



基于金刚石NV色芯极化-时间两自由度并行稳健非定域控制非门的远程实现

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摘要 网络不同节点间非定域量子门在量子通信网络与分布式量子计算中有重要应用. 相较于单自由度非定域量子门远程实现, 两自由度并行非定域量子门具有更好并行性并且具有更高信道容量. 利用金刚石单边光学腔平衡条件, 本文提出极化、时间两自由度并行稳健非定域控制非门远程实现方案. 通讯方Alice, Bob让光子依次通过金刚石NV色芯单边光学腔, 对电子与纠缠光子执行单粒子测量并执行与测量结果相应局域么正操作, 即可在光子极化、时间两自由度同时实现非定域控制非门操作. 基于极化、时间两自由度超纠缠态并行非定域量子门在远程量子通信中仅需使用单一传输路径即可完成两自由度并行非定域控制非门, 具有光路简洁且需要传送光子数少的优点. 方案可抑制基于金刚石NV色芯单边光学腔并行非定域控制非门远程实现时由于反射系数不等引起的噪声, 具有可行性强的优点.

关键词 量子通信, 并行非定域量子门, 超纠缠态, 金刚石NV色芯

1 引言

量子态在量子信息中的应用为远程量子通信提供一些新的信息处理方法, 如量子隐形传态(Quantum Teleportation)^[1-8]、量子密钥分发(Quantum Key Distribution)^[9-14]、量子安全直接通信(Quantum Secure Direct Communication)^[15-19]、量子密集编码(Quantum Dense Coding)^[20-22]、量子纠缠纯化与浓缩(Quantum Entanglement Purification and Concentration)^[23-27]、量子态远程制备(Quantum Remote State Preparation)^[28-30]、量子操

控远程实现(Quantum Operation Remote Implementation)^[31-33]、非定域量子门(Quantum Nonlocal Gate)^[34-36]等.

由于双量子比特控制非门(Controlled Not)和单量子比特门可构建普适量子门, 双量子比特控制非门非定域远程实现在量子通信网络和分布式量子计算有重要应用. 基于纠缠信道的非定域控制非门引起了广泛关注. 1999年, Gottesman和Chuang^[37]开展量子门的隐形传送研究. 2000年, Eisert等人^[34]建立了基于双粒子纠缠态的双粒子非定域控制非门以及三粒子非定域

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Toffoli门模型. 2004年, Huang等人^[38]完成了基于线性光学元件的双量子比特非定域控制非门实验. 2009年, Chen等人^[39]提出基于双粒子部分纠缠态的多目标位非定域控制非门概率远程实现方案, 完成了部分纠缠信道下以通讯方Alice粒子A量子态为控制位, 通信方Bob粒子B₁, B₂, B₃为目标位的多目标位非定域控制非门概率远程实现. 2011年, Chen等人^[40]提出基于多粒子Greenberger-Horne-Zeilinger (GHZ)态的高维非定域控制非门模型. 2013年, Wang等人^[41]提出了双光子纠缠信道下基于量子点双边光学腔的电子自旋比特非定域控制非门远程实现方案. 2018年, Chou等人^[42]完成了两个量子逻辑比特之间确定的非定域控制非门远程实现实验. 2019年, Wan等人^[43]完成了基于纠缠离子的确定非定域控制非门实验. 2021年, Daiss等人^[44]通过辅助光子完成了间隔60 m两个原子比特间非定域控制非门. 2023年, Chen等人^[45]基于提出基于量子点双边光学腔光子极化、路径两自由度并行非定域控制非门远程实现方案. 2024年, Liu等人^[46]基于双光子路径纠缠态 $\frac{1}{\sqrt{2}}(|SS\rangle + |LL\rangle)$ 完成了间隔7.0 km两光子极化态间非定域控制非门实验并利用非定域控制非门完成了两节点间Deutsch-Jozsa算法. Qiu等人^[47]完成了间隔64 m超导量子比特间非定域控制非门.

由于在室温条件下具有长相干时间, 基于金刚石NV色芯量子信息处理引起广泛关注^[48-60]. 2010年, Togan等人^[48]完成了金刚石NV色芯中电子与光子纠缠实验. 2012年, Zheng等人^[50]开展了基于金刚石NV色芯GHZ态制备研究. 2013年, Wei和Deng^[51]基于金刚石NV色芯, 完成电子自旋控制非门构建. 2017年, Ren和Deng^[52]利用金刚石NV色芯平衡条件, 构建可克制反射系数不等引起的噪声超并行稳健控制相位门. 2019年, Du和Shi^[55]提出可减少光子数耗散噪声影响的超并行稳健杂化控制非门. 2023年, Xie等人^[57]基于金刚石NV色芯实现了保真度为99.2%的控制非门. 2025年, Dong等人^[60]利用金刚石NV色芯构建了可克制联合噪声的两量子比特控制非门.

在远程量子通信中, 光子由于在光纤信道或自由空间容易传输已成为远程量子通信理想信息载体. 除了极化自由度之外, 光子有路径、角动量、时间、频率等多个自由度可编码量子信息. 基于多自由度同时纠缠的光子超纠缠态的远程量子通信可以提高远程量

子通信信道容量, 引起了研究者广泛关注. 2008年, Barreiro等人^[61]开展了基于超纠缠态量子密集编码研究. 2010年, Barreiro等人^[62]完成了基于光子极化、轨道角动量两自由度同时纠缠双光子超纠缠态 $(|HH\rangle + |VV\rangle)/\sqrt{2} \otimes (|lr\rangle + |rl\rangle)/\sqrt{2}$ 的单光子双量子比特纠缠态 $(|Hl\rangle + |Vr\rangle)/\sqrt{2}$ 远程制备实验, $|H\rangle(|V\rangle)$ 表示水平(垂直)极化光子, $|l\rangle(|r\rangle)$ 表示轨道角动量 $+\hbar(-\hbar)$ 光子. 2011年, Wang等人^[63]提出超纠缠信道下基于量子密集编码高容量量子安全直接通信协议. 2015年, Li和Ghose^[64]提出基于非线性克尔效应多粒子非定域超纠缠浓缩方案. Wang等人^[65]完成超纠缠信道下光子自旋、轨道角动量自由度同时编码多自由度量子隐形传态实验. Graham等人^[66]基于极化、轨道角动量两自由度同时纠缠超纠缠信道, 完成高维赤道态隐形传态. 2017年, Ren等人^[67]提出基于线性光学元件双光子六量子比特态非定域超纠缠浓缩方案. Wu等人^[68]提出基于双光子六量子比特超纠缠态高容量量子安全直接通信方案. 2018年, Nawaz等人^[69]提出基于超纠缠原子的量子态远程制备方案, Zhou等人^[70]提出基于极化、路径两自由度同时纠缠超纠缠光子的任意单量子比特态极化、路径两自由度并行远程制备方案. 2019年, Jiao等人^[71]提出超纠缠信道下基于线性光学元件任意单光子双量子比特杂化态远程制备方案, 讨论了部分超纠缠信道下量子态远程制备. 由于通讯方可依据已知需制备量子态信息对纠缠信道重复演化直到将纠缠信道成功演化为目标信道, 方案可提高远程量子态制备中纠缠信道使用效率. 在远程量子信息处理中选用路径自由度编码量子信息具有量子态可操控性强的优点, 但是需要使用多条路径完成光子传输. 选用时间自由度编码可以优化远程量子信息处理中光子传输线路. 2021年, Wang等人^[72]开展了时间自由度编码多自由度量子态并行远程制备研究. 2024年, Wang和Guo^[73]提出基于非线性克尔效应光子极化、时间自由度量子远程操控模型.

虽然已有一些基于光子系统多自由度并行非定域量子门远程实现协议, 但是大多数方案都是基于光子极化、路径两自由度并行非定域量子门远程实现^[45,73]. 本文提出超纠缠信道下基于金刚石NV色芯光子极化、时间两自由度并行量子控制非门非定域远程实现方案. 通讯方以双光子极化、时间两自由度超纠

缠态为量子纠缠信道, 让光子依次通过金刚石NV色芯单边光学腔, 对电子和超纠缠光子执行单粒子测量并执行与测量结果相应的局域么正操作即可完成光子极化、时间两自由度并行控制非门非定域远程实现. 基于极化、时间两自由度超纠缠态并行非定域量子门在远程量子通信中仅需使用单一传输路径即可完成两自由度并行非定域控制非门, 具有光路简洁且需要传送光子数少的优点.

2 金刚石NV色芯单边光学腔系统

金刚石NV色芯是金刚石晶体内部的一种点缺陷结构, 它由一个氮原子取代碳原子并捕获相邻的晶格空位而形成^[48–52]. 金刚石NV色芯包含6个来自氮原子和空穴周围三个碳原子的电子. 由于自旋相互作用, 导致金刚石NV色芯基态子能级 $|0\rangle(m_s=0)$ 与 $|\pm 1\rangle(m_s=\pm 1)$ 之间存在2.88 GHz能级劈裂. 由于自旋相互作用以及自旋-轨道相互作用, 金刚石NV色芯有6个激发态: $|A_1\rangle = (|E_-|+) - |E_+|-)/\sqrt{2}$, $|A_2\rangle = (|E_-|+) + |E_+|-)/\sqrt{2}$, $|E_x\rangle = |X\rangle|0\rangle$, $|E_y\rangle = |Y\rangle|0\rangle$, $|E_1\rangle = (|E_-|-) - |E_+|+)/\sqrt{2}$, $|E_2\rangle = (|E_-|-) + |E_+|+)/\sqrt{2}$. 其中 $|X\rangle = (|E_-| - |E_+|)/2$, $|Y\rangle = i(|E_-| + |E_+|)/2$. $|E_{\pm}\rangle$ 为角动量沿NV轴投影为 ± 1 的轨道态. 与文献^[52,53]类似, 金刚石NV色芯中的光学跃迁可通过金刚石NV色芯与频率简并、极化非简并光学腔耦合增强. 如图1(b)所示, 激发态 $|A_2\rangle$ 等概率向基态 $|-1\rangle(|+1\rangle)$ 跃迁并释放左旋(右旋)圆偏振光 $|L\rangle(|R\rangle)$. 如图1(a)所示, 金刚石NV色芯单边光学腔输入和输出过程可由腔场算子 $\hat{a}$ 、下降算子 σ_- 和海森堡运动方程描述^[49–51]:

$$\begin{aligned} \frac{d\hat{a}}{dt} &= -\left[i(\omega_c - \omega_p) + \frac{\eta}{2} + \frac{\kappa}{2}\right]\hat{a} - g\sigma_- - \sqrt{\eta}\hat{a}_{in}, \\ \frac{d\sigma_-}{dt} &= -\left[i(\omega_0 - \omega_p) + \frac{\gamma}{2}\right]\sigma_- - g\sigma_z\hat{a}, \\ \hat{a}_{out} &= \hat{a}_{in} + \sqrt{\eta}\hat{a}, \end{aligned} \quad (1)$$

其中, ω_p , ω_c 和 ω_0 分别表示输入光子频率、腔模频率和能级跃迁频率. γ 为NV色芯衰减率, η 为腔衰减率, κ 为腔固有损耗率. g 为金刚石NV色芯单边光学腔耦合强度. $\hat{a}_{in}$ 代表输入算子, $\hat{a}_{out}$ 代表输出算子. σ_z 为翻

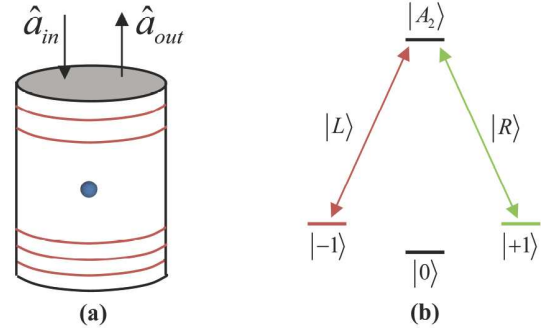


图 1 (网络版彩图) (a) 金刚石NV色芯单边光学腔系统. (b) 金刚石NV色芯能级. $|A_2\rangle$ 为金刚石NV色芯激发态, $|\pm 1\rangle, |0\rangle$ 为基态. 光学跃迁 $|-1\rangle \rightarrow |A_2\rangle$ 由左旋圆极化偏振光子 $|L\rangle$ 驱动, $|+1\rangle \rightarrow |A_2\rangle$ 由右极化偏振光 $|R\rangle$ 驱动

Figure 1 (Color online) (a) The schematic diagram for a diamond NV-center coupling to the optical cavity. (b) The energy level configuration of the diamond NV-center. $|A_2\rangle$ denotes the excited state of the diamond NV-center, $|\pm 1\rangle, |0\rangle$ denote the ground states. The optical transition $|-1\rangle \rightarrow |A_2\rangle$ is driven by the left-circular polarized photon $|L\rangle$, the optical transition $|+1\rangle \rightarrow |A_2\rangle$ is driven by the right-circular polarized photon $|R\rangle$.

转算符. 弱激发条件($\langle\sigma_z\rangle = -1$)下, 金刚石NV色芯单边光学腔反射系数可表示为

$$r(\omega) = \frac{\left[i(\omega_c - \omega_p) - \frac{\eta}{2} + \frac{\kappa}{2}\right]i(\omega_0 - \omega_p) + \frac{\gamma}{2} + g^2}{\left[i(\omega_c - \omega_p) + \frac{\eta}{2} + \frac{\kappa}{2}\right]i(\omega_0 - \omega_p) + \frac{\gamma}{2} + g^2}. \quad (2)$$

在共振条件 $\omega_c = \omega_p = \omega_0$ 下, 有

$$r(\omega) = \frac{(\kappa - \eta)\gamma + 4g^2}{(\kappa + \eta)\gamma + 4g^2}. \quad (3)$$

若耦合强度 $g = 0$, 即NV色芯与光学腔未耦合时反射系数可表示为

$$r_0(\omega) = \frac{\kappa - \eta}{\kappa + \eta}. \quad (4)$$

理想情况下, 由于 $\kappa \ll \eta$ 腔边泄漏可忽略. 此时强耦合($4g^2 \gg \eta\gamma$)状态下反射系数 $r(\omega)$ 趋近于1, 未耦合状态下反射系数 $r_0(\omega)$ 趋近于-1, 则圆偏振光与金刚石NV色芯单边光学腔相互作用可表示为

$$\begin{aligned} |R\rangle|-1\rangle &\rightarrow r_0|R\rangle|-1\rangle, |R\rangle|+1\rangle \rightarrow r|R\rangle|+1\rangle, \\ |L\rangle|-1\rangle &\rightarrow r|L\rangle|-1\rangle, |L\rangle|+1\rangle \rightarrow r_0|L\rangle|+1\rangle. \end{aligned} \quad (5)$$

由于理想状况($\kappa \ll \eta$, $4g^2 \gg \eta\gamma$)在金刚石NV色芯中往往难以实现, 反射系数 $|r(\omega)| \neq |r_0(\omega)|$ 导致非定域控制非门保真度降低. 与文献类似, 平衡条

件 $r = -r_0 = -\frac{F_P}{F_P+2}$ 可抑制由于反射系数不等而引起的基于金刚石NV色芯非定域控制非门保真度降低. 其中普赛尔因子 $F_P = \frac{1-\lambda}{\lambda}$, $\lambda = \kappa/\eta$. 平衡条件下, 圆偏振光与金刚石NV色芯单边光学腔相互作用可表示为^[52]

$$\begin{aligned} |R\rangle|-1\rangle &\rightarrow -r|R\rangle|-1\rangle, |R\rangle|+1\rangle \rightarrow r|R\rangle|+1\rangle, \\ |L\rangle|-1\rangle &\rightarrow r|L\rangle|-1\rangle, |L\rangle|+1\rangle \rightarrow -r|L\rangle|+1\rangle. \end{aligned} \quad (6)$$

在远程量子信息处理实际应用中, 噪声无法完全消除. 噪声影响非定域量子门保真度与远程量子信息处理结果. 为提升远程量子信息处理可靠性, 研究者开展噪声环境下实现高保真度非定域量子门方案研究^[74,75]. 利用金刚石NV色芯单边光学腔平衡条件有效抑制反射系数不等引起的噪声、提高量子门保真度, 本文基于金刚石NV色芯平衡条件构建可克制反射系数不等引起的噪声的两自由度并行稳健非定域控制非门^[72].

3 基于金刚石NV色芯单边光学腔极化、时间两自由度并行非定域控制非门

基于金刚石NV色芯单边光学腔极化、时间两自由度并行非定域控制非门原理如图2所示, 其中圆偏振光极化分束器CPBS (Circular Polarization Beam Splitter)可透射右旋圆极化偏振光 $|R\rangle$ 和反射左旋圆极化偏振光 $|L\rangle$. 分束器BS (Beam Splitter)可执行路径自由度Hadamard门操作^[73,76]:

$$\begin{aligned} |a_1\rangle &\xrightarrow{\text{BS}} \frac{1}{\sqrt{2}}(|a_1\rangle + |a_2\rangle), \\ |a_2\rangle &\xrightarrow{\text{BS}} \frac{1}{\sqrt{2}}(|a_1\rangle - |a_2\rangle), \end{aligned} \quad (7)$$

其中, $|a_1\rangle, |a_2\rangle$ 为光子两个路径态. 玻片 R_{45} 可执行极化自由度Hadamard门操作:

$$\begin{aligned} |R\rangle &\xrightarrow{R_{45}} \frac{1}{\sqrt{2}}(|R\rangle + |L\rangle), \\ |L\rangle &\xrightarrow{R_{45}} \frac{1}{\sqrt{2}}(|R\rangle - |L\rangle). \end{aligned} \quad (8)$$

玻片 X 用于执行极化自由度比特翻转操作:

$$\sigma_X = |R\rangle\langle L| + |L\rangle\langle R|. \quad (9)$$

光开关SW (Optical Switch)使光子依次进入光学腔.

与单一自由度非定域量子门类似, 通讯方Alice光子 A 极化、时间自由度量量子态处于任意单量子比特态:

$$|\varphi\rangle_A = (\alpha_1|R\rangle + \beta_1|L\rangle) \otimes (\gamma_1|t_1\rangle + \delta_1|t_2\rangle), \quad (10)$$

其中 $|\alpha_1|^2 + |\beta_1|^2 = 1$, $|\gamma_1|^2 + |\delta_1|^2 = 1$ 满足归一化条件, $|t_1\rangle, |t_2\rangle$ 为光子 A 两个时间态. 通讯方Bob光子 B 极化、时间自由度量量子态为

$$|\varphi\rangle_B = (\alpha_2|R\rangle + \beta_2|L\rangle) \otimes (\gamma_2|t_1\rangle + \delta_2|t_2\rangle), \quad (11)$$

其中 $|\alpha_2|^2 + |\beta_2|^2 = 1$, $|\gamma_2|^2 + |\delta_2|^2 = 1$ 满足归一化条件. 通讯方希望执行以光子 A 极化、时间自由度量量子态为控制位, 光子 B 极化、时间自由度量量子态为目标位的非定域控制非门操作.

为完成极化、时间两自由度并行非定域控制非门, Alice, Bob共享极化、时间自由度同时双光子纠缠超纠缠态:

$$|\psi\rangle_{CD} = \frac{1}{2}(|RR\rangle + |LL\rangle) \otimes (|t_3t_3\rangle + |t_4t_4\rangle), \quad (12)$$

其中光子 C 发送给通信方Alice, 光子 D 发送给通信方Bob. $|t_3\rangle, |t_4\rangle$ 为光子 C, D 两个时间态.

超纠缠信道下基于金刚石NV色芯单边光学腔极化、时间两自由度并行非定域控制非门原理如图2所示.

光子 A, C, D, B 以及金刚石NV色芯 NV_i ($i = 1, 2, 3, 4$)中电子 e_i 组成复合系统状态为(未归一化):

$$\begin{aligned} |\Psi_0\rangle &= (\alpha_1|R\rangle + \beta_1|L\rangle)(\gamma_1|t_1\rangle + \delta_1|t_2\rangle)_A \\ &\otimes (|RR\rangle + |LL\rangle)(|t_3t_3\rangle + |t_4t_4\rangle)_{CD} \\ &\otimes (\alpha_2|R\rangle + \beta_2|L\rangle)(\gamma_2|t_1\rangle + \delta_2|t_2\rangle)_B \\ &\otimes (|-1\rangle + |+1\rangle)_{e_1} \otimes (|-1\rangle + |+1\rangle)_{e_2} \\ &\otimes (|-1\rangle + |+1\rangle)_{e_3} \otimes (|-1\rangle + |+1\rangle)_{e_4}. \end{aligned} \quad (13)$$

玻片 R_{45} 可执行极化自由度Hadamard门操作. 光子 C, D 经过玻片 R_{45} 后, 复合系统状态转化为(未归一化):

$$\begin{aligned} |\Psi_1\rangle &= (\alpha_1|R\rangle + \beta_1|L\rangle)(\gamma_1|t_1\rangle + \delta_1|t_2\rangle)_A \\ &\otimes (|RR\rangle + |LR\rangle + |RL\rangle - |LL\rangle)(|t_3t_3\rangle + |t_4t_4\rangle)_{CD} \end{aligned}$$

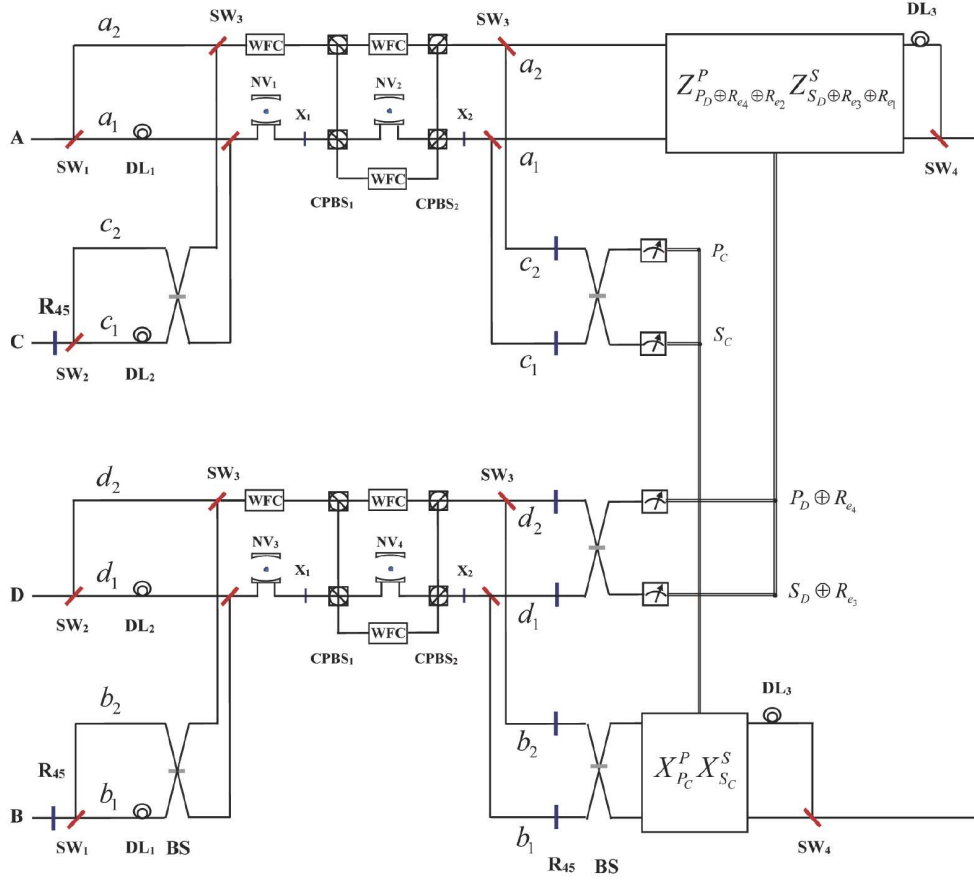


图 2 (网络版彩图) 基于金刚石 NV 色芯并行稳健非定域控制非门原理图. R_{45} 为玻片用于执行极化自由度 Hadamard 操作, BS 为分束器可执行路径自由度 Hadamard 门操作, WFC 为波形矫正器可将量子态 $|a_2\rangle$ 转化为 $|r|a_2\rangle$

Figure 2 (Color online) Scheme diagram for remote implementation of hyperparallel robust nonlocal CNOT operation via diamond NV center. The wave plate R_{45} can implement Hadamard in polarization DoF and BS denotes the beam splitter which can implement Hadamard operation in spatial-mode DoF, WFC represents waveform corrector which transforms state $|a_2\rangle$ to $|r|a_2\rangle$.

$$\begin{aligned} & \otimes (\alpha'_2 |R\rangle + \beta'_2 |L\rangle) (\gamma_2 |t_1\rangle + \delta_2 |t_2\rangle)_B \\ & \otimes (|-1\rangle + |1\rangle)_{e_1} \otimes (|-1\rangle + |1\rangle)_{e_2} \\ & \otimes (|-1\rangle + |1\rangle)_{e_3} \otimes (|-1\rangle + |1\rangle)_{e_4}, \end{aligned}$$

其中,

$$\alpha'_2 = \frac{\alpha_2 + \beta_2}{\sqrt{2}}, \beta'_2 = \frac{\alpha_2 - \beta_2}{\sqrt{2}}. \quad (14)$$

与文献[77,78]类似, 光开关 SW_1 和 SW_2 可以使 t_1 时间到达的光子 A , B 和 t_3 时间到达的光子 C , D 继续沿原来路径 a_1 , b_1 和 c_1 , d_1 传输, t_2 时间到达的光子 A , B 和 t_4 时间到达的光子 C , D 传输路径切换为 a_2 , b_2 和 c_2 , d_2 . 延迟 DL_1 和 DL_2 使得路径 a_1 , b_1 和 c_1 , d_1 中光子到达时间转

换为 t_2 和 t_4 . 光子 A , B 和 C , D 经过 SW_1 , DL_1 和 SW_2 , DL_2 后, 复合系统状态转化为

$$\begin{aligned} |\Psi_2\rangle = & (\alpha_1 |R\rangle + \beta_1 |L\rangle) (\gamma_1 |a_1\rangle + \delta_1 |a_2\rangle) |t_2\rangle_A \\ & \otimes (|RR\rangle + |LR\rangle + |RL\rangle - |LL\rangle) \\ & (|c_1 d_1\rangle + |c_2 d_2\rangle) |t_4 t_4\rangle_{CD} \\ & \otimes (\alpha'_2 |R\rangle + \beta'_2 |L\rangle) (\gamma_2 |b_1\rangle + \delta_2 |b_2\rangle) |t_2\rangle_B \\ & \otimes (|-1\rangle + |1\rangle)_{e_1} \otimes (|-1\rangle + |1\rangle)_{e_2} \\ & \otimes (|-1\rangle + |1\rangle)_{e_3} \otimes (|-1\rangle + |1\rangle)_{e_4}. \end{aligned} \quad (15)$$

与极化自由度类似, 分束器 BS 用于执行路径自由度 Hadamard 操作. 光子 C , B 经过 BS_1 后, 光子 A , C 复合

系统状态转化为

$$\begin{aligned}
 |\Psi_3\rangle = & (\alpha_1|R\rangle + \beta_1|L\rangle)(\gamma_1|a_1\rangle + \delta_1|a_2\rangle)|t_2\rangle_A \\
 & \otimes (|RR\rangle + |LR\rangle + |RL\rangle - |LL\rangle) \\
 & (|c_1d_1\rangle + |c_2d_1\rangle + |c_1d_2\rangle - |c_2d_2\rangle)|t_4t_4\rangle_{CD} \\
 & \otimes (\alpha'_2|R\rangle + \beta'_2|L\rangle)(\gamma'_2|b_1\rangle + \delta'_2|b_2\rangle)|t_2\rangle_B \\
 & \otimes (|-1\rangle + |1\rangle)_{e_1} \otimes (|-1\rangle + |1\rangle)_{e_2} \\
 & \otimes (|-1\rangle + |1\rangle)_{e_3} \otimes (|-1\rangle + |1\rangle)_{e_4}.
 \end{aligned} \quad (16)$$

其中,

$$\gamma'_2 = \frac{\gamma_2 + \delta_2}{\sqrt{2}}, \quad \delta'_2 = \frac{\gamma_2 - \delta_2}{\sqrt{2}}. \quad (17)$$

与文献[77,78]类似, 光开关SW₃可使光子A, C和D, B依次进入金刚石NV色芯. 路径a₂中波形矫正器WFC (WaveForm Corrector)将量子态|a₂⟩转化为|a₂⟩用于消除平衡状态下反射系数对非定域量子门保真度影响. 为执行两自由度并行非定域控制非门, Alice和Bob使手中光子A, C和D, B依次进入金刚石NV色芯NV₁, NV₂和NV₃, NV₄, 对金刚石NV色芯中电子自旋、光子C, D执行正交投影测量并依据测量结果对光子A, B执行相应局域么正操作即可完成极化, 时间两自由度并行非定域控制非门操作. 如图2所示, 光子A和D分别经过NV₁, X₁, CPBS₁, NV₂, CPBS₂, X₂和NV₃, X₁, CPBS₁, NV₄, CPBS₂, X₂后, 复合系统状态演化为

$$\begin{aligned}
 |\Phi\rangle = & \left[|-1\rangle_{e_1} (\gamma_1|a_1\rangle - \delta_1|a_2\rangle) \right. \\
 & \left. + |1\rangle_{e_1} (\gamma_1|a_1\rangle + \delta_1|a_2\rangle) \right] \\
 & \otimes \left[|-1\rangle_{e_2} (\alpha_1|R\rangle + \beta_1|L\rangle) + |1\rangle_{e_2} \right. \\
 & \left. (\alpha_1|R\rangle - \beta_1|L\rangle) \right] |t_2\rangle_A \\
 & \otimes \left[|-1\rangle_{e_3} (|c_1d_1\rangle + |c_2d_1\rangle - |c_1d_2\rangle + |c_2d_2\rangle) \right. \\
 & \left. + |1\rangle_{e_3} (|c_1d_1\rangle + |c_2d_1\rangle + |c_1d_2\rangle - |c_2d_2\rangle) \right] \\
 & \otimes \left[|-1\rangle_{e_3} (|RR\rangle + |LR\rangle + |RL\rangle - |LL\rangle) \right. \\
 & \left. + |1\rangle_{e_3} (|RR\rangle + |LR\rangle - |RL\rangle + |LL\rangle) \right] |t_4t_4\rangle_{CD} \\
 & \otimes (\alpha'_2|R\rangle + \beta'_2|L\rangle)(\gamma'_2|b_1\rangle + \delta'_2|b_2\rangle)|t_2\rangle_B.
 \end{aligned} \quad (18)$$

随后, Alice和Bob先对色芯NV_i (i = 1, 2, 3, 4)中电子e_i自旋执行Hadamard门操作:

$$\begin{aligned}
 |-1\rangle & \rightarrow \frac{1}{\sqrt{2}}(|-1\rangle + |1\rangle), \\
 |1\rangle & \rightarrow \frac{1}{\sqrt{2}}(|-1\rangle - |1\rangle).
 \end{aligned} \quad (19)$$

然后使光子C和B分别进入NV₁, X₁, CPBS₁, NV₂, CPBS₂, X₂和NV₃, X₁, CPBS₁, NV₄, CPBS₂, X₂. 复合系统状态由|Φ⟩转化为|Φ₁⟩.

$$\begin{aligned}
 |\Phi_1\rangle = & \left\{ |-1\rangle_{e_1} \delta_1|a_2\rangle \left[|-1\rangle_{e_3} (|c_1d_2\rangle + |c_2d_2\rangle) \right. \right. \\
 & \left. (\gamma'_2|b_1\rangle - \delta'_2|b_2\rangle) + |1\rangle_{e_3} \right. \\
 & \left. (|c_1d_1\rangle - |c_2d_1\rangle)(\gamma'_2|b_1\rangle + \delta'_2|b_2\rangle) \right] \\
 & \left. + |1\rangle_{e_1} \gamma_1|a_1\rangle \left[|-1\rangle_{e_3} (|c_1d_2\rangle - |c_2d_2\rangle) \right. \right. \\
 & \left. (\gamma'_2|b_1\rangle - \delta'_2|b_2\rangle) + |1\rangle_{e_3} (|c_1d_1\rangle + |c_2d_1\rangle) \right. \\
 & \left. (\gamma'_2|b_1\rangle + \delta'_2|b_2\rangle) \right] \Big\} \\
 & \otimes \left\{ |-1\rangle_{e_2} \alpha_1|R\rangle \left[|-1\rangle_{e_4} (|RR\rangle + |LR\rangle) \right. \right. \\
 & (\alpha'_2|R\rangle + \beta'_2|L\rangle) + |1\rangle_{e_4} (|RL\rangle - |LL\rangle) \\
 & (\alpha'_2|R\rangle - \beta'_2|L\rangle) \Big] + |1\rangle_{e_2} \beta_1|L\rangle \\
 & \left[|-1\rangle_{e_4} (|RR\rangle - |LR\rangle)(\alpha'_2|R\rangle + \beta'_2|L\rangle) \right. \\
 & \left. + |1\rangle_{e_4} (|RL\rangle + |LL\rangle)(\alpha'_2|R\rangle - \beta'_2|L\rangle) \right] \Big\} \\
 & \otimes |t_2t_4t_4t_2\rangle_{ACDB}.
 \end{aligned} \quad (20)$$

为完成极化、时间两自由度并行非定域控制非门, Alice和Bob对光子C和B执行极化, 路径自由度Hadamard门操作. 复合系统状态转化为

$$\begin{aligned}
 |\Phi_2\rangle = & \left\{ |-1\rangle_{e_1} \delta_1|a_2\rangle \left[|-1\rangle_{e_3} |c_1d_2\rangle (\gamma_2|b_2\rangle + \delta_2|b_1\rangle) \right. \right. \\
 & \left. + |1\rangle_{e_3} |c_2d_1\rangle (\gamma_2|b_1\rangle + \delta_2|b_2\rangle) \right] \\
 & \left. + |1\rangle_{e_1} \gamma_1|a_1\rangle \left[|-1\rangle_{e_3} |c_2d_2\rangle (\gamma_2|b_2\rangle + \delta_2|b_1\rangle) \right. \right. \\
 & \left. + |1\rangle_{e_3} |c_1d_1\rangle (\gamma_2|b_1\rangle + \delta_2|b_2\rangle) \right] \Big\} \\
 & \otimes \left\{ |-1\rangle_{e_2} \alpha_1|R\rangle \left[|-1\rangle_{e_4} |RR\rangle (\alpha_2|R\rangle + \beta_2|L\rangle) \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & +|+1\rangle_{e_4} |LL\rangle (\alpha_2|L\rangle + \beta_2|R\rangle) \\
 & +|+1\rangle_{e_2} \beta_1|L\rangle \left[|-1\rangle_{e_4} |LR\rangle (\alpha_2|R\rangle + \beta_2|L\rangle) \right. \\
 & \left. +|+1\rangle_{e_4} |RL\rangle (\alpha_2|L\rangle + \beta_2|R\rangle) \right] \} \otimes |t_2 t_4 t_2\rangle_{ACDB}. \quad (21)
 \end{aligned}$$

与文献[73]类似, Alice和Bob对电子 e_i ($i=1, 2, 3, 4$) 自旋执行Hadamard操作, Bob对光子 D 执行极化、路径自由度Hadamard操作. 复合系统状态转化为

$$\begin{aligned}
 |\Phi_3\rangle = & \left\{ (|-1\rangle + |+1\rangle)_{e_1} \delta_1 |a_2\rangle \left[(|-1\rangle + |+1\rangle)_{e_3} \right. \right. \\
 & |c_1\rangle (|d_1\rangle - |d_2\rangle) (\gamma_2 |b_2\rangle + \delta_2 |b_1\rangle) \\
 & + (|-1\rangle - |+1\rangle)_{e_3} |c_2\rangle (|d_1\rangle + |d_2\rangle) (\gamma_2 |b_1\rangle + \delta_2 |b_2\rangle) \left. \right] \\
 & + (|-1\rangle - |+1\rangle)_{e_1} \gamma_1 |a_1\rangle \left[(|-1\rangle + |+1\rangle)_{e_3} |c_2\rangle \right. \\
 & (|d_1\rangle - |d_2\rangle) (\gamma_2 |b_2\rangle + \delta_2 |b_1\rangle) + (|-1\rangle - |+1\rangle)_{e_3} \\
 & |c_1\rangle (|d_1\rangle + |d_2\rangle) (\gamma_2 |b_1\rangle + \delta_2 |b_2\rangle) \left. \right] \} \\
 & \otimes \left\{ (|-1\rangle + |+1\rangle)_{e_2} \alpha_1 |R\rangle \right. \\
 & \left[(|-1\rangle + |+1\rangle)_{e_4} |R\rangle (|R\rangle + |L\rangle) \right. \\
 & (\alpha_2 |R\rangle + \beta_2 |L\rangle) + (|-1\rangle - |+1\rangle)_{e_4} |L\rangle \\
 & (|R\rangle - |L\rangle) (\alpha_2 |L\rangle + \beta_2 |R\rangle) \left. \right] \\
 & + (|-1\rangle - |+1\rangle)_{e_2} \beta_1 |L\rangle \\
 & \left[(|-1\rangle + |+1\rangle)_{e_4} |L\rangle (|R\rangle + |L\rangle) (\alpha_2 |R\rangle + \beta_2 |L\rangle) \right. \\
 & + (|-1\rangle - |+1\rangle)_{e_4} |R\rangle (|R\rangle - |L\rangle) \\
 & (\alpha_2 |L\rangle + \beta_2 |R\rangle) \left. \right] \} \otimes |t_2 t_4 t_2\rangle_{ACDB}. \quad (22)
 \end{aligned}$$

Alice和Bob对电子 e_i ($i=1, 2, 3, 4$) 自旋, 光子 C, D 极化, 路径自由度量子态执行正交投影测量, 通过经典信道发送测量结果并依据测量结果执行相应么正操作即可完成极化, 时间两自由度并行非域控制非门. 若光子 C 极化自由度测量结果为 $|R\rangle\langle L|$, 则 P_C 值为0(1). 若光子 C 路径自由度测量结果为 $|c_1\rangle\langle c_2|$, 则 S_C 值为0(1). 若电子 e_i ($i=1, 2, 3, 4$) 自旋测量结果为 $|-1\rangle\langle +1|$, 则 R_{e_i} 值为0(1). 若光子 D 极化自由度测量结果为

$|R\rangle\langle L|$, 则 P_D 值为0(1). 若光子 D 路径自由度测量结果为 $|d_1\rangle\langle d_2|$, 则 S_D 值为0(1). Alice将光子 C 测量结果 P_C, S_C ($P_C, S_C = 0, 1$) 用经典信道发送给Bob. Bob将电子 e_3, e_4 , 光子 D 测量结果 $R_{e_3} \oplus S_D, R_{e_4} \oplus S_D$ ($R_{e_3}, R_{e_4}, S_D, P_D = 0, 1$) 用经典信道发送给Alice. 其中 $R_{e_3} \oplus S_D$ 表示 R_{e_3} 模2加 S_D . Bob依据测量结果 P_C, S_C 对光子 B 极化、路径自由度执行么正操作 $X_{P_C}^P, X_{S_C}^S$. 其中 $X_0^P = X_0^S = I$ 为 2×2 单位矩阵表示不需执行么正操作, $X_1^P = |R\rangle\langle L| + |L\rangle\langle R|$ 表示对光子 B 极化自由度量子态执行比特翻转操作, $X_1^S = |b_1\rangle\langle b_2| + |b_2\rangle\langle b_1|$ 表示对光子 B 路径自由度量子态执行比特翻转操作. Alice依据测量结果 P_C, S_C 对光子 B 极化、路径自由度执行么正操作 $Z_{P_D \oplus R_{e_4} \oplus R_{e_2}}^P, Z_{S_D \oplus R_{e_3} \oplus R_{e_1}}^S$. 其中 $Z_0^P = Z_0^S = I$ 为 2×2 单位矩阵, $Z_1^P = |R\rangle\langle R| - |L\rangle\langle L|$ 表示对光子 A 极化自由度执行相位翻转操作, $Z_1^S = |a_1\rangle\langle a_1| - |a_2\rangle\langle a_2|$ 表示在光子 A 路径自由度执行相位翻转操作.

与测量结果相应局域么正操作可将光子 A, B 状态转化为

$$\begin{aligned}
 |\phi_1\rangle = & [\gamma_1 |a_1\rangle (\gamma_2 |b_1\rangle + \delta_2 |b_2\rangle) \\
 & + \delta_1 |a_2\rangle (\gamma_2 |b_2\rangle + \delta_2 |b_1\rangle)] \\
 & \otimes [\alpha_1 |R\rangle (\alpha_2 |R\rangle + \beta_2 |L\rangle) \\
 & + \beta_1 |L\rangle (\alpha_2 |L\rangle + \beta_2 |R\rangle)] \\
 & \otimes |t_2 t_2\rangle. \quad (23)
 \end{aligned}$$

延迟 DL_3 可将路径 a_2, b_2 中光子到达时间转换为 t_3 .

$$\begin{aligned}
 |\phi_2\rangle = & [\gamma_1 |a_1 t_2\rangle (\gamma_2 |b_1 t_2\rangle + \delta_2 |b_2 t_3\rangle) \\
 & + \delta_1 |a_2 t_3\rangle (\gamma_2 |b_2 t_3\rangle + \delta_2 |b_1 t_2\rangle)] \\
 & \otimes [\alpha_1 |R\rangle (\alpha_2 |R\rangle + \beta_2 |L\rangle) \\
 & + \beta_1 |L\rangle (\alpha_2 |L\rangle + \beta_2 |R\rangle)]. \quad (24)
 \end{aligned}$$

光开关 SW_5 可使时间 t_2 到达光子继续沿原路径 a_1, b_1 传输, 时间 t_3 到达光子传输路径由 a_2, b_2 切换为 a_1, b_1 . 经过光开关 SW_5 后, 光子 A, B 状态由 $|\phi_2\rangle$ 转换为目标态 $|\phi\rangle$.

$$\begin{aligned}
 |\phi\rangle = & [\gamma_1|t_2\rangle(\gamma_2|t_2\rangle + \delta_2|t_3\rangle) + \delta_1|t_3\rangle(\gamma_2|t_3\rangle + \delta_2|t_2\rangle)] \\
 & \otimes [\alpha_1|R\rangle(\alpha_2|R\rangle + \beta_2|L\rangle) + \beta_1|L\rangle(\alpha_2|L\rangle + \beta_2|R\rangle)] \\
 & \otimes |a_1b_1\rangle. \quad (25)
 \end{aligned}$$

4 结论

与文献[52]类似, 在基于金刚石NV色芯并行非定域控制非门远程实现中若不满足条件 $|r| = -|r_0| = 1$, 光子 A , D 经过 NV_1 , NV_3 后复合系统量子态转化为

$$\begin{aligned}
 |\Psi\rangle = & \left\{ \gamma_1|a_1\rangle \left[|-1\rangle_{e_1} (\alpha_1 r_0|R\rangle + \beta_1 r|L\rangle) \right. \right. \\
 & \left. \left. + |+1\rangle_{e_1} (\alpha_1 r|R\rangle + \beta_1 r_0|L\rangle) \right] + \delta_1|a_2\rangle \right. \\
 & \left. \left[|-1\rangle_{e_1} (\alpha_1|R\rangle + \beta_1|L\rangle) + |+1\rangle_{e_1} (\alpha_1|R\rangle + \beta_1|L\rangle) \right] \right\} \\
 & \otimes \left\{ (|c_1d_1\rangle + |c_2d_1\rangle) \left[|-1\rangle_{e_3} (r_0|RR\rangle \right. \right. \\
 & \left. \left. + r_0|LR\rangle + r|RL\rangle - r|LL\rangle) + |+1\rangle_{e_3} \right. \right. \\
 & \left. \left. (r|RR\rangle + r|LR\rangle + r_0|RL\rangle - r_0|LL\rangle) \right] \right. \\
 & \left. + (|c_2d_1\rangle - |c_2d_2\rangle) \left[|-1\rangle_{e_3} (|RR\rangle + |LR\rangle \right. \right. \\
 & \left. \left. + |RL\rangle - |LL\rangle) + |+1\rangle_{e_3} (|RR\rangle + |LR\rangle + |RL\rangle - |LL\rangle) \right] \right\} \\
 & \otimes (\alpha'_2|R\rangle + \beta'_2|L\rangle)(\gamma'_2|b_1\rangle + \delta'_2|b_2\rangle) \\
 & \otimes (|-1\rangle + |+1\rangle)_{e_2} \otimes (|-1\rangle + |+1\rangle)_{e_4} \\
 & \otimes |t_2t_4t_4t_2\rangle_{ACDB}. \quad (26)
 \end{aligned}$$

基于金刚石NV色芯量子门操作中由于耦合和非耦合状态下不满足反射系数 $|r| = -|r_0| = 1$ 可导致输出量子态失真, 影响量子门保真度[52,74]. 在构建基于金刚石NV色芯并行非定域控制非门中, 通过引入Ren等人[52]提出的平衡条件设置耦合与非耦合状态下反射系数相等 $|r| = -|r_0|$ 即可消除反射系数不等对复合系统量子态影响:

$$\begin{aligned}
 |\Psi\rangle = & \left\{ \gamma_1|a_1\rangle \left[|-1\rangle_{e_1} (-\alpha_1 r|R\rangle + \beta_1 r|L\rangle) \right. \right. \\
 & \left. \left. + |+1\rangle_{e_1} (\alpha_1 r|R\rangle - \beta_1 r|L\rangle) \right] + \delta_1|a_2\rangle \right. \\
 & \left. \left[|-1\rangle_{e_1} (\alpha_1 r|R\rangle + \beta_1 r|L\rangle) + |+1\rangle_{e_1} (\alpha_1 r|R\rangle + \beta_1 r|L\rangle) \right] \right\}
 \end{aligned}$$

$$\begin{aligned}
 & \otimes \left\{ (|c_1d_1\rangle + |c_2d_1\rangle) \left[|-1\rangle_{e_3} (-r|RR\rangle - r|LR\rangle \right. \right. \\
 & \left. \left. + r|RL\rangle - r|LL\rangle) + |+1\rangle_{e_3} (r|RR\rangle + r|LR\rangle \right. \right. \\
 & \left. \left. - r|RL\rangle + r|LL\rangle) \right] + (|c_2d_1\rangle - |c_2d_2\rangle) \right. \\
 & \left. \left[|-1\rangle_{e_3} (r|RR\rangle + r|LR\rangle + r|RL\rangle - r|LL\rangle) \right. \right. \\
 & \left. \left. + |+1\rangle_{e_3} (r|RR\rangle + r|LR\rangle + r|RL\rangle - r|LL\rangle) \right] \right\} \\
 & \otimes (\alpha'_2|R\rangle + \beta'_2|L\rangle)(\gamma'_2|b_1\rangle + \delta'_2|b_2\rangle) \\
 & \otimes (|-1\rangle + |+1\rangle)_{e_2} \otimes (|-1\rangle + |+1\rangle)_{e_4} \\
 & \otimes |t_2t_4t_4t_2\rangle_{ACDB}. \quad (27)
 \end{aligned}$$

构建基于金刚石NV色芯极化、时间两自由度超并行稳健非定域控制非门.

为完成两自由度超并行非定域控制非门, 通讯双方共享双光子时间、极化两自由度超纠缠态, 执行两个时间、极化两自由度超并行控制非门以及传送4比特经典信息. 与基于单一自由度非定域控制非门的远程实现不同, 通讯方仅需共享一个双光子超纠缠态即可完成光子时间、极化两自由度超并行稳健非定域控制非门, 减少远程量子信息处理所需传送纠缠光子数, 提高远程量子通信信道容量. 与文献[51,52]类似, 非定域控制非门效率可定义经过非定域控制非门操作后光子被检测到的概率. 强耦合条件($g \sim 2\pi \times 2$ GHz, $\eta \sim 2\pi \times 3$ GHz, $\gamma \sim 2\pi \times 13$ MHz, $\lambda \sim 0.0025$)下, 两自由度并行稳健非定域控制非门效率为98%. 与其他方案相比, 平衡条件下两自由度并行稳健非定域控制非门具有可行性强的优点.

本文以时间、极化两自由度同时纠缠超纠缠态为量子纠缠信道, 利用入射光子与金刚石NV色芯单边光学腔相互作用, 构建时间、极化两自由度超并行稳健非定域控制非门. 通讯方Alice和Bob使光子 A , C 和 D , B 依次进入金刚石NV色芯, 对NV色芯中电子和光子 C , D 执行单量子比特正交投影测量并依据测量结果执行相应局域么正操作即可完成时间、极化两自由度超并行稳健非定域控制非门. 方案使用金刚石NV色芯光学腔平衡条件消除反射系数不同对非定域量子门保真度影响, 不需要满足理想条件即可实现时间、路径两自由度超并行稳健非定域控制非门, 与其他方案相比具有可行性强、信道容量高的优点.

参考文献

- 1 Bennett C H, Brassard G, Crepeau C, et al. Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. *Phys Rev Lett*, 1993, 70: 1895–1899
- 2 Karlsson A, Bourennane M. Quantum teleportation using three-particle entanglement. *Phys Rev A*, 1998, 58: 4394–4400
- 3 Bouwmeester D, Pan J W, Mattle K, et al. Experimental quantum teleportation. *Nature*, 1997, 390: 575–579
- 4 Lago-Rivera D, Rakonjac J V, Grandi S, et al. Long distance multiplexed quantum teleportation from a telecom photon to a solid-state qubit. *Nat Commun*, 2023, 14: 1889
- 5 Shen S, Yuan C, Zhang Z, et al. Hertz-rate metropolitan quantum teleportation. *Light Sci Appl*, 2023, 12: 115
- 6 Tan X, Ye T Y. Semiquantum proxy blind signature based on quantum teleportation (in Chinese). *Sci Sin-Phys Mech Astron*, 2024, 54: 230313, arXiv: [2310.19327](#) [谈笑, 叶天语. 基于量子隐形传态的半量子代理盲签名. 中国科学: 物理学 力学 天文学, 2024, 54: 230313, arXiv: [2310.19327](#)]
- 7 Thomas J M, Yeh F I, Chen J H, et al. Quantum teleportation coexisting with classical communications in optical fiber. *Optica*, 2024, 11: 1700, arXiv: [2404.10738](#)
- 8 Lv M Y, Hu X M, Gong N F, et al. Demonstration of controlled high-dimensional quantum teleportation. *Sci China-Phys Mech Astron*, 2024, 67: 230311
- 9 Bennett C H, Brassard G. Quantum cryptography: Public key distribution and coin tossing. *Theor Comput Sci*, 2014, 560: 7–11
- 10 Ekert A K. Quantum cryptography based on Bell's theorem. *Phys Rev Lett*, 1991, 67: 661–663
- 11 Kwek L C, Cao L, Luo W, et al. Chip-based quantum key distribution. *AAPPS Bull*, 2021, 31: 15
- 12 Li W, Zhang L, Tan H, et al. High-rate quantum key distribution exceeding 110 Mb s^{-1} . *Nat Photon*, 2023, 17: 416–421, arXiv: [2307.02364](#)
- 13 Yin H L, Chen Z B. Cost-efficient quantum access network boosts practical deployment of quantum key distribution network. *Sci China-Phys Mech Astron*, 2024, 67: 240332
- 14 Li H W, Hao C P, Chen Z J, et al. Security of quantum key distribution with virtual mutually unbiased bases. *Sci China-Phys Mech Astron*, 2024, 67: 270313
- 15 Long G L, Liu X S. Theoretically efficient high-capacity quantum-key-distribution scheme. *Phys Rev A*, 2002, 65: 032302, arXiv: [quant-ph/0012056](#)
- 16 Deng F G, Long G L, Liu X S. Two-step quantum direct communication protocol using the Einstein-Podolsky-Rosen pair block. *Phys Rev A*, 2003, 68: 042317, arXiv: [quant-ph/0308173](#)
- 17 Zhang W, Ding D S, Sheng Y B, et al. Quantum secure direct communication with quantum memory. *Phys Rev Lett*, 2017, 118: 220501, arXiv: [1609.09184](#)
- 18 Zhu F, Zhang W, Sheng Y, et al. Experimental long-distance quantum secure direct communication. *Sci Bull*, 2017, 62: 1519–1524, arXiv: [1710.07951](#)
- 19 Bennett C H, Wiesner S J. Communication via one- and two-particle operators on Einstein-Podolsky-Rosen states. *Phys Rev Lett*, 1992, 69: 2881–2884
- 20 Liu X S, Long G L, Tong D M, et al. General scheme for superdense coding between multiparties. *Phys Rev A*, 2002, 65: 022304
- 21 Hu X M, Guo Y, Liu B H, et al. Beating the channel capacity limit for superdense coding with entangled ququarts. *Sci Adv*, 2018, 4: eaat9304, arXiv: [1807.10452](#)
- 22 Leng Y, Chen F. Improving the capacity of quantum dense coding via non-Hermitian operation. *Laser Phys*, 2023, 33: 125203
- 23 Bennett C H, Brassard G, Popescu S, et al. Purification of noisy entanglement and faithful teleportation via noisy channels. *Phys Rev Lett*, 1996, 76: 722–725, arXiv: [quant-ph/9511027](#)
- 24 Pan J W, Simon C, Brukner Č, et al. Entanglement purification for quantum communication. *Nature*, 2001, 410: 1067–1070, arXiv: [quant-ph/0012026](#)
- 25 Sheng Y B, Deng F G, Zhou H Y. Efficient polarization-entanglement purification based on parametric down-conversion sources with cross-Kerr nonlinearity. *Phys Rev A*, 2008, 77: 042308, arXiv: [0805.0032](#)
- 26 Sheng Y B, Deng F G. Deterministic entanglement purification and complete nonlocal Bell-state analysis with hyperentanglement. *Phys Rev A*, 2010, 81: 032307, arXiv: [0912.0079](#)

- 27 Yan P S, Zhou L, Zhong W, et al. Advances in quantum entanglement purification. *Sci China-Phys Mech Astron*, 2023, 66: 250301, arXiv: [2304.12679](#)
- 28 Pati A K. Minimum classical bit for remote preparation and measurement of a qubit. *Phys Rev A*, 2000, 63: 014302
- 29 Lo H K. Classical-communication cost in distributed quantum-information processing: A generalization of quantum-communication complexity. *Phys Rev A*, 2000, 62: 012313, arXiv: [quant-ph/9912009](#)
- 30 Bennett C H, Divincenzo D P, Shor P W, et al. Remote state preparation. *Phys Rev Lett*, 2001, 87: 077902, arXiv: [quant-ph/0006044](#)
- 31 Huelga S F, Vaccaro J A, Chefles A, et al. Quantum remote control: Teleportation of unitary operations. *Phys Rev A*, 2001, 63: 042303, arXiv: [quant-ph/0005061](#)
- 32 Huelga S F, Plenio M B, Vaccaro J A. Remote control of restricted sets of operations: Teleportation of angles. *Phys Rev A*, 2002, 65: 042316, arXiv: [quant-ph/0107110](#)
- 33 Xiang G Y, Li J, Guo G C. Teleporting a rotation on remote photons. *Phys Rev A*, 2005, 71: 044304
- 34 Eisert J, Jacobs K, Papadopoulos P, et al. Optimal local implementation of nonlocal quantum gates. *Phys Rev A*, 2000, 62: 052317, arXiv: [quant-ph/0005101](#)
- 35 Chen L B, Lu H. Efficient nonlocal M -control and N -target controlled unitary gate using non-symmetric GHZ states. *Int J Theor Phys*, 2018, 57: 706–714
- 36 Li J, Xie Z, Li Y, et al. Heralded entanglement between error-protected logical qubits for fault-tolerant distributed quantum computing. *Sci China-Phys Mech Astron*, 2024, 67: 220311
- 37 Gottesman D, Chuang I L. Demonstrating the viability of universal quantum computation using teleportation and single-qubit operations. *Nature*, 1999, 402: 390–393, arXiv: [quant-ph/9908010](#)
- 38 Huang Y F, Ren X F, Zhang Y S, et al. Experimental teleportation of a quantum controlled-NOT gate. *Phys Rev Lett*, 2004, 93: 240501, arXiv: [quant-ph/0408007](#)
- 39 Chen L B, Tan P, Dong S G, et al. Controlled implementation of a nonlocal and open-target destination quantum controlled-NOT (CNOT) gate using partially entangled pairs (in Chinese). *Acta Phys Sin*, 2009, 58: 6772 [陈立冰, 谭鹏, 董少光, 等. 利用二粒子部分纠缠态实现开靶目标非局域量子可控非(CNOT)门的受控操作. *物理学报*, 2009, 58: 6772]
- 40 Chen L B, Bai Y H, Lu H. Nonlocal d -dimensional controlled-NOT with multi-target gate: A controlled and local implementation. *J Korean Phy Soc*, 2011, 58: 411–416
- 41 Wang H F, Zhu A D, Zhang S, et al. Optically controlled phase gate and teleportation of a controlled-NOT gate for spin qubits in a quantum-dot-microcavity coupled system. *Phys Rev A*, 2013, 87: 062337, arXiv: [1306.4737](#)
- 42 Chou K S, Blumoff J Z, Wang C S, et al. Deterministic teleportation of a quantum gate between two logical qubits. *Nature*, 2018, 561: 368–373, arXiv: [1801.05283](#)
- 43 Wan Y, Kienzler D, Erickson S D, et al. Quantum gate teleportation between separated qubits in a trapped-ion processor. *Science*, 2019, 364: 875–878, arXiv: [1902.02891](#)
- 44 Daiss S, Langenfeld S, Welte S, et al. A quantum-logic gate between distant quantum-network modules. *Science*, 2021, 371: 614–617, arXiv: [2103.13095](#)
- 45 Chen M F, Zhou P, Lan Q, et al. Hyper-parallel nonlocal CNOT operation assisted by quantum-dot spin in a double-sided optical microcavity. *J Opt Soc Am B*, 2023, 40: 3291
- 46 Liu X, Hu X M, Zhu T X, et al. Nonlocal photonic quantum gates over 7.0 km. *Nat Commun*, 2024, 15: 8529, arXiv: [2307.15634](#)
- 47 Qiu J, Liu Y, Hu L, et al. Deterministic quantum state and gate teleportation between distant superconducting chips. *Sci Bull*, 2025, 70: 351–358
- 48 Togan E, Chu Y, Trifonov A S, et al. Quantum entanglement between an optical photon and a solid-state spin qubit. *Nature*, 2010, 466: 730–734
- 49 Eto Y, Noguchi A, Zhang P, et al. Projective measurement of a single nuclear spin qubit by using two-mode cavity QED. *Phys Rev Lett*, 2011, 106: 160501, arXiv: [1012.1724](#)
- 50 Zheng A, Li J, Yu R, et al. Generation of Greenberger-Horne-Zeilinger state of distant diamond nitrogen-vacancy centers via nanocavity input-output process. *Opt Express*, 2012, 20: 16902
- 51 Wei H R, Deng F G. Compact quantum gates on electron-spin qubits assisted by diamond nitrogen-vacancy centers inside cavities. *Phys Rev A*, 2013, 88: 042323, arXiv: [1310.0197](#)
- 52 Ren B C, Deng F G. Robust hyperparallel photonic quantum entangling gate with cavity QED. *Opt Express*, 2017, 25: 10863

- 53 Li M, Zhang M. Robust universal photonic quantum gates operable with imperfect processes involved in diamond nitrogen-vacancy centers inside low-Q single-sided cavities. [Opt Express](#), 2018, 26: 33129
- 54 Wu Y M, Fan G, Du F F. Error-detected three-photon hyperparallel Toffoli gate with state-selective reflection. [Front Phys](#), 2022, 17: 51502, arXiv: [2206.07336](#)
- 55 Du F F, Shi Z R. Robust hybrid hyper-controlled-NOT gates assisted by an input-output process of low-Q cavities. [Opt Express](#), 2019, 27: 17493
- 56 Song G Z, Guo J L, Liu Q, et al. Heralded quantum gates for hybrid systems via waveguide-mediated photon scattering. [Phys Rev A](#), 2021, 104: 012608
- 57 Xie T, Zhao Z, Xu S, et al. 99.92%-fidelity CNOT gates in solids by noise filtering. [Phys Rev Lett](#), 2023, 130: 030601, arXiv: [2212.02831](#)
- 58 Zhang J X, Sun X T, Wang L X, et al. Heralded hyper-CNOT gates for two-photon systems assisted by quantum scattering in waveguides. [Commun Theor Phys](#), 2024, 76: 115105
- 59 Sun X T, Zhang J X, Gu Y Y, et al. Heralded high-fidelity photonic hyper-CNOT gates with quantum scattering in one-dimensional waveguides. [Quantum Inf Process](#), 2024, 23: 326
- 60 Dong L, Yuan Z Q, Zhao Z L, et al. Logic-qubit CNOT gates in decoherence-free subspaces assisted by the nitrogen-vacancy center. [Opt Lett](#), 2025, 50: 722
- 61 Barreiro J T, Wei T C, Kwiat P G. Beating the channel capacity limit for linear photonic superdense coding. [Nat Phys](#), 2008, 4: 282–286, arXiv: [1009.5128](#)
- 62 Barreiro J T, Wei T C, Kwiat P G. Remote preparation of single-photon “hybrid” entangled and vector-polarization states. [Phys Rev Lett](#), 2010, 105: 030407, arXiv: [1009.5412](#)
- 63 Wang T J, Li T, Du F F, et al. High-capacity quantum secure direct communication based on quantum hyperdense coding with hyperentanglement. [Chin Phys Lett](#), 2011, 28: 040305, arXiv: [1103.0471](#)
- 64 Li X H, Ghose S. Efficient hyperconcentration of nonlocal multipartite entanglement via the cross-Kerr nonlinearity. [Opt Express](#), 2015, 23: 3550, arXiv: [1601.03756](#)
- 65 Wang X L, Cai X D, Su Z E, et al. Quantum teleportation of multiple degrees of freedom of a single photon. [Nature](#), 2015, 518: 516–519
- 66 Graham T M, Bernstein H J, Wei T C, et al. Superdense teleportation using hyperentangled photons. [Nat Commun](#), 2015, 6: 7185, arXiv: [1505.05615](#)
- 67 Ren B C, Wang H, Alzahrani F, et al. Hyperentanglement concentration of nonlocal two-photon six-qubit systems with linear optics. [Ann Phys](#), 2017, 385: 86–94
- 68 Wu F Z, Yang G J, Wang H B, et al. High-capacity quantum secure direct communication with two-photon six-qubit hyperentangled states. [Sci China-Phys Mech Astron](#), 2017, 60: 120313
- 69 Nawaz M, ul-Islam R, Ikram M. Remote state preparation through hyperentangled atomic states. [J Phys B-At Mol Opt Phys](#), 2018, 51: 075501
- 70 Zhou P, Jiao X F, Lv S X. Parallel remote state preparation of arbitrary single-qubit states via linear-optical elements by using hyperentangled Bell states as the quantum channel. [Quantum Inf Process](#), 2018, 17: 298
- 71 Jiao X F, Zhou P, Lv S X, et al. Remote preparation for single-photon two-qubit hybrid state with hyperentanglement via linear-optical elements. [Sci Rep](#), 2019, 9: 4663
- 72 Wang M Y, Yan F, Gao T. Remote preparation for single-photon state in two degrees of freedom with hyper-entangled states. [Front Phys](#), 2021, 16: 41501, arXiv: [2103.07671](#)
- 73 Wang M, Guo H. Quantum remote control utilizing multiple degrees of freedom. [Opt Laser Tech](#), 2024, 169: 110075
- 74 Reiserer A, Kalb N, Rempe G, et al. A quantum gate between a flying optical photon and a single trapped atom. [Nature](#), 2014, 508: 237–240, arXiv: [1404.2453](#)
- 75 Yi K, Hai Y J, Luo K, et al. Robust quantum gates against correlated noise in integrated quantum chips. [Phys Rev Lett](#), 2024, 132: 250604, arXiv: [2401.01810](#)
- 76 Zhou P, Lv L. Hyper-parallel nonlocal CNOT operation with hyperentanglement assisted by cross-Kerr nonlinearity. [Sci Rep](#), 2019, 9: 15939
- 77 Brendel J, Gisin N, Tittel W, et al. Pulsed energy-time entangled twin-photon source for quantum communication. [Phys Rev Lett](#), 1999, 82: 2594–2597, arXiv: [quant-ph/9809034](#)
- 78 Lan Q, Huang C M, Zhou P. Parallel remote preparation of quantum states with polarization-frequency-time-bin hyperentangled state. [Quantum Inf Process](#), 2024, 23: 388

Robust hyperparallel nonlocal operation assisted by diamond nitrogen-vacancy center coupled to a single-side optical microcavity

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Remote implementation of nonlocal quantum gates between different nodes in quantum communication network plays an important role in distributed quantum computation. Compared to protocol for nonlocal implementation of quantum operation in one degree of freedom (DoF), the protocol for parallel remote implementation of nonlocal operations simultaneously in two DoFs has the advantage of having high channel capacity. Under the balance condition of diamond nitrogen-vacancy center coupled to a single-side optical microcavity, we present a protocol for remote implementation of robust hyperparallel nonlocal operation in polarization, time-bin DoFs. Alice and Bob let photons pass through the diamond nitrogen-vacancy center system successively, perform single-qubit measurement on the entangled photons and the electrons. The robust nonlocal CNOT operation can be simultaneously implemented by applying unitary operations in accordance with the measurement results. The protocol has the advantage of transmitting less entangled photons for remote implementation of nonlocal CNOT operations in two DoFs. Moreover, the protocol is more convenient in application since the noise in the protocol for hyperparallel nonlocal CNOT operation via diamond nitrogen-vacancy center can be efficiently depressed.

quantum communication, hyperparallel nonlocal operation, hyperentangled state, diamond nitrogen-vacancy center

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