外, 摆结构由于其固有频率低、实现形式简单等特点, 逐步在低频吸振中得到了研究^[17-20]. Chung等人^[21]考虑小振幅情况对摆式调谐质量阻尼器采用近似线性化处理. 研究表明, 该吸振器吸振效果主要受摆的质量、长度及阻尼影响, 对调谐频率尤为敏感, 而对阻尼参数敏感度较低. Deraemaeker等人^[22]基于等峰优化方法, 对摆式调谐质量阻尼器进行了参数优化, 得到了最优摆长与阻尼. Soltani等人^[23]采用定点理论研究了摆式调谐质量阻尼器在几种不同优化准则下的参数优化问题, 得到摆长和阻尼比的最优解析解, 如图2(a)所示. Kecik等人^[24]提出了一种具有非线性弹簧摆的自参数摆式吸振器, 研究了该吸振器对系统振动抑制性能的影响. 为了进一步拓宽摆式动力吸振器的有效吸振频带, Brzeski等人^[25]将黏性阻尼与摆式动力吸振器结合, 提出了一种耦合黏性阻尼摆式吸振器, 如图2(b)所示. 通过参数分析确定了最优阻尼. Ha等人^[26]提出一种由弹簧和质量组成的弹簧摆式吸振器, 用于降低结构在两个频率上的振动, 如图2(c)所示. Zhang等人^[27]提出了一种参数激励摆式吸振器, 对其动力学响应进行了分析, 研究发现, 与传统摆式吸振器相比, 该结构

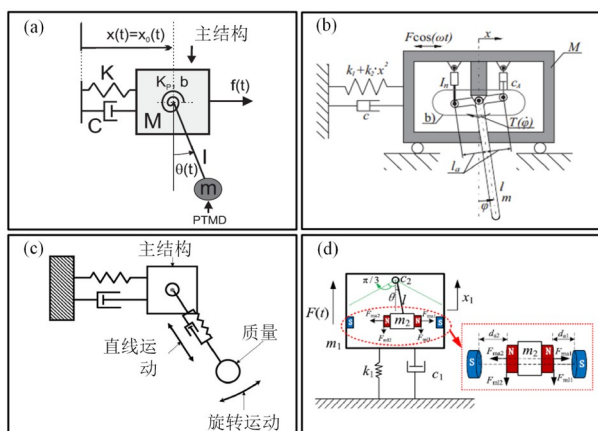


图2 (网络版彩图)摆式动力吸振器。(a) 摆式吸振器; (b) 耦合黏性阻尼摆式吸振器; (c) 弹簧摆式吸振器; (d) 参数激励摆式吸振器

Figure 2 (Color online) Pendulum dynamic vibration absorber. (a) Pendulum DVA; (b) DVA with additional viscous damper and inerter attached to the pendulum; (c) spring pendulum DVA; (d) autoparametric pendulum absorber.

可实现更小的振幅响应和更宽的有效频带, 如图2(d)所示。

在吸振器中引入惯容器^[28~30]、滚子^[31,32]、杠杆^[33,34]等结构以提高系统等效惯量, 也是实现低频吸振的一种有效途径。Barredo等人^[19]设计了一种惯容式吸振器, 如图3(a)所示, 并采用 H_∞ 和 H_2 对系统参数进行了优化, 其吸振性能优于普通动力吸振器。随后, Wang等人^[35]设计了四种含有惯容器的负刚度吸振器, 基于 H_∞ 优化准则分别推导了在谐波激励和随机激励下的最优参数。Zuo等人^[36]对比了三种惯容式吸振器, 并给出了最优参数估算公式。Chen等人^[32]提出一种调谐滚子吸振器, 该装置通过单个或多个钢球的滚动进而降低风致结构振动。Bian等人^[37]提出了一种基于X型结构的非线性吸振器, 如图3(b)所示, 可以有效抑制低频振动。Shen等人^[38]提出了一种杠杆式动力吸振器, 克服了动力吸振器质量比受实际条件限制而影响振动抑制效果的问题。结果表明, 杠杆式动力吸振器能够在更小质量比条件下实现比传统吸振器更好的吸振效果。随后, 通过将负刚度引入杠杆式吸振器, 进一步提升了低频吸振效果^[39], 如图3(c)所示。Yu等人^[40]将杠杆式吸振器与准零刚度结合, 以增强低频振动抑制能力。Su等人^[41]在主结构和惯性阻尼系统之间连接柔性杠杆, 设计了一种杠杆式惯性阻尼吸振器, 如图3(d)所示。并结合 H_∞ 与 H_2 优化的优点, 提出了一种混合优化方

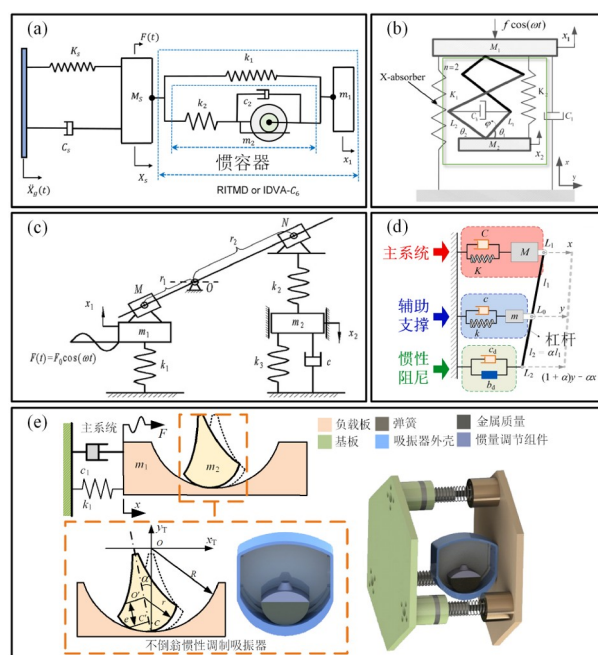


图3 (网络版彩图)惯性调谐式吸振器。(a) 惯容式吸振器; (b) X型吸振器; (c) 杠杆式吸振器; (d) 杠杆式惯性阻尼吸振器; (e) 不倒翁惯性式吸振器

Figure 3 (Color online) Tuned inertia dynamic vibration absorbers. (a) Inerter-based DVA; (b) X-absorber; (c) lever-type DVA; (d) lever-type inerter-based vibration absorber; (e) tumbler-inspired DVA.

法^[42]。随后, 将该方法应用于具有负刚度的杠杆式吸振器、压电分流阻尼系统以及负刚度调谐质量阻尼器上, 从而获得了响应的最优解析解。除了上述结构外, 不倒翁作为一种惯性式结构, 其特征在于结构的质心位置低于几何形心, 且其摆动中心并非固定点, 这与传统的摆或倒立摆结构有所不同。在初始倾角或外力作用下, 不倒翁将发生低频摆动^[43]。基于该特性, Pan等人^[44]提出了一种不倒翁惯性式吸振器, 并采用了多目标遗传算法对系统参数进行了优化, 提升其低频吸振性能, 如图3(e)所示。研究表明, 该吸振器在0.25~1的固有频率比范围内均表现出良好的振动抑制性能。

被动式吸振器的刚度、阻尼及质量等参数在工作时一般保持恒定, 因此其有效吸振频带相对固定。尽管通过引入惯量等手段可在一定程度上拓展其吸振频带, 还难以对激励频率的变化进行自适应调节。此外, 被动式吸振器由于结构形式简单、性能稳定、可靠性高及鲁棒性好等优势, 适用于激励频率相对稳定的工程结构。

3 可调谐式吸振器

被动式吸振器存在工作频率敏感性强、频率适应性差等缺陷, 基于此, 研究人员通过对吸振器结构与参数调控等方式发展了能够实现固有频率可控调节的可调谐式吸振器, 以提升其宽频吸振性能。根据吸振器参数调谐方式的不同, 可调谐式吸振器通常可分为刚度调谐、阻尼可调和质量调谐三类, 如图4所示。下文将围绕上述三类可调谐吸振器的调控机制、原理及应用等进行归纳与总结。

3.1 刚度调谐式吸振器

刚度调谐是吸振器常见的调谐方式, 其实现途径主要包括采用智能材料和设计调谐机构两种形式。常见的智能材料有压电材料^[45-48]、磁流变弹性体^[49-52]、形状记忆合金^[53-56]等。在压电材料应用方面, Bonello等人^[57]利用压电材料设计了一种半主动吸振器, 通过压电片控制片簧的曲率, 实现吸振器的频率调谐。Yamada等人^[58]提出了一种压电式二自由度吸振器, 并对吸振器进行了参数优化。Lossouarn等人^[59]研究了一类具有电路非线性为主结构非线性的压电式吸振器。磁流变弹性体在外加磁场作用下力学性能可快速变化。尹帅虎等人^[60]基于磁流变弹性体构建了变刚度吸振器。Song等人^[61]通过在磁流变弹性体中添加相容剂降低阻尼, 结合刚度调控策略实现了对目标频段振动的有效抑制。形状记忆合金则利用温度诱导相变改变材料刚度, Wang等人^[62]研制了一种镍钛合金钢丝绳吸振器, 最大振动抑制效果可达55.42%。

调谐机构通常通过改变弹性元件参数实现刚度可调。电磁调控技术因其磁力大小可通过电流或磁场强度实现连续调节, 成为刚度调谐的常用手段之一。Lo Feudo等人^[63]基于磁结构提出了一种非线性吸振器, 通过调节磁体间距实现刚度调谐。Ding等人^[64]针对镜面铣过程的振动问题, 提出半主动电磁吸振器, 通过调控电流实现刚度调节, 如图5(a)所示。Liu等人^[65]利用电磁结构设计了一种半主动吸振器, 获得了21~25 Hz的吸振频带。电磁刚度调谐技术也可被用来抑制扭转振动。Xiang等人^[66]将电磁吸振器应用于旋转机械以降低扭转振动, 通过调节刚度与阻尼系数, 可在整个工作频率范围内实现振动抑制。为提高吸振器低频减振性能, Jin等人^[67]将磁悬浮技术与动力吸振器相结合, 提出了

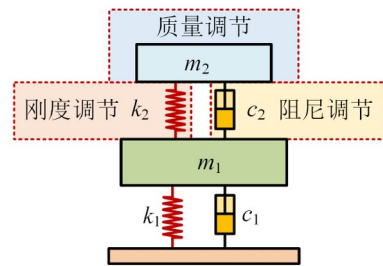


图4 (网络版彩图)可调谐式吸振器的基本原理

Figure 4 (Color online) Principle of tunable dynamic vibration absorbers.

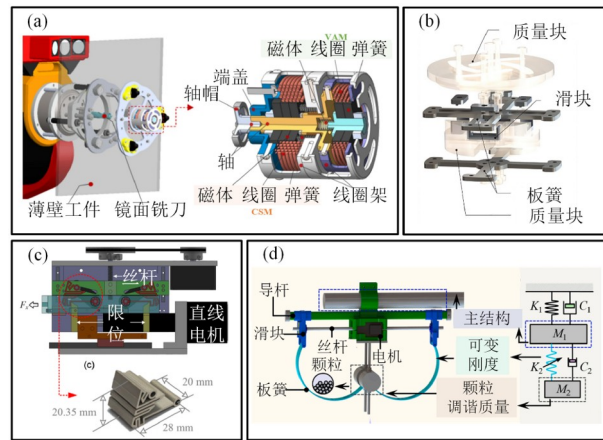


图5 (网络版彩图)刚度调谐式吸振器。(a) 电磁吸振器; (b) 旋转调谐刚度吸振器; (c) 可变弹簧刚度吸振器; (d) 变刚度颗粒阻尼器

Figure 5 (Color online) Stiffness-tuned dynamic vibration absorber. (a) Electromagnetic absorber; (b) rotational stiffness-tuned absorber; (c) variable spring stiffness absorber; (d) variable-stiffness particle damper.

磁悬浮式动力吸振器, 显著降低了主系统在一阶模态频率下的振动。除电磁调谐外, 刚度机构构型设计也是实现吸振器刚度可调的重要途径。Guo等人^[68]通过改变拱形结构厚度实现等效刚度调节, 用于抑制轴承转子振动。Shui等人^[69]通过带有两个限位块的滑块限制弹性元件的偏转, 实现了较宽频率范围的固有频率调节。Ma等人^[70]提出了一种快拆吸盘式旋转调谐吸振器, 如图5(b)所示, 该吸振器可有效降低薄壁结构加工过程中的刀具振动, 其原理是通过旋转改变板簧的有效弦长, 从而调节系统刚度。Guo等人^[71]受尺蠖爬行启发, 设计了一种由屈曲板簧阵列组合的弹性单元, 可在电机驱动下改变板簧曲率, 实现刚度调节。Wu等人^[72]提出了一种可变弹簧刚度吸振器, 调节两个并联弹簧的对称旋转实现在8.1~17.2 Hz频率范围内的有效减

振, 如图5(c)所示. Zhu等人^[73]将屈曲板簧与颗粒阻尼相结合, 构建了一种变刚度颗粒阻尼器, 实现了5~10 Hz的宽频吸振, 如图5(d)所示.

3.2 吸振器的阻尼调控

吸振器的阻尼调控主要通过调节系统阻尼配比, 使得系统达到最佳吸振效果. 阻尼调控主要包括磁流变阻尼^[74~77]、连续阻尼控制(continuous damping control, CDC)^[78,79]、电磁分支电路阻尼^[80~82]等方式. 磁流变阻尼通过外加磁场改变其流变特性, 实现阻尼力的调节. Hu等人^[83]设计了一种混合磁流变阻尼器, 在有限体积条件下实现了较大的阻尼调节范围, 如图6(a)所示. Gao等人^[84]提出了自供能磁流变阻尼器, 并将其应用于车辆悬架系统, 降低了外部能量需求, 提高了系统工程适用性. 吴源良等人^[85]针对刚度与阻尼均可调的磁流变吸振器, 提出了相应的半主动控制策略, 实现了频率同调与阻尼寻优的协同控制, 在6~30 Hz范围内取得了良好的振动抑制效果. He等人^[86]将磁流变阻尼器应用于轧机系统, 抑制其非线性垂直振动. CDC阻尼器具有成本低、性能稳定等优点. 针对CDC阻尼器非线性特征精确建模困难的问题, Li等人^[87]提出了一种参数模型与神经网络相结合的混合建模方法, 实现了阻尼器参数的高精度辨识.

电磁分支电路阻尼可通过调节电路参数提升电磁阻尼力, 改善吸振效果, 近年来在振动抑制领域得到了广泛关注^[88]. Kassem等人^[89]将气动弹性系统的全无量纲数学模型引入主动控制吸振器, 有效提升了系统的振动抑制效果. 李斌等人^[90]针对飞机垂尾抖振问题, 基于电涡流耗能机理设计了一种可调阻尼式动力吸振器, 在保持结构紧凑的同时实现了最高达98%的振幅衰减. 为进一步增强电磁阻尼效果, 研究者围绕机电耦合机制展开了多种改进. 简彬等人^[91]分析了两种非线性电磁吸振器, 通过优化机电耦合系数和电阻等参数, 实现了系统吸振性能的最优配置, 如图6(b)所示. Yan等人^[92]提出了负电阻电磁分支电路的概念, 通过负电阻电路抵消线圈内阻, 显著增强了电路对振动抑制效果. Sun等人^[93]将库伦摩擦阻尼器、负阻抗转换器与电磁阻尼器相结合, 扩展了可调谐阻尼范围, 如图6(c)所示. Vashisht^[94]将负阻抗电磁分支电路阻尼应用于柔性转子系统, 通过调节电阻与线圈匝数实现最优阻尼匹配, 如图6(d)所示. 此外, 在结构与控制方法

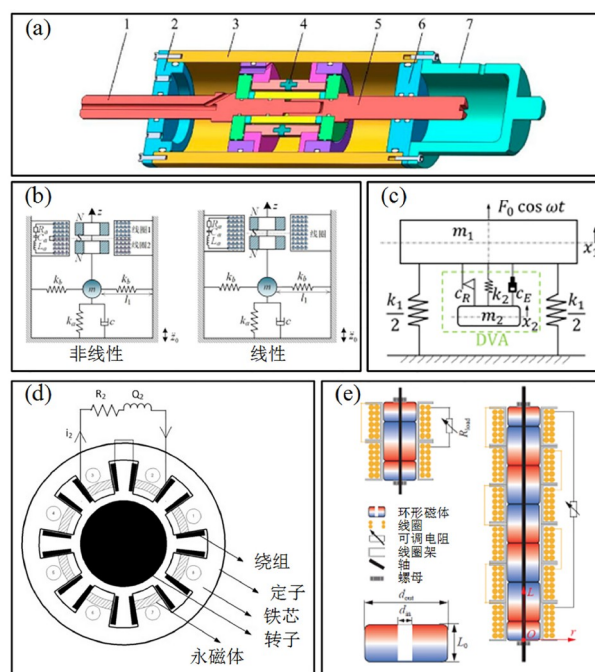


图6 (网络版彩图)吸振器阻尼调控技术. (a) 磁流变阻尼器; (b) RLC吸振电路; (c) 库伦摩擦和电磁耦合阻尼器; (d) 转子负阻抗电磁分支电路; (e) 对置式电磁分支电路阻尼器
Figure 6 (Color online) Damping of the dynamic vibration absorber. (a) Magnetorheological damper; (b) RLC shunt circuit; (c) coulomb friction-electromagnetic coupled damper; (d) negative-impedance electromagnetic shunt circuit with rotor; (e) the electromagnetic shunt damper with opposing magnet pairs.

协同优化方面, Jung等人^[95]提出结合滚动时域控制的电磁同步开关阻尼器, 用于改善悬臂梁的阻尼特性. Sun等人^[96,97]采用永磁体同极对置的方法提高电磁分支电路阻尼的机电耦合系数, 并将其应用于悬置式和接地式吸振系统, 实现了基于 H_∞ 优化准则的最优吸振效果^[98], 如图6(e)所示.

3.3 质量调谐式吸振器

质量调谐最简单的方式就是改变质量单元的质量或惯量. 高强等人^[99]研究了变质量吸振器的模糊控制策略, 使系统始终处于最佳调谐状态. Shakib等人^[100]提出一种自适应质量调谐吸振器, 通过调节支撑杆位置改变转动惯量, 实现吸振器固有频率的连续可调, 如图7所示. Megahed等人^[101]针对二自由度主系统振动问题, 提出可变惯量吸振器, 结合优化算法, 使吸振系统参数始终维持在最优区域附近. 为增强吸振器低频振动抑制能力, 研究者引入负刚度或惯容等机制.

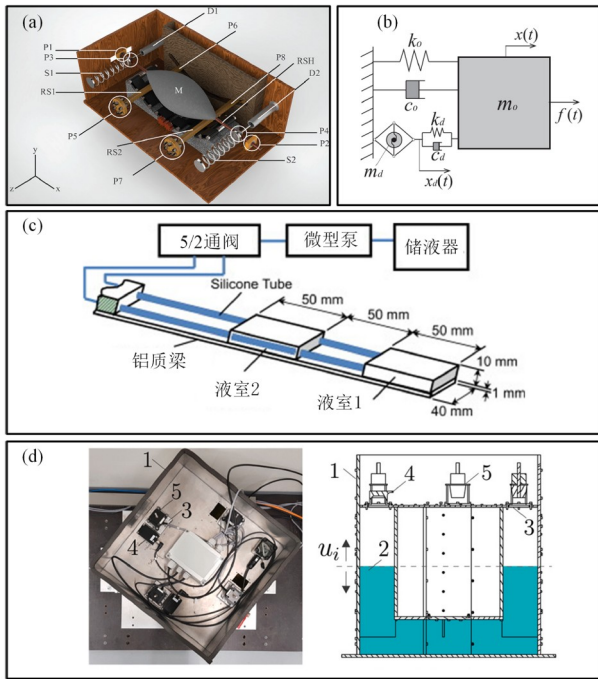


图 7 (网络版彩图)质量调谐吸振器. (a) 变质量惯性矩吸振器; (b) 惯容耦合菱形结构吸振器; (c) 液体质量可调吸振器; (d) 半主动全向调谐液柱吸振器

Figure 7 (Color online) Tuned mass dynamic vibration absorber. (a) Variable mass moment absorber; (b) inerter-based DVA with a rhombus truss structure; (c) vibration absorber with tunable liquid mass; (d) semi-active omnidirectional tuned liquid column damper.

刘刚等人^[102]提出变质量-负刚度吸振器,显著提升了吸振器在低频区的振动抑制效果. Dong等人^[103]基于磁流变效应构建可变惯性飞轮吸振器,实现惯量的半主动调节. Wang等人^[104]提出自调节可变摆式调谐质量阻尼器,通过改变摆的长度实现频率调谐. Chowdhury等人^[105]设计了一种负刚度惯性吸振器,通过改变惯性质量比可实现等效质量的调控. Alotta等人^[106]将惯容与菱形结构相结合,提出了改进型惯容吸振器.通过改变菱形结构角度,可以显著调节吸振器的惯量,并实现在一定频率范围内的调谐,如图7(b)所示. Lei等人^[107]提出一种带有颗粒阻尼的复合吸振器,并基于固定点理论对参数进行优化,减轻了吸振器的附加质量并改善了系统吸振性能.

由于液体质量易于调节且质量分布灵活,液体型结构也被广泛用于质量调谐吸振器设计. Lee等人^[108]通过调控吸振器液腔之间的流体分配,实现吸振质量的变化,从而调节吸振器固有频率,如图7(c)所示. 随

后他们将该液体调谐吸振器与压电换能器相结合,在质量调谐吸振的同时实现了振动能量俘获^[109]. Yao等人^[110]提出了一种基于液体弹性腔的可调谐吸振器,利用主系统管路中的液体作为吸振质量. Guo等人^[111]设计了一种填充液态金属的可调谐吸振器,实现了能量俘获与半主动振动抑制的协同控制. Mehrkian等人^[112]提出了一种半主动全向调谐液柱吸振器,可在不改变液体总质量时调节其固有频率,如图7(d)所示. Gao等人^[113]提出了一种悬臂式吸振器,通过向悬臂梁端部玻璃瓶中注入或抽出液体实现吸振器质量的连续调节.

综上所述,国内外研究者已对可调谐式吸振器开展了深入研究,一般通过调节刚度、阻尼或等效质量等方式实现与主系统的动态适配,从而达到最佳吸振效果. 可调谐式吸振器具有结构简单,调谐响应速度快,稳定性高等特点. 然而,其调控与控制过程往往需要外部传感器对振动信号进行信号采集与状态识别,这对实际工况下的系统鲁棒性提出了更高要求.

4 非线性能量汇

非线性能量汇最早由Roberson^[114]提出,是通过非线性动力学设计消除线性刚度构建的强非线性吸振器,如图8所示. 相较于传统线性吸振器, NES可以自适应捕获宽频范围内的振动,靶向转移主结构的振动能量,从而高效抑制结构振动,且具有不新增、不改变主结构共振频率的优势^[115]. 非线性刚度提供非线性恢复力,使得系统表现出不恒定的自振频率,因此NES可与主结构发生瞬时共振捕获(transient resonance capture),从而扩大其有效频带. NES与主体结构之间的能量传递称为靶向能量传递,其特点是快速且单向^[116]. 当共振捕获发生时,振动能量迅速从主体结构传递到NES,并通过阻尼耗散. Gourdon等人^[117]采用两个线性弹簧的组合产生立方非线性刚度,构建了NES,实现了

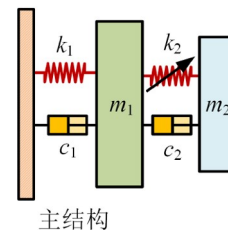


图 8 (网络版彩图)非线性能量汇原理图
Figure 8 (Color online) Principle of nonlinear energy sinks.

振动的靶向能量传递. 通过合理设计非线性恢复力形式、附加质量或惯量分布, NES的动力学特性可向低频区间延展, 实现低频宽带振动抑制. 为提高NES振动抑制性能, 学者们采用多种非线性刚度机制构建NES结构, 主要可分为多稳态式、轨道式、旋转式以及非光滑非线性能量汇. 下文将对这几类NES构型与减振机制进行详细分析.

4.1 多稳态式非线性能量汇

非线性能量汇通常需要在外部激励达到一定阈值后, 才能触发有效的靶向能量传递. 为降低这一能量阈值, 研究者们提出了多稳态式NES. Chen等人^[118]提出并联耦合非线性能量汇模型, 将两个NES通过弹簧连接形成并联系统. 在特定参数范围内, 该并联结构可获得优于单自由度NES的振动抑制效果. Chen等人^[119]引入了两个水平浮能结构以降低双稳态势垒, 从而在较低的激发阈值下实现靶向能量传递. 基于永磁体或屈曲结构也是构建多稳态的常见形式, Fang等人^[120]采用永磁体构建了双稳态NES, 分析了其在冲击激励下的非线性动力学行为与靶向能量传递特性, 如图9(a)所示. Lo Feudo等人^[63]设计了一种可调参数的双稳态NES, 实现了对三层结构的振动抑制, 如图9(b)所示. Yao等人^[121]采用屈曲梁设计了一种双稳态NES用于抑制转子振动. 随后提出由永磁体提供负刚度、屈曲梁提供分段线性正刚度的三稳态NES. 该结构具有良好的减振性能, 并能降低微幅振动^[122]. Zeng等人^[123]基于屈曲梁振子构建双稳态NES, 通过双稳态降低NES的靶向能量传递阈值, 实现了微振动的有效抑制, 如图9(c)所示. Zeng等人^[124]进一步提出可在单稳态、双稳态与三稳态之间切换的多稳态NES, 如图9(d)所示.

4.2 轨道式非线性能量汇

一般而言, NES结构的非线性刚度通过弹性元件几何变形产生. 通过合理结构与动力学设计, 曲形轨道同样能够构建非线性刚度. Wang等人^[125]利用轨道与滚子设计了一种轨道式NES, 通过设计轨道形状产生所需的非线性刚度, 如图10(a)所示. 随后, 引入单侧冲击使NES在有效的空间中实现更好的振动抑制效果^[126]. Lu等人^[127]设计了一种轨道式NES, 在0.02的质量比下可使主系统响应降低约50%. Zuo等人^[128,129]通过设计了一种具有二阶和四阶多项式恢复力的轨道式

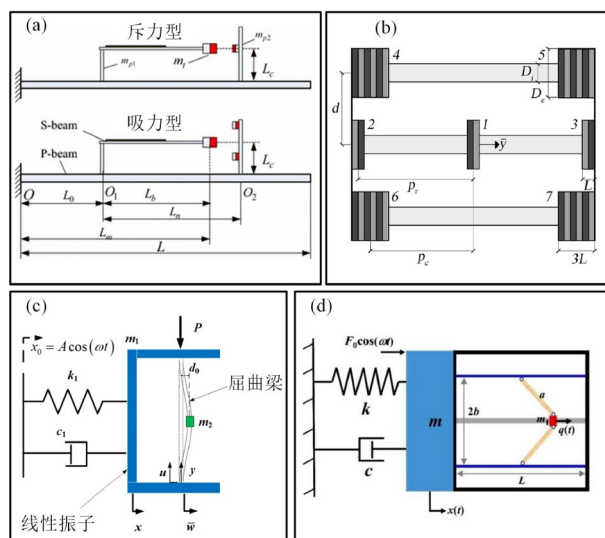


图9 (网络版彩图)多稳态式NES结构. (a) 双稳态NES; (b) 永磁式双稳态NES; (c) 屈曲梁式双稳态NES; (d) 多稳态式NES

Figure 9 (Color online) Multi-stable NES. (a) Bistable NES; (b) magnet-based bistable NES; (c) bistable NES constructed by a buckling beam; (d) multi-stable NES.

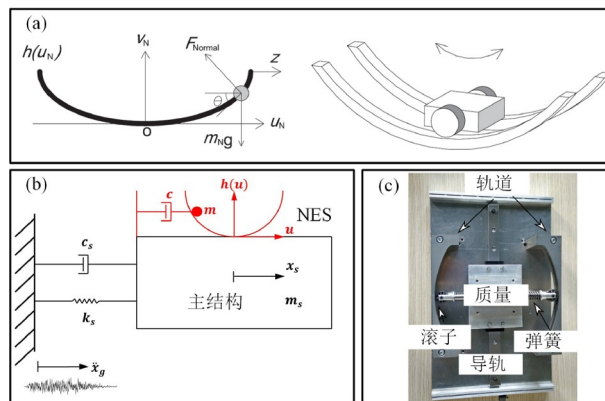


图10 (网络版彩图)轨道式NES结构. (a) 轨道式NES; (b) 二阶和四阶多项式恢复力NES; (c) 弹簧轨道式NES

Figure 10 (Color online) Track NES. (a) Track NES; (b) track NES with second-order and fourth-order polynomials; (c) NES combined with curved track and coil spring.

NES, 使其在小激励下即可实现较好的振动抑制, 如图10(b)所示. Zou等人^[130]通过预压弹簧和滚子提出了一种轨道式NES, 该装置不仅能够实现振动抑制, 还兼具隔振功能. 随后提出了可定制化的轨道式NES, 其轨道形状可根据需求任意设计^[131]. Dekemele等人^[132]将弹簧与轨道式NES相结合, 提出一种弹簧轨道式NES, 该结构具有抑制多模态振动的能力, 如图10(c)所示.

4.3 旋转式非线性能量汇

除了上述直线运动的非线性能量汇外, 非线性能量汇也可被设计为旋转式结构. Gendelman等人^[133]和 Benarous等人^[134]设计了偏心质量式NES. 与相同质量的纯立方刚度NES相比, 该结构能够在更宽的初始能量范围内保持有效的振动抑制性能, 如图11(a)所示. Saeed等人^[135]提出了一种带有旋转阻尼和径向阻尼的旋转-振荡型NES, 用于抑制多层结构的振动, 如图11(b)所示. Wang等人^[136]在圆环上设计了六个凸轮, 凸轮旋转并挤压周围的弹簧, 构造非线性能量汇, 如图11(c)所示. Yao等人^[137]设计了一种具有分段刚度特性的接地式NES, 用于抑制转子系统的瞬态冲击振动. Friskney等人^[138]采用圆盘作为惯性式NES, 产生非线性恢复力, 以减小小齿轮轴的扭转. Mao等人^[139]提出了一种用于抑制柔性简支结构的旋转式非线性能量汇, 将其安装在结构端部, 可使振动幅值降低约90%, 如图11(d)所示. Tumkur等人^[140]耦合了非线性旋转阻尼和弹性圆柱体来抑制涡激振动. Hubbard等人^[141]设计了一种紧凑型旋转式NES, 能够减小机翼的第一扭转与第二阶弯曲模态振动.

4.4 非光滑非线性能量汇

上述结构大多采用非线性连续刚度, 然而, 引入非光滑刚度同样也可以构建具有强非线性特征的NES结构. Cao等人^[142]设计了非光滑NES, 用于抑制齿轮轴系的扭转振动. Wang等人^[143]构建了非光滑NES模型, 并推导得到了其有效近似解, 如图12(a)所示. Yao等人^[144]提出了一种梳齿式NES, 该结构简单紧凑, 在中等强度激励下振幅衰减可达80%, 如图12(b)所示. 随后, 曹焱博等人^[145]将该结构用于抑制转子-叶片系统的振动, 振幅衰减可达62%. Li等人^[146]提出了一种具有分段式刚度的NES, 在实现有效振动抑制的同时兼具振动能量俘获功能.

此外, 利用碰撞也可构造非光滑非线性能量汇. Nucera等人^[147]采用碰撞式NES抑制了三层结构的振动, 降低了振动对主结构的损伤程度. Geng等人^[148]分析了碰撞式NES对悬臂梁前三阶模态振动的控制效果, 其减振性能与具体模态特征相关, 需要针对模态特性进行优化设计. Wei等人^[149]将碰撞式NES与经典NES相结合, 并采用两种评价指标对其性能进行了评

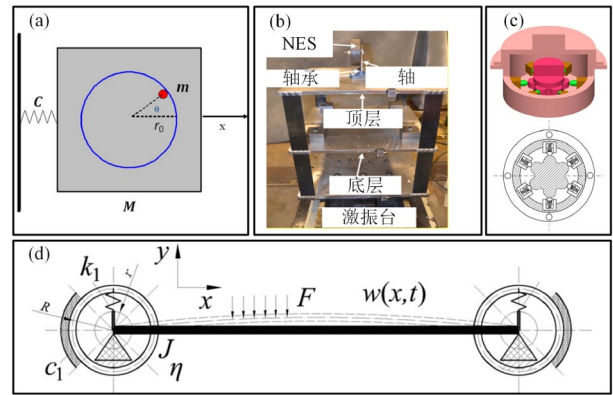


图11 (网络版彩图) 旋转式NES结构. (a) 偏心质量式NES; (b) 用于多层结构的旋转式NES; (c) 扭转式NES; (d) 用于抑制柔性简支梁振动的NES结构

Figure 11 (Color online) Rotatory NES. (a) Eccentric rotational NES; (b) rotatory NES for multi-layer structure vibration control; (c) torsional NES; (d) torsional NES for flexible structures.

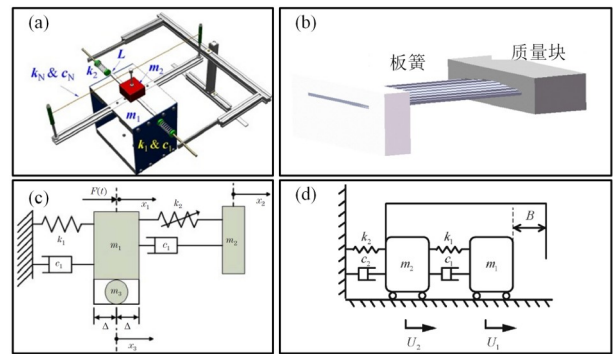


图12 (网络版彩图) 非光滑非线性能量汇. (a) 非光滑NES; (b) 梳齿式NES; (c) 碰撞耦合经典NES模型; (d) 碰撞式NES

Figure 12 (Color online) NES with non-smooth stiffness feature. (a) Nonlinear structure with discontinuous stiffness; (b) comb-shape NES; (c) parallel NES; (d) vibro-impact NES.

估, 如图12(c). Li等人^[150]探讨了单侧碰撞式NES的能量传递机理, 并提出冲击指标来评估其减振性能, 如图12(d)所示. AL-Shudeifat等人^[151]提出了一种非对称碰撞式NES, 实现了能量的快速吸收. 总体而言, 非光滑刚度NES在振动抑制方面展现出较强的非线性能量耗散能力, 但刚度不连续性也使系统动力学方程更加复杂, 对理论分析与数值求解提出了更高要求.

非线性能量汇无需精确调谐即可与主系统形成共振俘获, 具有优异的宽频吸振性能. 然而, 非线性能量汇对阻尼条件较为敏感, 频率调控难度较大, 同时, 非线性容易诱发混沌、分岔、突跳等复杂动力学行为,

给系统稳定性带来了一定的挑战。

从吸振频带、优缺点及应用场景等方面对比了上述三类吸振器如表1所示。被动式吸振器结构简单、可靠性高、工程实现性强,适用于激励频率相对稳定且对鲁棒性要求较高的工况。可调谐式吸振器通过对系统参数的动态调节实现与主系统的频率匹配,能够在工况变化和频率漂移条件下维持较优的振动抑制性能,适合工况多变、振动抑制性能要求高的场景。非线性能量汇则在宽频激励、冲击载荷和强非线性等复杂动力学环境中表现出独特优势。总体而言,不同类型吸振器在振动抑制机理、性能边界和工程适用性方面各具侧重,其选型应综合考虑激励特性、系统复杂度、控制需求与工程约束条件,从而实现振动抑制性能与系统稳定性之间的平衡。

5 吸振器优化方法

吸振器参数优化是提升吸振性能的关键,主要包括质量比、刚度比以及阻尼比等。基于系统幅频响应在不同阻尼条件下存在固定点这一特性,研究者提出了以固定点理论为基础的参数优化方法^[152]。在此基础上,发展了以系统最大幅值最小化为目标的 H_∞ 优化准则^[153],并在吸振器优化中得到了广泛应用。Cheung等人^[154]对接地式动力吸振器进一步分析后发现,利用固定点理论得到的最优参数只是在一定质量比范围内的局部最优参数,当系统的频率比超过特定阈值时,主系

统的最大振幅仍有进一步降低的可能。从系统能量角度出发,Crandall等人提出了以系统在全频域范围内振动能量均值最小为目标的 H_2 优化准则^[155],其精确解于2002年由Asami等人给出^[156]。此外,Nishihara等人建立了以主结构振动衰减时间最短为目标的最大稳定性优化准则^[157]。上述三种经典优化准则在吸振器设计中具有代表性,其优化目标与数学定义如表2所示。

在某些简化条件下,上述优化准则可获得解析解。然而,对于大多数工程问题,由于系统强非线性、多自由度、参数耦合复杂等因素,获取解析解过程往往十分复杂,甚至不存在解析解^[42]。因此,相较于解析方法,数值优化算法在吸振器参数优化中具有更强的通用性和实用价值。近年来,研究人员采用多种优化算法对吸振器进行了参数调谐。Habib等人^[158]将等峰法推广为非线性吸振器参数优化。Shi等人^[159]提出了一种基于位移和动能的等峰优化方法,用于非线性调谐惯性阻尼器。

元启发式优化算法(metaheuristic algorithms)作为一类能够有效解决复杂优化问题的算法,通过搜索逐步逼近问题的最优解。因此,相关学者也将该类算法用于寻找吸振系统的全局最优解,主要包括蚁群优化(ant colony optimization, ACO)^[160]、粒子群优化(particle swarm optimization, PSO)^[30,161~163]、和声搜索(harmony search, HS)算法^[164,165]、遗传算法(genetic algorithm, GA)^[20,44,166,167]、人工蜂群(artificial bee colony, ABC)算法^[168]、模拟退火(simulated annealing,

表 1 不同类型吸振器对比

Table 1 Comparison of different types of vibration absorbers

吸振器类型	有效频带	优势	局限性	应用场景特征
被动式吸振器	窄	结构简单,高可靠性	频带窄、激励适应性差	激励频率稳定、可靠性要求高
可调谐式吸振器	宽	能适应变化工况,控制灵活	依赖传感器与控制系统	宽频、变频工况
非线性能量汇	宽	宽频吸振能力强	阻尼敏感、系统复杂、鲁棒性不足	冲击、宽频振动

表 2 吸振器优化准则

Table 2 Dynamic vibration absorber optimization criteria

优化准则	优化目标	数学定义
H_2	减低随机或宽带激励下的平均响应	最小化系统传递函数的 H_2 范数
H_∞	频域内系统最大振动响应幅值最小	最小化系统频率响应函数的最大幅值
最大稳定性	系统在最短时间内达到稳定状态	最大化系统闭环特征根的最小实部

SA)算法^[169,170]、布谷鸟搜索(cuckoo search, CS)算法^[171]、引力搜索算法(gravitational search algorithm, GSA)^[172]和蝙蝠算法(bat algorithm, BA)^[173]等. Hadi等人^[174]使用遗传算法对调谐质量阻尼器参数进行了优化. 周龙华等人^[175]采用遗传算法对含放大机构的负刚度动力吸振器进行了优化, 如图13(a)所示. Ma等人^[176]采用模拟退火算法对多自由度吸振器进行了参数优化, 如图13(b)所示. Xu等人^[177]基于非支配排序遗传算法II(non-dominated sorting genetic algorithm II, NSGA-II)对非线性吸振器性能进行了多目标优化. Wang等人^[115]利用多目标粒子群优化对非线性能量汇参数进行优化, 如图13(c)所示. Jin等人^[178]分别采用和声搜索算法、遗传算法和粒子群算法对调谐质量阻尼器的刚度与阻尼进行优化.

元启发式优化算法相较于传统解析方法具有全局搜索能力强, 对初值和参数的设定要求不高的特点, 特别适用于解决复杂、高维度的优化问题. 然而, 该类方法仍普遍依赖系统的动力学模型, 在迭代过程中计算成本较高, 同时不同元启发式优化算法在不同问题求解上的实用性也不相同, 这使其应用受到一定的限制.

近年来, 机器学习(machine learning, ML)方法作为一种前沿的技术手段, 逐渐被引入工程问题求解当中. 与传统依赖解析模型推导和数值迭代计算的优化方法不同, 机器学习模型通过对大量实验数据或仿真数据的学习, 能够在不依赖精确动力学模型的情况下建立系统输入与输出之间的映射关系, 从而实现对复杂系统行为的预测与参数优化. 由于其对强非线性和高维

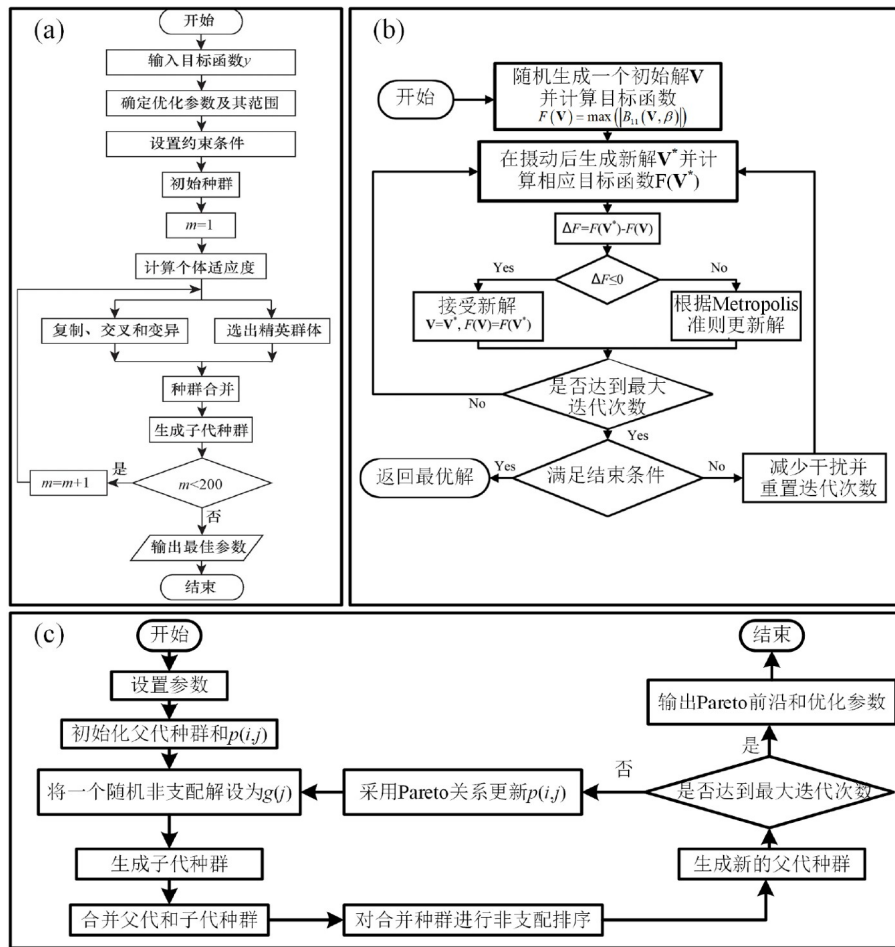


图 13 元启发式优化算法. (a) 遗传算法优化流程; (b) 模拟退火算法多自由度吸振器优化流程; (c) 多目标粒子群优化流程

Figure 13 Metaheuristic algorithms. (a) The flow diagram of GA; (b) the flow diagram of SA; (c) the flow diagram of multi-objective particle swarm optimization.

耦合问题具有良好的适应性, 为复杂吸振系统的参数优化设计提供了一种新的研究思路. Yucel等人^[179]将人工神经网络应用于调谐质量阻尼器的参数优化, 但该方法需借助花朵授粉优化算法(flower pollination algorithm, FPA)获得所有参数样本以构建训练数据集. Yang等人^[180]提出了两种基于机器学习的优化算法, 用于优化吸振器的固有频率与阻尼比, 优化流程如图14所示. Kang等人^[181]结合智能优化算法和反向传播(back propagation, BP)神经网络对串联粘滞质量阻尼器、调谐惯性质量阻尼器、调谐粘滞质量阻尼器和调谐质量阻尼器的最优参数进行了优化. 总体而言, 机器学习方法在降低对动力学模型依赖度方面具有潜在优势, 但其在吸振系统参数优化中的直接应用仍处于探索阶段. 未来研究可进一步聚焦于减少模型训练所需的数据样本规模, 提高算法的收敛效率与泛化能力, 提升机器学习方法在吸振器参数优化中的效率与实用性.

综合考虑优化方法对动力学模型的依赖程度、适用系统类型、是否具备全局最优能力及典型应用场景, 对上述三类优化方法进行对比分析, 如表3所示. 经典优化方法更适用于结构简单、线性或弱非线性系统; 元启发式优化算法在处理强非线性与复杂系统方面具有明显优势, 但其性能对具体问题类型具有较强的依赖性, 不同算法在不同优化问题上收敛速度、稳定性存在差异, 实用性与有效性难以统一评价; 基于机器学习的优化方法以数据驱动为核心, 在完成模型训练后能够实现快速预测, 因此被认为是未来吸振器优化研究的重要发展方向之一.

6 结论与展望

本文系统归纳了近期在被动式吸振器、可调谐式吸振器、非线性能量汇及吸振器参数优化方法等方面的研究进展, 剖析了各类吸振技术的基本特性、调控

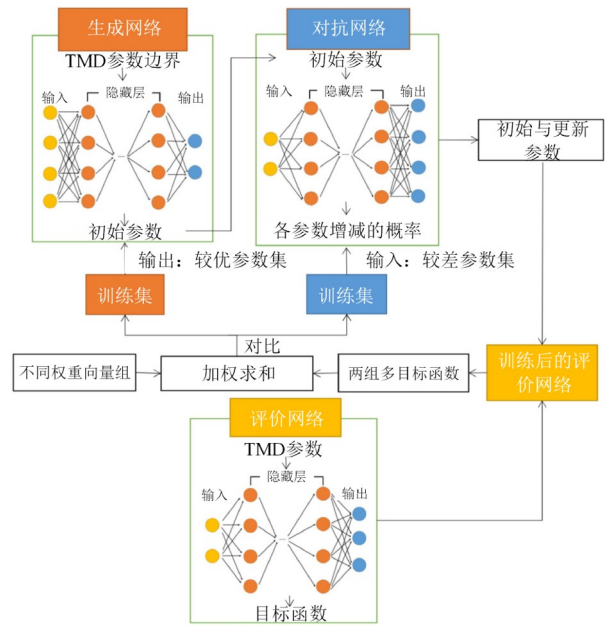


图 14 (网络版彩图)基于神经网络的多目标参数优化流程
Figure 14 (Color online) Neural network flowchart for the multi-objective optimization.

机理与工程应用等方面的发展脉络, 系统分析了其优势与局限性.

吸振技术已由传统的线性被动形式逐步向非线性、可调谐与主动自适应方向发展, 相关研究不断向多学科交叉拓展, 推动了吸振理论体系的持续完善. 不同类型吸振器呈现出互补与融合的发展趋势, 为高端装备的低频宽带振动抑制提供了多样化技术路径. 尽管相关研究已取得显著进展, 但在复杂工况与高性能应用需求的驱动下, 还存在若干关键科学与工程问题亟待深入探索, 未来可重点从以下几个方面开展系统研究.

(1) 多场耦合吸振机制: 高端装备在实际运行中普遍受到力、电、磁、热等多物理场耦合作用, 其振动响应呈现显著的非线性和时变特征. 亟需研究多物理场耦合条件下吸振系统的能量传递与耗散机制, 揭示

表 3 吸振器不同优化算法对比

Table 3 Comparison of optimization algorithms of DVAs

优化方法	模型依赖	适用系统	全局最优性	典型应用场景
经典优化方法	很高	线性或弱非线性、低自由度	局部最优	传统吸振系统
元启发式优化算法	较高	强非线性、多自由度	全局最优	非线性吸振器
机器学习优化算法	较低	强非线性、多自由度	全局最优	自适应吸振

多场作用对吸振性能、系统稳定性及鲁棒性的影响规律, 构建多场耦合动力学建模与分析框架, 以提升吸振系统在复杂工况下的整体性能。

(2) 轻量化、集成化与结构功能一体化: 面向装备对空间尺寸、质量与性能的综合约束需求, 通过几何构型优化、惯性增强机制设计以及新型功能材料的引入, 推动吸振结构向轻量化与高度集成化方向发展, 实现减振、传感、能量俘获等多功能的一体化融合, 提

升系统整体能效与工程实用性。

(3) 智能化与先进控制算法一体化控制: 将吸振结构与先进控制算法深度融合, 构建集感知、决策与控制于一体的智能吸振系统。通过引入自感知技术、机器学习和深度强化学习方法, 实现系统状态的在线识别、参数的自适应调谐与性能的实时优化控制, 进一步提升低频宽带吸振能力以及复杂工况下的稳定性与可靠性。

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A state-of-the-art review and research progress on low-frequency and broadband vibration absorption technology

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As equipment in aerospace and related fields becomes larger, lighter, and more precise, structural modes become increasingly dense. Low-frequency vibration problems become more prominent and seriously affect equipment safety, reliability, and operational stability. Vibration absorption technology suppresses vibration at target frequencies by attaching an absorber to the primary structure and tuning its parameters. It has been widely used in engineering applications because it does not change the stiffness of the primary structure. This paper reviews recent research progress and development trends in low-frequency broadband vibration absorption technologies. It highlights the current state of passive absorbers, tunable vibration absorbers, and nonlinear energy sinks. It also summarizes and compares absorber parameter optimization methods and design criteria. Finally, this paper discusses future development trends in low-frequency broadband vibration absorption. It provides theoretical references and technical support for vibration suppression in high-end equipment.

vibration absorption, dynamic vibration absorber, low-frequency vibration, nonlinear energy sink, vibration suppression

doi: [10.1360/SST-2025-0443](https://doi.org/10.1360/SST-2025-0443)