

The Early Transmission of Calculus in China and Japan: A Comparative Study

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Abstract: Drawing upon a wealth of primary documents and research materials, this paper presents a systematic and comprehensive examination of the introduction and transmission of calculus in China and Japan from the mid-nineteenth century to the early twentieth century. It also brings to light newly identified historical documents. Building on this foundation, the paper offers a comparative analysis of the transmission and development of calculus and other forms of Western mathematical knowledge in the two countries and proposes new perspectives on why modern mathematics advanced rapidly in Japan but developed more slowly in China.

Keywords: calculus, mathematics in modern China, mathematics in modern Japan, history of mathematics, comparative study

摘 要: 本文依据大量原始文献和研究资料对 19 世纪中期至 20 世纪初中日两国引进和传播微积分知识的情况进行了系统和全面的考察, 发掘了新的史料。在此基础上, 本文对微积分和西方数学知识在两国的传播与发展情况进行了比较分析, 对近代日本数学发展迅速而中国数学发展缓慢的原因提出了新的看法。

关键词: 微积分, 中国近代数学, 日本近代数学, 数学史, 比较研究

Calculus stands as one of the most emblematic achievements of modern mathematics and constitutes some of the highest-level Western mathematical knowledge introduced into China and Japan in the late nineteenth century. The ways in which calculus was introduced, absorbed, and assimilated in the two countries reflect

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their respective transitions from traditional to modern mathematical frameworks and encapsulate the broader process of mathematical modernization in China and Japan. To explore this process and undertake comparative inquiry is undoubtedly a crucial task in the study of modern mathematical history in China and Japan. This paper focuses primarily on the transmission and development of calculus in the two countries from the mid-nineteenth century to the early twentieth century and offers comparative analyses on that basis.

1 The early transmission of calculus in China

Calculus entered China earlier than it reached Japan. Chinese scholars first encountered and studied calculus through Chinese translations of Western treatises. *Daiweiji sheji* 代微积拾级 (*Stepping-Stones to Algebra and Calculus*), was the first Chinese translation of a Western work on calculus. Published in 1859 by the Mohai Bookstore in Shanghai, the work comprises eighteen *juan*. It was translated by the eminent late-Qing mathematician Li Shanlan 李善兰 (1811–1882) and the British missionary Alexander Wylie (1815–1887) from the American mathematician Elias Loomis's (1811–1889) *Elements of Analytical Geometry and of the Differential and Integral Calculus* (second edition published by Harper & Brothers in New York in 1852) (Gao 2014). Loomis's treatise was not a cutting-edge scholarly monograph in the Western world but rather a widely used textbook. In the preface, Loomis (1852, "Preface," iii) made this intention explicit: "It was written, not for mathematicians, nor for those who have a peculiar talent or fondness for the mathematics, but rather for the mass of college students of average abilities." Nevertheless, for Chinese readers encountering calculus for the first time, the book introduced an astonishing range of unprecedented knowledge. In his preface, Li Shanlan (1859, "Preface") wrote: "In the reign of our Emperor Kangxi, Western scholars Leibniz and Newton established the arts of differentiation and integration. Their methods rely upon algebraic techniques and their principles reveal mysteries never conceived in all antiquity."¹ The sense of intellectual novelty felt by Chinese mathematicians is easy to imagine.

Loomis's original text consists of three parts: "Analytical Geometry" (sections i–ix), "Differential Calculus" (sections i–vii), and "Integral Calculus" (sections i–ii), followed by "Miscellaneous Examples" containing ninety problems with solutions. *Daiweiji sheji* mirrors this structure: *juan* 1–9 correspond to Part 1 "Analytical Geometry"; *juan* 10–16 to Part 2 "Differential Calculus"; and *juan* 17–18 to Part 3 "Integral Calculus." The translation includes answers to all exercises, even those for which Loomis provided none, although "Miscellaneous Examples" were omitted entirely. In terms of

1 “我朝康熙时，西国来本之、奈端二家又创微分、积分二术。其法也是借助于代数，其理实发千古未有之奇秘。”

mathematical content, the book presents the elementary methods and theories of calculus at a level comparable to that of the era before Augustin-Louis Cauchy (1789–1857) established his eponymous analytical theorem in the 1820s; it did not reflect the contemporary state of Western calculus. Yet for the Chinese encountering calculus for the first time, the text served as an excellent introductory manual—indeed, this was primarily why Wylie and Li chose Loomis’s treatise as the initial work to translate.

During the translation process, Li and Wylie coined an unprecedented body of Chinese mathematical terminology. Following the English preface that Wylie wrote in July 1859 for the first edition of the Chinese translation, they appended an extensive glossary of more than 350 English-Chinese mathematical terms and symbols. The translated terms and methods of translation they established were largely adopted by later generations, and many terms have remained in use to this day. Commonly used Chinese expressions—such as differential (微分), integral (积分), differential equation (微分方程式), total differential (全微分), series (级数), abscissa (横轴), ordinate (纵轴), curve (曲线), curvature (曲率), tangent (切线), function (函数), explicit function (阳函数), implicit function (阴函数), maximum (极大), and minimum (极小)—all originated from their translation. The establishment of Chinese translations for calculus and its associated mathematical terms marked the initial completion of mathematical conceptual transformation, clearing a major barrier to the transmission and popularization of calculus in China.

Li Shanlan and Alexander Wylie, in their handling of symbols and formulas, adopted some of the mathematical symbols used in the English original, such as $\times$, $\div$, $\sqrt{\quad}$, $=$, $<$, and $>$, but more often Sinicized Western notation, transforming it into a system based on Chinese characters. Arabic numerals were replaced with Chinese numeral characters 一, 二, 三, . . . 九, and 〇. To avoid confusion between the marks “+” and “−” and the Chinese numerals “一” and “十,” they used seal-script characters “上” and “下” (written as “⊥” and “⊥”) to denote addition and subtraction. Their notation for fractions and rational fractions followed the method in the late-Ming *Rules of Arithmetic Common to Cultures* (*Tongwen suanzhi* 同文算指): the denominator was written above the fraction line and the numerator below it. The twenty-six letters of the English alphabet were substituted by the celestial stems and terrestrial branches together with the four characters “天,” “地,” “人,” and “物” in sequence to replace the lowercase letters; all were written as Chinese characters. Uppercase letters were indicated by prefixing these characters with the “口” radical. Greek letters were replaced by the names of the twenty-eight lunar mansions (二十八宿). The circle constant π was denoted by “周,” the base of natural logarithms e by “訥,” and the function symbol f by “函.” The differential and integral signs were replaced respectively by the radicals “彳” and “禾,” components taken from the characters “微”

and “积.” For example, the expression $\int f(x)dx$ was rendered in their notation as “禾函彳天.” To a modern reader this system of notation seems esoteric and difficult to decipher, yet its adoption was deliberate: by recasting Western mathematical symbols into Chinese characters, Wylie and Li sought to accommodate traditional Chinese reading habits and thereby make foreign mathematics more accessible to Chinese learners. Below are two further examples that juxtapose Li (1859, *juan* 17:2a) and Wylie’s Chinese symbolic formulas with the corresponding formulas in the English original:

$$\text{禾卯天彳天} = \frac{\text{卯天}}{\text{卯天}} \perp \text{卯丙}$$

$$\text{i.e.,} \quad \int nx dx = \frac{1}{2}nx^2 + C$$

$$\text{禾卯天彳天} = \frac{\text{卯天}}{\text{卯天}} \text{卯乙丁} \frac{\text{卯甲}}{\text{卯甲}} = \frac{\text{卯(乙-甲)}}{\text{卯(乙-甲)}}$$

$$\text{i.e.,} \quad \int_a^b nx dx = \frac{1}{2}nb^2 - \frac{1}{2}na^2 = \frac{n(b^2 - a^2)}{2}$$

After *Daiweiji sheji* was published in Shanghai, it exerted considerable influence: Chinese mathematicians and mathematics enthusiasts at the time learned calculus primarily through this work. Some mathematicians, after studying the book, went on to identify problems that merited further investigation. For example, the mathematician Xia Luanxiang 夏鸾翔 (1823–1864) noticed that *Daiweiji sheji* contained only the formula for the total surface area generated when an ellipse is revolved about its major axis; it did not provide algorithms for computing the surface area generated by revolving a portion of the ellipse (an elliptical arc) about its major or minor axis. This lacuna prompted Xia to undertake additional research. In the early 1860s, Xia composed *Procedures for Curves* (*Zhiqu shu* 致曲术), and *A Myriad Images—A Single Origin* (*Wanxiang yiyuan* 万象一原, 1862), in which he set out methods for computing arc lengths of various curves, the areas they enclose, the surface areas generated by revolving those arcs, and the volumes enclosed by such surfaces. He solved a number of computational problems concerning ellipses, hyperbolas, parabolas, logarithmic curves, and various spirals. For example, in *Procedures for Curves* he established a series expansion for the surface area formed by revolving a portion of an ellipse about its major or minor axis. For the surface area produced by revolving the arc of the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ from the point $(0, b)$ to the point (x, y) about the major axis (what he called the “elliptic major-axis cap surface area” [椭圆求大径端截盖壳积]), the result he presented is equivalent to:

$$2\pi b \int_0^x \left(1 - \frac{c^2 x^2}{a^4}\right)^{\frac{1}{2}} dx = 2\pi b \left(x - \frac{c^2 x^3}{3!a^4} - \frac{1 \cdot 3c^4 x^5}{5!a^8} - \frac{1 \cdot 3^2 c^6 x^7}{7!a^{12}} - \dots \right)$$

Similarly, the surface area obtained by revolving that arc about the minor axis (the “elliptic minor-axis cap surface area” [椭圆求小径端截盖壳积]) is:

$$2\pi a \int_0^y \left(1 - \frac{c^2 y^2}{b^4}\right)^{\frac{1}{2}} dy = 2\pi a \left(y - \frac{c^2 y^3}{3!b^4} - \frac{1 \cdot 3c^4 y^5}{5!b^8} - \frac{1 \cdot 3^2 c^6 y^7}{7!b^{12}} - \dots \right)$$

Xia Luanxiang also derived a series expansion for the length of the segment of the ellipse from the point $(0, b)$ to the point (x, y) :

$$\begin{aligned} s &= \int_0^x \frac{\sqrt{a^4 - c^2 x^2}}{a\sqrt{a^2 - x^2}} dx = \int_0^x \left(1 - \frac{c^2 x^2}{a^2}\right)^{\frac{1}{2}} \left(1 - \frac{x^2}{a^2}\right)^{-\frac{1}{2}} dx \\ &= x + \frac{1^2 x^3}{3!a^2} + \frac{1^2 \cdot 3x^5}{5!a^4} + \frac{1^2 \cdot 3^2 x^7}{7!a^6} + \dots - \\ &\quad - \left(\frac{c^2 x^2}{2 \cdot 3a^4} + \frac{c^2 x^5}{2 \cdot 2 \cdot 5a^6} + \frac{1 \cdot 3c^2 x^7}{2 \cdot 2 \cdot 4 \cdot 7a^7} + \dots \right) - \\ &\quad - \left(\frac{c^4 x^5}{2 \cdot 4 \cdot 5a^8} - \frac{c^4 x^7}{2 \cdot 4 \cdot 2 \cdot 7a^{10}} + \dots \right) - \\ &\quad - \left(\frac{c^6 x^7}{2 \cdot 4 \cdot 6 \cdot 7a^{12}} + \dots \right) - \dots \end{aligned}$$

The problems discussed above fall into the class of elliptic-integral problems, and Xia addressed them by means of infinite series. His methods resemble Xiang Mingda’s 项名达 and Dai Xu’s 戴煦 methods for finding the circumference of an ellipse (椭圆求周术) and Li Shanlan’s methods for finding the volume of a pointed cone (尖锥求积术); Xia’s work is therefore regarded as an extension of the approaches developed by Xiang, Dai, and Li (Qian 1964, 332–333). One point that historians of mathematics have often overlooked, however, is that Xia’s results depended on his command of basic calculus methods, and without the transmission of calculus, these achievements could not have been realized. Xia (n.d., “Preface”) himself made this point explicit:

Circles emerge from squares, but circular forms are not uniform; hence the names of curves are varied. The methods of the ancients, such as they were, applied only to the simple circle; when it came to elliptic curves, the techniques of antiquity had long been lost. My teacher, Xiang Meilü (i.e., Xiang Mingda), with clear thought and boundless reflection, devised a method to determine the ellipse’s circumference. Those who followed—Dai Eshi (i.e., Dai Xu) and Xu Junqing (i.e., Xu Youren 徐有壬)—each established a procedure, and the ellipse thereby became a figure to which methods could

be applied. Yet [these methods] could only yield the ellipse's perimeter; they could not compute elliptic arcs, nor the arcs, surfaces, areas, and volumes associated with curves, which is a regrettable limitation. Since Newton and Leibniz fashioned horizontal and vertical lines to govern curves and coined the names differential and integral, what was once without method is now methodized. Although forms are manifold, the governing principle is one; this is truly the contribution to mathematics and a joy in one's life. In recent years, while seeking refuge from turmoil in Wumen, Pinghu, Nanhui, and Tiehe, I used my leisure to probe the subtleties of calculus, setting them down and working them out. In all I developed more than a hundred methods, and the techniques thus gradually became complete. With this, the science of geometry stands firm, shaken no longer even in the slightest.²

Xia also produced many formulas that *Daiweiji sheji* did not include. These include expressions for the elliptic major-axis cap volume (椭圆求大径端截盖体积) and the elliptic minor-axis cap volume (椭圆求小径端截盖体积), as well as formulas for the semi-cubical paraboloid cap surface area (半立方抛物线截盖壳积), the semi-cubical paraboloid cap volume (半立方抛物线截盖体积), and the cubic paraboloid cap surface area (立方抛物线截盖壳积). Where *Daiweiji sheji* made no mention of problems concerning surfaces and solids generated by revolving hyperbolas, Xia supplied a broad suite of formulas, such as the constant surface area for a rectangular hyperbola (正双曲线求常曲面积), the cap surface area for a rectangular hyperbola (正双曲线求截盖壳积), the constant volume for a rectangular hyperbola (正双曲线求常体积), the cap volume for a rectangular hyperbola (正双曲线求截盖体积), the major-axis curve-area for an oblique hyperbola (斜双曲线求大径端曲面积), the minor-axis curve-area for an oblique hyperbola (斜双曲线求小径端曲面积), the major-axis cap surface area for an oblique hyperbola (斜双曲线求大径端截盖壳积), the minor-axis cap surface area for an oblique hyperbola (斜双曲线求小径端截盖壳积), the major-axis cap volume for an oblique hyperbola (斜双曲线求大径端截盖体积), and the minor-axis cap volume for an oblique hyperbola (斜双曲线求小径端截盖体积).

Xia Luanxiang was one of the earliest Qing mathematicians to adopt calculus as a working tool and obtain results by its means. At the same time, after absorbing the calculus knowledge he himself translated, Li Shanlan also began to employ calculus techniques in his own mathematical research. Many of his writings produced after the

2 “圆出于方，而圆形不一，曲线之名因而万殊焉。昔人所谓有法者，只一平圆，至椭圆曲线，古已遗之。吾师项梅吕先生，澄思渺虑，立术以求椭圆。继之者鄂士戴氏、君青徐氏，各立一术，而椭圆乃为有法之形。然只能求椭圆，不能截椭圆，且不能求诸曲线之弧与曲面与面积与体积，亦憾事也。自奈端、来本之二家作横直二线以驭曲线，创名曰微分、积分，于是昔所无法者，今皆有法。形虽万，法则一，诚算学之功臣也，亦人生之快事也。余逐年避乱于吴门，于平湖，于南汇，于铁河，暇则细寻微积分奥窍，疏而演之，凡一百余术，法乃浸备。几何之学至是无纤芥之憾矣。”

publication of his translations, *Daiweiji sheji* and *Algebra* (*Daishu xue* 代数学), manifest an integration of Chinese and Western mathematics. For example, Li's two works on power series, *Derivations for Pointed Cones* (*Jianzhui bianfa* 尖锥变法) and *Series Reversion* (*Jishu huiqiu* 级数回求), make use of both algebraic notation and calculus knowledge. *Derivations for Pointed Cones* shows that the derivation of the "product between a hyperbola and its asymptote equaling the logarithmic product" (双曲线与渐近线中间之积即对数积) discussed in *Algebra* is essentially equivalent to the pointed-cone method (尖锥术) he used in *Principles of Logarithms* (*Duishu tanyuan* 对数探原). *Series Reversion* derives power-series expansions by algebraic symbols and presents procedures for inverting functions and their independent variables that were more concise and effective than those of Minggatu 明安图 and Dong Youcheng 董祐诚. Published in 1867, Li Shanlan's collected monograph *Zeguxi Studio's Mathematics* (*Zeguxi zhai suanxue* 则古昔斋算学) contained two new works, *New Methods for the Ellipse* (*Tuoyuan xinshu* 椭圆新术) and *Picking Up Omissions Concerning the Ellipse* (*Tuoyuan shiyi* 椭圆拾遗). The former marked the first Chinese work to introduce infinite-series methods into the computation of elliptic orbits, presenting a power-series expansion for Kepler's equation; the latter applied calculus and infinite series to resolve numerous concrete problems in elliptic orbit computation (Feng and Niu 1992, 100–111). The eminent late-Qing mathematician Hua Hengfang 华蘅芳 (1833–1902) first encountered calculus while visiting Shanghai in his youth around the time Li Shanlan and Alexander Wylie were translating Loomis's work. Hua's *Discussion on the Learning of Mathematics* (*Xuesuan bitan* 学算笔谈), published in twelve *juan* in 1882, includes *juan* 11 "On Calculus" (*Tan weijifen* 谈微积分) which summarizes his early insights into calculus and contains some distinctive interpretations of its concepts. Hua also identified and corrected numerical errors in the answers to the last three problems of item 3 in *juan* 10 and the last two problems of item 6 in *juan* 17 of *Daiweiji sheji*.

A decade or so after the appearance of *Daiweiji sheji*, new Chinese treatises on calculus began to be published. In 1862, Feng Guifen 冯桂芬 and Chen Zijin 陈子璿 published *Direct Explanation of New Methods of Western Mathematics* (*Xisuan xinfa zhijie* 西算新法直解), a work in three parts: "Algebra" (代数), "Differential Calculus" (微分), and "Integral Calculus" (积分), whose calculus sections (*juan* 5–8) drew heavily on *Daiweiji sheji*. While it added little new knowledge, its aim was to render calculus "accessible and learnable to all" (人人可知, 人人可学), and its clearer, more popular style helped disseminate calculus more widely.

In 1874, Hua Hengfang, in collaboration with the Englishman John Fryer (1839–1928), published the eight-*juan* *On the Origins of Calculus* (*Weiji suyuan* 微积溯源). This was the second relatively systematic treatise on calculus to be translated into Chinese, rendered from a work by the British mathematician William Wallace (1768–1843).

Although the terminology and notational system in this translation followed those established in *Daiweiji sheji*, its contents were richer, adding such topics as bivariate calculus, line integrals, and differential equations. In his preface, Hua stated that the purpose of the translation was to remedy the omissions (所略) in *Daiweiji sheji*. Nonetheless, in overall mathematical quality it did not substantially surpass *Daiweiji sheji* and shared its shortcomings, most notably a lack of rigorous exposition and proof; Cauchy's rigorization of analysis is entirely absent.

Although *Daiweiji sheji* and *On the Origins of Calculus* did not reflect the cutting edge of contemporary Western mathematics, they nonetheless introduced entirely new bodies of knowledge to Chinese readers and, as the only two complete Chinese translations of works on calculus in the nineteenth century, played a pivotal role in its early diffusion in China. Over the subsequent decades up to the Xinhai Revolution, more than twenty calculus books were published as calculus gradually became more widespread in China. Nevertheless, throughout this period the calculus texts and calculus knowledge that Chinese scholars actually acquired still stemmed primarily from the two translations co-authored by Li Shanlan and Hua Hengfang with their Western collaborators; many Chinese-authored calculus texts were adaptations, expositions, or supplemented solutions based on these two earlier translations. Representative late-Qing Chinese works on calculus include:

(1) *"Daiweiji sheji" with Supplemented Solutions* (*Daiweiji sheji bucao* 代微积拾级补草), 2 *juan*, Zhang Bingshu 张秉枢, Guangxu 22 (1897), block-printed edition.

(2) *Collected Proofs in Calculus* (*Weiji jizheng* 微积集证), 4 *juan*, Lin Chuanjia 林传甲, Guangxu 26 (1900), block-printed edition.

(3) *"Daiweiji sheji" with Supplemented Solutions*, 4 *juan*, Lu Jing 卢靖 (authored and commented), manuscript.

(4) *On the Origins of Calculus with Supplemented Solutions* (*Weiji suyuan bucao* 微积溯源补草), 4 *juan*, Lu Jing, manuscript.

(5) *Multiple Integrals with Supplemented Solutions* (*Die jifen bucao* 叠积分补草), Lu Jing, Guangxu 28 (1902), manuscript.

(6) *Detailed Exposition of Calculus* (*Weiji chanxiang* 微积阐详), 5 *juan*, Chen Zhijian 陈志坚, Guangxu 31 (1904), block-printed edition.

(7) *Draft of "Daiweiji sheji" with Detailed Solutions* (*Daiweiji sheji xiangcao* 代微积拾级详草), Zhou Fan 周藩, Guangxu 31 (1905), block-printed edition.

(8) *Comprehensive Interpretation of Calculus* (*Weiji tongquan* 微积通詮), 5 *juan*, Huang Qiming 黄启明, Guangxu 31 (1905), block-printed edition.

(9) *A Simple Explanation of the Method of Integration* (*Jifen fa qianshi* 积分法浅释), Qu Fangmei 瞿方梅, letterpress edition.

Calculus was gradually incorporated into school curricula and official education

plans from the 1860s and 1870s onward. For example, in 1867 the School of Combined Learning 同文馆 established the Department of Astronomy and Mathematics 天文算学馆 and appointed Li Shanlan as its first mathematics instructor. The school soon began teaching elementary Western mathematics and later added calculus to its syllabus. The eight-year curriculum, promulgated in the second year of the Guangxu reign (1876), stipulated that students in the Western-language track should study calculus in their sixth year, while students in the Chinese-language track were to take it in their fourth year (Zhu 1983, 72). Many late-Qing schools and academies subsequently introduced calculus courses. The Foochow Navy Yard School 福建船政学堂, founded in 1867, listed calculus among its courses in the shipbuilding (French-language) division, taught by a foreign mathematics instructor, Louis Médard (Zhu 1983, 464). Established in 1895, the Sino-Western School 中西学堂 (first-level school 头等学堂) included a course in differential calculus in its second-year program (Zhu 1986, 493). Zhang Zhidong's 张之洞 Lianghu Academy, founded in 1890, offered a mathematics program that contained calculus components. In 1897, Zhang even wrote to Hua Hengfang, asking him to recommend a mathematics assistant: "The Lianghu Academy has this year revised its regulations so that everyone studies mathematics. [. . .] [We] intend to add one assistant in mathematics to collaborate in instruction. The person must be proficient in algebra and calculus; if you know of such a capable candidate to serve as my assistant, please inform me at once" (Zhu 1986, 410).³ This correspondence indicates that Hua was by then teaching mathematics (including calculus) at the academy, and indeed the Lianghu Academy's teaching materials included a "Table for Mutual Seeking of Differentials and Integrals" (Weiji huqiu biao 微积互求表). The Shaoxing Sino-Western School 绍兴中西学堂, established in 1897, set up a Mathematics Department whose curriculum required that "[students] coming to the Mathematics Department should first study arithmetic and, if they had mastered arithmetic, can proceed to geometry; those proficient in geometry are to study algebra, after which the 'eight trigonometric lines,' logarithms, and the theory of various curves would be taken up as preparation for differential and integral calculus. [. . .] Once a student has completed arithmetic, they must attain proficiency in *Daiweiji* before they are considered finished" (Zhu 1986, 742-743).⁴ The curriculum also specified required textbooks, and the stipulation for calculus read:

3 “两湖书院迄今年改章，令人人皆习算学，……拟添算学帮分教一位，协同分教分任讲授，须通代数、微积分者方可分阁下讲授之劳，如素知有此等好手，可为台端助理者，请速示。”

4 “凡入算学馆，先习数学，已通数学者习几何，已通几何者习代数，然后讨论三角八线、对数、诸曲线之理，以进于微分、积分。……惟学者既习算学，必求通代微积而后已。”

For the study of calculus, one should first learn *Daiweiji sheji*, the translation by the Westerner Wylie, and thereafter study the calculus translated by John Fryer. (Zhu 1986, 744)⁵

Thus, the principal calculus textbooks in use at the time remained *Daiweiji sheji* and *On the Origins of Calculus*. From the twenty-fourth to the twenty-sixth years of the Guangxu reign (1898–1900), Lin Chuanjia taught at the Xihu Refined Lodge 西湖精舍 in Hengzhou 衡州, the Qiushi Academy 求是书院 in Changning 常宁, and the Hunan Education Commission 湘学使署 in Changsha 长沙, lecturing at all three institutions on calculus, with *Daiweiji sheji* still serving as the primary textbook (Wu 1900).

Only in the early years of the twentieth century did new Chinese translations appear. To date, the major late-Qing translations of calculus known to us include:

(1) *Calculus (Weiji xue 微积学)*, E. Loomis; orally interpreted by Alvin Pierson Parker 潘慎文 with notes by Xie Honglai 谢洪赉; Shanghai: The Commercial Press, Guangxu 32 (1906), third print.

(2) *Calculus (Weiji xue 微积学)*, 15 chapters, E. Loomis; translated by Liu Guangzhao 刘光照; Shanghai: American Presbyterian Mission Press (letterpress edition), Guangxu 31 (1905); reprinted in the first year of the Republic of China (1912).

(3) *Outline of Calculus (Weijifen xue gangyao 微积分学纲要)*, Sawada Goichi 泽田吾一; translated by Zhao Liao 赵縯; Shanghai: Yiqun Press, Guangxu 33 (1907).

(4) *Calculus (Weiji xue 微积学)*, 1 vol., G. A. Osborne; translated by Zheng Jiabin 郑家斌; Shanghai: Shanghai Science Society (letterpress), Xuanton 1 (1909).

These newer translations introduced improvements in notation. For instance, Liu Guangzhao's edition adopted Arabic numerals consistently, used the symbols “+” and “−” directly, and employed “ π ” for the circle constant. Nevertheless, variables and the symbols for differential and integral operations in many of these texts still followed the conventions first established by Li Shanlan and Hua Hengfang. It was not until around the Xinhai Revolution that the usage of Li's symbolic system began to decline in favor of standard Western notation. More than half a century elapsed from the 1859 publication of *Daiweiji sheji* to the wholesale transplantation of Western algebraic and calculus notation into China.

The new translations however still remained only elementary textbooks; they presented the basic contents of calculus rather than research-level material. In 1904, the Qing government promulgated a new set of educational regulations, the Presented School Regulations (Zouding xuetang zhangcheng 奏定学堂章程), which not only mandated calculus courses in the newly established imperial colleges (大学堂) but also required the teaching of calculus in the higher-level schools (高等学堂). This created an

5 “习微积，宜先习西士伟烈亚力所译之《代微积拾级》，再习傅兰雅所译之微积术。”

urgent demand for calculus textbooks. The translation and publication of these works were undertaken precisely to meet this pressing need. Although the regulations were not immediately implemented in full, many Chinese schools and mission schools began to offer elementary calculus courses. Because these newer translations contained little knowledge that was theoretically profound, Chinese understanding of calculus—despite its gradually widening acceptance over more than half a century of transmission—remained fairly limited in scope and depth throughout the late Qing; it was not until after the fall of the Qing dynasty that this low level of mathematical proficiency was largely overcome.

In the autumn of 1913, the Department of Mathematics at Peking University 北京大学数学门 admitted its first mathematics-major students, marking the formal start of China's first modern university mathematics program. Two students comprised the inaugural cohort and graduated in 1916, at which time the department's faculty included Feng Zuxun 冯祖荀 and Hu Junji 胡浚济. By early 1917, the department had expanded to seven instructors and several dozen students. Courses in analysis had been taught by Feng, who had returned from study in Japan, and the curriculum was increasingly aligned with Japanese and Western standards. New knowledge was introduced—among them courses in integral equations, a discipline then newly emergent in Germany—using lecture notes by D. Hilbert as a textbook. In the November 1917 revision, the department's curriculum listed calculus, differential equations, and harmonic analysis. In subsequent years additional offerings followed, including differential geometry, integral equations, the calculus of variations, infinite series, elliptic functions, and elliptic modular functions. At the end of 1917, an institute affiliated with the Mathematics Department 数学门研究所 was established. Under the leadership of Feng Zuxun and Qin Fen 秦汾 (who had returned from Harvard University with graduate training in astronomical mathematics, receiving his MA degree in 1909), the institute began to conduct scholarly activities. In October 1918, Peking University founded its Mathematico-Physical Society 北京大学数理学会, and in January 1919, the society began publishing *Mathematico-Physical Journal of the Government University Peking* (北京大学数理杂志). Articles on calculus and related topics soon started to appear—for example, Feng Zuxun published papers on differential equations, integral equations, and elliptic modular functions (Ding 1999, 532–536). In 1917, Hu Mingfu 胡明复 completed his doctoral dissertation in integral equations at Harvard University; that same year, he was awarded his PhD. In October 1918, the *Bulletin of the American Mathematical Society* (vol. 19, no. 4) published his doctoral dissertation in full under the title “Linear Integro-Differential Equation with a Boundary Condition.” Hu Mingfu was the first Chinese to earn a PhD in mathematics, and his paper stands among the earliest modern mathematical papers published by a Chinese scholar (Zhang 1991). Returning to China in 1918, Hu held professorships in

mathematics at Shanxi Datong University and at Southeast University. He authored textbooks on calculus and advanced analysis and in collaboration with editors at the Commercial Press, organized mathematics teachers in Nanjing and Shanghai to compile several popular mathematics books, laying important groundwork for the diffusion of mathematics in China. In the 1920s and 1930s, a large cohort of Chinese scholars returned from overseas studies in the West and took up teaching and research positions in Chinese universities. Their work accelerated the introduction of new mathematical knowledge and brought China's calculus education and transmission increasingly into line with international developments of mathematics.

2 The transmission and impact of calculus in Japan

Calculus was first introduced into Japan during the Bakumatsu 幕末 period, originating from two principal channels: Chinese translations of Western mathematical works, and calculus works imported from the Netherlands. Of these two, Chinese translations were the primary source of calculus knowledge in Japan during the Bakumatsu and early Meiji 明治 eras. *Daiweiji sheji* played an especially significant role in the early transmission of calculus to Japan.

Shortly after its publication in China, *Daiweiji sheji* made its way to Japan and quickly became a key text for Japanese scholars learning calculus. Ono Tomogoro 小野友五郎 (1817–1898) is likely the earliest known Japanese mathematician to have studied the book. A prominent disciple of the Wasan 和算 mathematician Hasegawa Hiromu 长谷川弘, Ono had edited Wasan works in his youth. In the second year of the Ansei 安政 era (1855), he entered the Nagasaki Naval Training Center 长崎海军传习所, an institution established to train naval specialists and staffed by Dutch instructors, regarded as one of the earliest systematic venues in Japan for instruction in Western mathematics (Editorial Committee 1983, 75, 23). The mathematics taught there focused on subjects relevant to navigation—arithmetic, algebra, logarithms, geometry, and trigonometry. Because Ono already possessed a comparatively advanced command of Wasan, he absorbed Western mathematics more rapidly than his peers and was able to study more advanced knowledge (Ogura 1941, 129). In an address reprinted in the Meiji mathematical periodical *Sugaku Hochi* 数学报知 (May 1894, issue 89), Ono recalled that, starting from the end of the second year of the Ansei era (1856), he had spent four or five years studying Western mathematics. He specifically mentioned having studied a Chinese work titled *Daiweiji* 代微积 that covered algebra, differential calculus, and integral calculus (Ogura 1948, 209). The calculus text Ono studied was almost certainly the Chinese publication *Daiweiji sheji*. His fourth year of study would have been in 1860, only about one year after the treatise's publication in Shanghai, suggesting that it reached Japan very shortly after being published.

In 1862, the feudal retainers 藩士 Takasugi Shinsaku 高杉晋作 and Nakamuta Kuranosuke 中牟田倉之助 purchased *Daiweiji sheji* during a visit to Shanghai. Nakamuta had been Ono's classmate at the Nagasaki Naval Training Center and later rose to the rank of vice admiral. He was, along with Ono, among the founding members of the Tokyo Mathematical Society 東京数学会社 (Tokyo Mathematical Society 1877), the earliest Japanese mathematical association that later evolved into the Mathematical Society of Japan 日本数学会. As an enthusiast of mathematics, Nakamuta's acquisition of *Daiweiji sheji* was evidently motivated by a desire to study or propagate advanced mathematical knowledge.

Throughout the Bakumatsu and early Meiji periods, numerous Japanese scholars studied *Daiweiji sheji*. Some of these figures would later play important roles in the history of modern Japanese mathematics. A variety of handwritten copies and translations of *Daiweiji sheji* from this era still survive in Japan. A close examination of these copies and their contextual materials provides a valuable window into how calculus first circulated and took root in Japan during its early phase (Feng 1999).

(1) A fair autograph copy by Kanda Takahira

This manuscript is now held in the Tohoku University Library and is bound in three volumes (labelled 甲, 乙, and 丙) (Figure 1). Because Kanda Takahira 神田孝平 generally dated his copies at the end of each *juan*, the manuscript can be dated to the period from the seventh month of the first year of the Genji 元治 era (1864) to the first month of the first year of the Keio 庆应 era (1865).

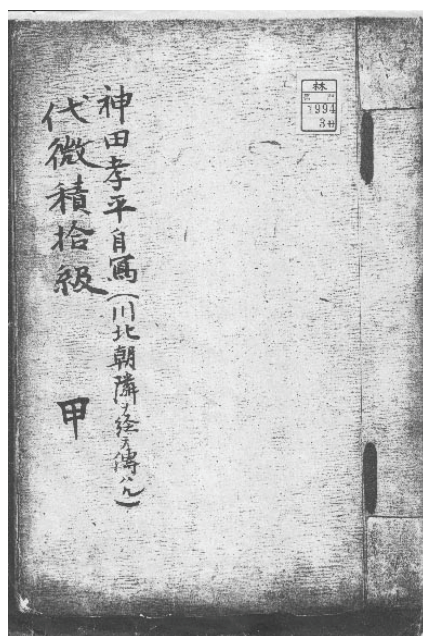


Figure 1: Cover of Kanda Takahira's fair autograph copy of *Daiweiji sheji*.

Kanda Takahira (1830–1898) was a prominent scholar of Western learning during the Bakumatsu and early Meiji eras. He studied Chinese classics in his youth, subsequently turned to *rangaku* 兰学 (Dutch learning), and played a significant role in the introduction of Western learning in Japan. He held important posts such as Vice Minister in the Ministry of Education and in 1862 was appointed mathematics professor at the Institute for the Study of Western Books 洋书调所 (later renamed 开成所 Kaiseijo [School of Foreign Studies]). Although he later accepted other appointments, he continued to teach at Kaiseijo part-time and served as director of the Kaiseijo school dormitory between July and December 1864. Kaiseijo combined the tasks of translating Western works and training specialists in Western learning, and later became the Kaisei School 开成学校, a predecessor of the University of Tokyo. Several noted Meiji mathematicians studied there. The most distinguished among Kanda's students was Kikuchi Dairoku 菊池大麓 (1855–1917), who later served as president of the University of Tokyo and Minister of Education. Kanda was also one of the founders of Japan's earliest mathematical association—the Tokyo Mathematical Society—and served as its first president (Komatsu 1986). Kanda's copy of *Daiweiji sheji* was produced during his teaching period at Kaiseijo. Although his instruction focused mainly on elementary mathematics, he himself began to study more advanced subjects such as calculus. The manuscript contains Kanda's marginalia in the head and tail margins of certain leaves: Dutch mathematical terms and formulas written in Western notation, which are of considerable historical value. On some pages, he wrote the Dutch equivalents of certain important mathematical terms in the margins and, on a few leaves, supplied English equivalents for the Chinese terms. Elsewhere he added headnotes restoring the Chinese-style mathematical symbols and formulas originally used by Li Shanlan to their Western-style expressions.

(2) Omura Isshu's Japanese translation of *Daiweiji sheji*

This work is a manuscript housed in the Tohoku University Library, with no explicit date of translation recorded in the copy (Figure 2). The translation was made by the Wasan mathematician Omura Isshu 大村一秀, and the prefaces to most *juan* carry the inscription: "Authored by the American Loomis; interpreted by the English A. Wylie; recorded by Li Shanlan of Haining; translated by the Japanese Omura Isshu."⁶ The manuscript bears Omura's seal and appears to be his autograph manuscript. It is a complete translation that reproduces the entirety of the Chinese edition. The symbols and formulas follow the Chinese edition verbatim, and even the book title and technical terminology are taken directly from the Chinese text.

6 “米利坚罗密士撰，英国伟烈亚力口译，海宁李善兰笔述，日本大村一秀译述”

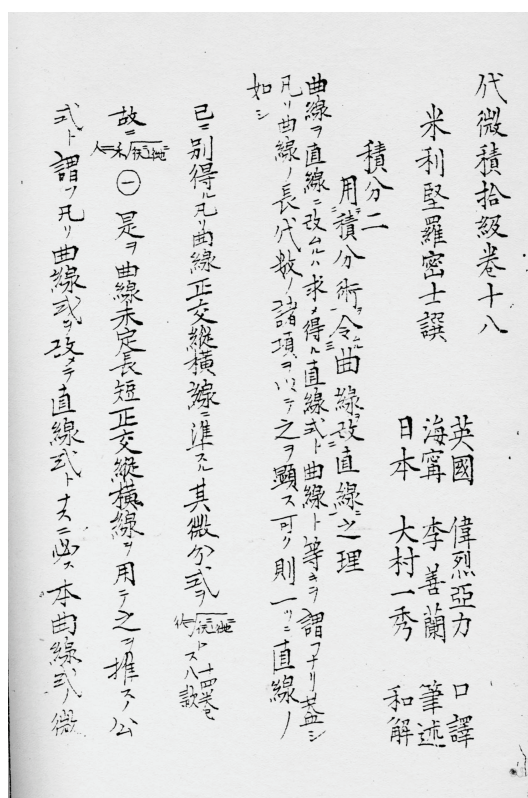


Figure 2: Omura Isshu's translation of *Daiweiji sheji* (front page of juan 18).

Omura Isshu (1842–1890) was a well-known Wasan mathematician in the Bakumatsu period and authored several Wasan works. He was also an early member of the Tokyo Mathematical Society and served as the founding editor of its journal. In the *Journal of the Tokyo Mathematical Society* 东京数学会社杂志 from the tenth to the twelfth years of the Meiji era (1877–1879), Omura published numerous articles on algorithms related to calculus; in those papers he employed fully Western-style formulas and notation, indicating that by the early Meiji period he had mastered Western mathematical expression. Because of Omura's background in Wasan and the lack of proficiency in Western languages, he initially learned calculus from the Chinese edition *Daiweiji sheji*. It can be inferred that his Japanese translation was made in the 1860s or the early 1870s.

(3) Translated Examples of “*Daiweiji sheji*” 代微积拾级译例

This is a concise English manuscript that contains a short English text followed by an English-Chinese glossary of mathematical terms (Figure 3). The manuscript carries Wylie's English name at the end of the glossary and is dated “written on the tenth month of the sixth (*Chenyau*) year of the Meiji era” (明治六〔辰酉〕年十月写之). The copy is also held at the Tohoku University Library.

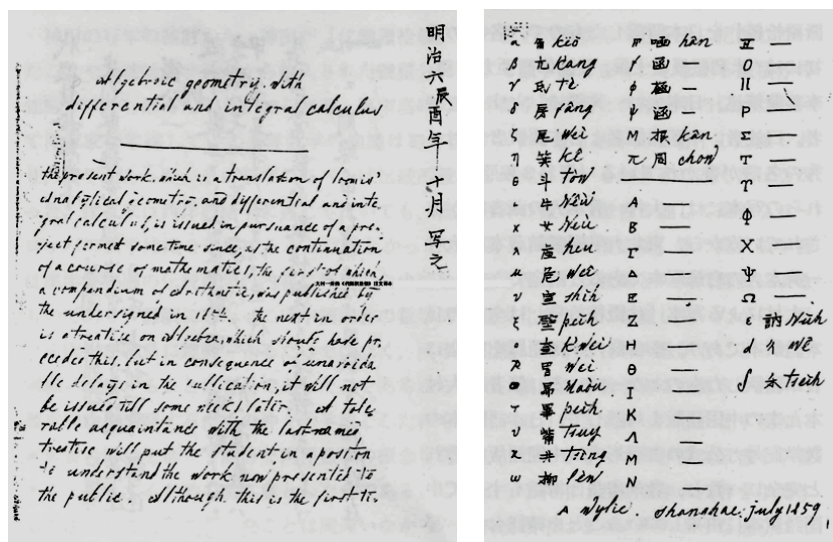


Figure 3: Translated Examples of "Daiweiji sheji" (front and back pages).

By textual comparison we have confirmed that the English text in this manuscript is Alexander Wylie's English preface (Figure 4), written in July 1859, for the first Chinese edition of *Daiweiji sheji*. Some extant first-edition copies of *Daiweiji sheji* include this English preface and the English-Chinese mathematical glossary—for example, the version in the Kano Collection 狩野文库 at Tohoku University—showing that early copies circulating in Japan sometimes contained the bilingual glossary.

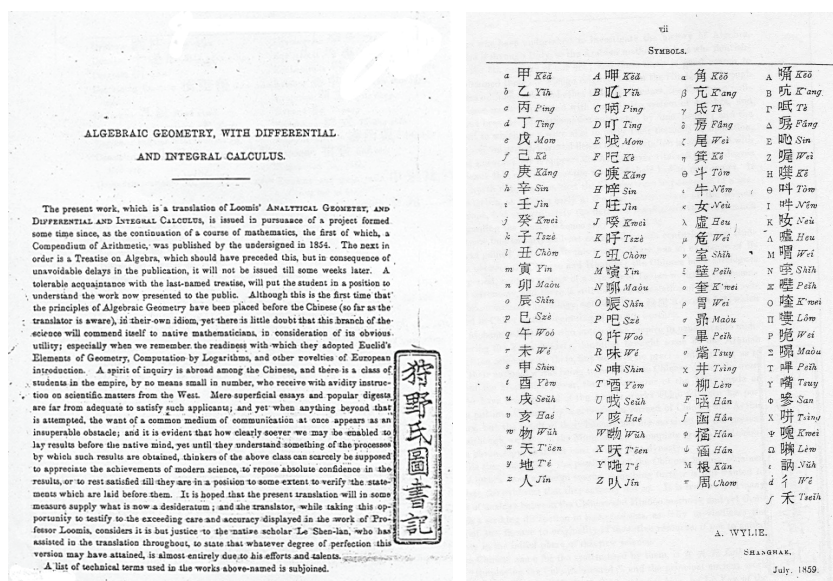


Figure 4: The English preface by Alexander Wylie in the copy of *Daiweiji sheji* held in the Kano Collection.

(4) Fukuda Han's Translated Explanations of "Daiweiji sheji" 代微积拾级译解

This Translated Explanations of "Daiweiji sheji" (1 *juan*) is a block-printed edition in Japanese issued in the fifth year of the Meiji era (1872) (Figure 5). It is primarily a translation of the Chinese edition of *Daiweiji sheji*. From Fukuda Han's 福田半 preface, it is clear that the translator consulted Loomis's English original and a Kanda manuscript.

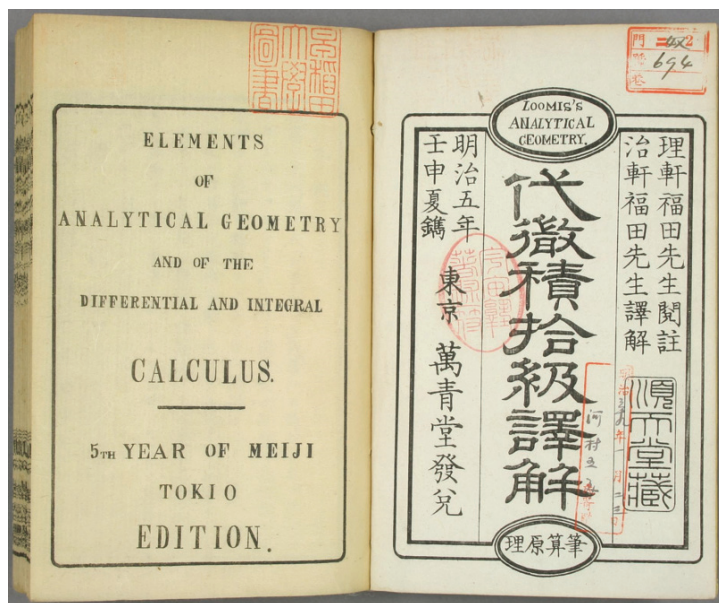


Figure 5: Translated Explanations of "Daiweiji sheji."

According to historical records, only the first *juan* of Fukuda's translation was ever published; that *juan* covers only a limited portion of the original work, mainly the material on analytical geometry that concerns the circle. Fukuda Han (Jiken 治軒), eldest son of the noted Wasan mathematician Fukuda Izumi 福田泉 (Riken 理軒) (1815–1889), studied Wasan at his father's Junten Kyugosha School 顺天求合社 and later turned to Western learning, receiving instruction from British teachers and serving as a naval instructor. The edition was issued by Banseido 万青堂 in Tokyo in the fifth year of the Meiji era. According to two Junten Kyugosha publishing catalogues—the "Junten Kyugosha School Mathematical Works" 顺天求合社算学书目 appended to *New Book of Mathematics for Elementary School Students* 明治小学塵劫记 (the twelfth year of the Meiji era) and the *Catalogue of Standard Arithmetic School Textbooks* 普通测算学校课书目 attached to *Introduction for Algebra and Calculus* 笔算微积入门 (the thirteenth year of the Meiji era)—Fukuda Han's *Translated Explanations of "Daiweiji sheji"* was issued in only one *juan*.

Fukuda's edition adopts Western-style symbols and formulas throughout, a marked departure from the Chinese edition, although the narrative text and many technical terms remain those of the Chinese translation. Despite its incompleteness, Fukuda's work was the first publicly printed Japanese translation of *Daiweiji sheji* and thus had substantial influence: it became one of the principal textbooks for the study of Western mathematics in Japan at the time.

(5) Outline of Integral Calculus 积分学提要

This is a manuscript now held in the Library of the Japan Academy (Figure 6). From the signature on the front, it is identified as a work of the Suri Kenkyusha 数理研究舎 (Institute for Mathematical Research). The manuscript is in Chinese, and a comparison with *Daiweiji sheji* shows that it is an excerpt containing only the integral-calculus portion of that work. Its narrative text reproduces the Chinese original verbatim, but the notation and formulas show modest improvements: Arabic numerals are employed, and the fraction convention is reversed from the Chinese edition, with the denominator placed below the numerator. The Suri Kenkyusha was a mathematical-education institution founded by the Takeda school 武田派 of Wasan and the Wasan mathematician Takeda Kenzo. In the early Meiji period, the institute produced several mathematical works (Japan Academy 1979, 371–372). The Library of the Japan Academy also holds a three-juan *Geometry Questions and Answers* 几何学问答 compiled by the Suri Kenkyusha, which is dated as “completed in the first month of the tenth year of the Meiji era” (明治十年一月稿成).

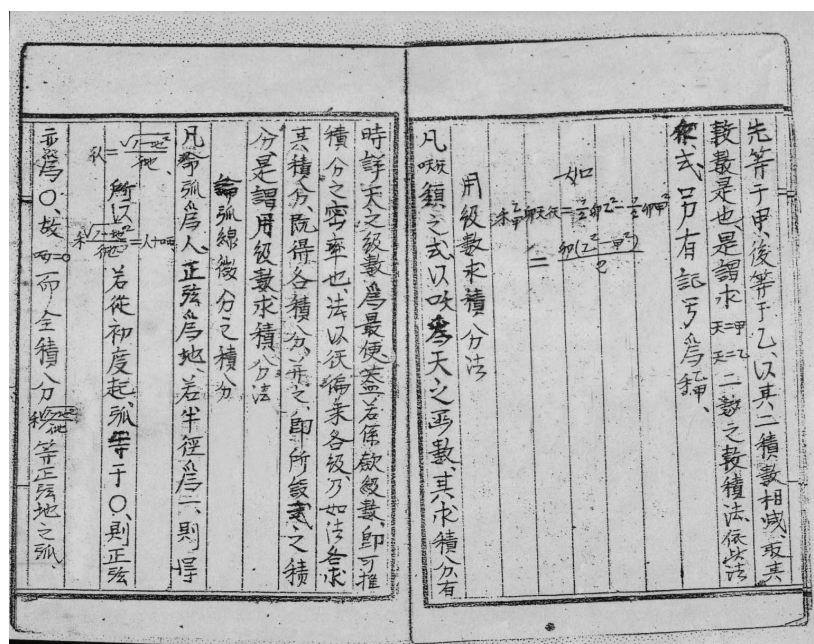


Figure 6: Outline of Integral Calculus.

Takeda Kenzo 武田谦藏 (?–1906), born Mori Keisuke 毛利惠助 and later taking the surname Takeda after being adopted by the Wasan master Takeda Shingen 武田真元, was a disciple of the prominent Bakumatsu-era Western-learning scholar Uchida Gokan 内田五观 and thus received some training in Western mathematics. He authored *Collection of Elementary Calculation Methods* 算法幼学集 in 1854. Although his works are mainly in Wasan, he also produced treatises on Western mathematics, for example *Principles of Western Written Calculation* 西洋の笔算之主义 in the twenty-ninth year of the Meiji era (1896). Accordingly, *An Outline of Integral Calculus* 积分学提要 should be dated to the early Meiji period.

Shortly after its Chinese publication, *On the Origins of Calculus* (Weiji suyuan 微积溯源) also arrived in Japan and exerted a measurable influence. Japanese-authored and -translated Western calculus texts began to appear in the 1880s. Prior to that decade, the only physically and language-accessible calculus works available in Japan were the Chinese translations *Daiweiji sheji* and *On the Origins of Calculus*. These two works therefore circulated widely in Japan and were generally well received. Even into the 1880s, they remained the principal reference works for Japanese students of calculus—a fact reflected in contemporaneous prefaces to Japanese mathematical treatises. For example, in his preface to his translation of *Differential Calculus* 微分学 in the fourteenth year of the Meiji era (1881), the noted mathematician Nagasawa Kamenosuke 长泽龟之助 observed that:

In recent times more mathematical works have appeared than ever before. Yet these publications are overall elementary; genuinely advanced treatises are entirely lacking. [. . .] Moreover, there are scarcely any precedents for translating mathematical terminology. Consequently, we have relied chiefly on two or three Chinese translations, *Daiweiji sheji* and *On the Origins of Calculus*, with occasional reference to Davies's *Mathematical Dictionary*. (Nagasawa 1881, "Preface")⁷

The eminent Japanese mathematics historian Mikami Yoshio 三上义夫 (1913, 301) has argued that for Wasan scholars, "the Chinese translation of Loomis; was perhaps the best source of information he could avail himself of." Mikami (1923), discussing the impact of Chinese translations of Western mathematics on Japan, observed: "The Chinese *Daiweiji sheji* played an especially significant role in Japan's initial study of Western calculus." Both *Daiweiji sheji* and *On the Origins of Calculus* contain abundant Chinese renderings of mathematical terminology. These Chinese terms are often natural, concise, and apt. They were adopted not only by contemporary Chinese

7 “算书之出，近世为最。然其书率皆卑近而已，至高等之书则缺然无有。……且如算语之译字，世有先例者鲜矣。故仅据支那译之《代微积拾级》、《微积溯源》等二、三书。或参考代威斯氏数学字典。”

readers but also by Meiji-period Japanese mathematicians, and many of the terms have persisted into modern usage.

During the Bakumatsu era, many Dutch-language mathematical works, including several on calculus, were imported into Japan. The extant titles that can be securely identified as Dutch imports of calculus from that period include (Ogura 1948, 176–178):

- (1) I. R. Schmidt, *Beginselen der differentiaal- en integraalrekening*, 1822.
- (2) J. Badon Ghijben, *Beginselen der differentiaal- en integraalrekening*, 1847.
- (3) R. Lobatto, *Lessen over de differentiaal- en integraalrekening*, 1851.

These texts on calculus were originally held at the Kaiseijo 开成所 and later entered the collection of Yamamoto Nobuzane 山本信実 (1851–1936), a noted early-Meiji mathematician and former Kaiseijo student. Because so few people at the time were proficient in both Dutch and mathematics, the influence of calculus works transmitted via the Netherlands was relatively limited compared with that of the Chinese translations. Earlier Japanese scholarship had found little direct evidence that the Japanese learned calculus from Dutch treatises, and given the generally modest mathematical competence among *rangaku* scholars in the Bakumatsu period, the impact of the Dutch calculus texts has seldom been discussed. One of the authors of this article rediscovered Kanda Takahira's autograph manuscript of *Daiweiji sheji* in the holdings of the Tohoku University Library, providing direct evidence on this point. It can be inferred that some individuals who engaged in Dutch or Western studies also began to learn calculus from Dutch treatises. Komatsu Atsuo 小松醇郎 (1990, 70), director of the editorial committee of *100 Years of Mathematics in Japan* 日本の数学 100 年史 and distinguished mathematician and historian of mathematics, originally believed that Kanda's mathematical proficiency amounted at best to a rudimentary grasp of the concept of coordinates and a very limited acquaintance with analytic geometry. The discovery of Kanda's autograph manuscript compels a reassessment of the mathematical levels of scholars of Western learning of the period.

Kanda, in his marginalia, solved several of the exercises in *Daiweiji sheji* (the original work gives no answers). In his notes he records the step-by-step solution procedures. One example is cited here to illustrate the point.

Exercise 14 on page 159 of the English original is an extremum problem stated as follows (Loomis 1852, 159):

Require the maximum cone which can be inscribed in a given sphere.

E. Loomis's English edition gives no solution. Although the Chinese translation does provide an answer, it supplies no procedures. In the Chinese translation (see *Daiweiji sheji*, *juan* 12, "Differential III" 微分三), the problem is phrased as follows:

Now, given a sphere, find the largest cone that can be inscribed in it; what is the proportion of its height? Answer: The height is three-quarters of the radius of the sphere.⁸

Kanda's annotations provide the following solution procedure (Figure 7):

$x(2r - x) = r^2$ (Let r denote the radius of the sphere and x the height of the cone inscribed in it; the r^2 on the right-hand side of the equation is incorrect. The correct notation should be: $x(2r - x) = r_{\text{锥}}^2$)

$$y = \frac{1}{3}x \times x(2r - x)\pi \quad (\text{Let } y \text{ be the volume of the cone inscribed in the sphere.})$$

$$y = \frac{1}{3}\pi(2rx^2 - x^3)$$

$$4rx - 3x^2 = 0$$

$$4r = 3x$$

$$x = \frac{4}{3}r$$

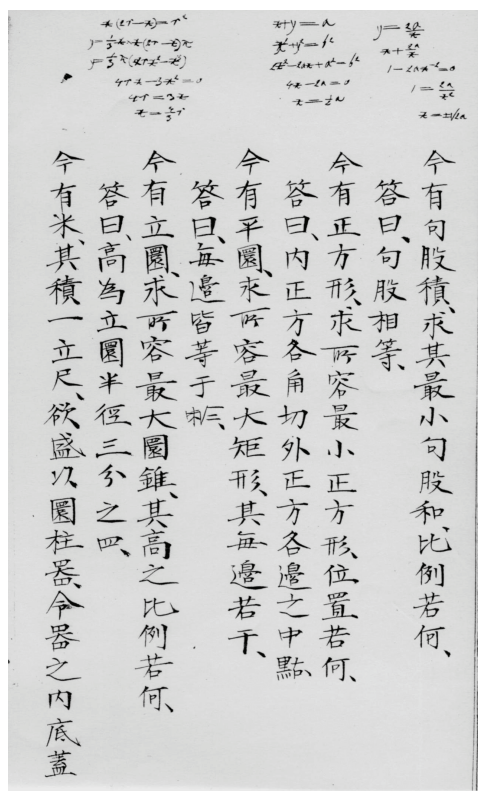


Figure 7: A page of the *Daiweiji sheji* manuscript annotated with the solution steps.

8 “今有立圆，求所容最大圆锥，其高之比例若何。答曰：高为立圆半径三分之四。”

By comparing the notation Kanda employed with that of the English original and with several Dutch calculus treatises printed before 1865 that are extant in Japan, we found that Kanda's symbols accord with the Dutch works. From Kanda's manuscript, it is evident that he had not only mastered several important parts of *Daiweiji sheji*, but had also begun studying calculus by consulting Dutch calculus texts. During the Bakumatsu and early Meiji periods, scholars in Western learning and mathematics who learned and taught Western mathematics possessed only modest knowledge of it and could not rival the level of traditional Wasan specialists. Although many of them could read Western languages, most found it difficult to acquire advanced Western mathematics directly from original sources. They therefore relied on Chinese translations of Western mathematical works such as *Daiweiji sheji* to comprehend more sophisticated concepts, and only then proceeded to the original Western texts. Chinese translations of Western mathematical books thus functioned as an intermediary tool, through which scholars in Western learning digested and assimilated high-level Western mathematical knowledge—a shortcut to learning modern Western mathematics.

In the 1870s, Japanese scholars began translating original Western calculus treatises directly into Japanese. Several such calculus translations appeared, though they circulated in manuscript form. A brief introduction to three extant manuscript calculus works is presented below.

(1) *Differential Calculus* 微分術, translated by Sekiguchi Hiraki 关口开; the manuscript was completed in the seventh year of the Meiji era (1874). Its source text was H. N. Robinson's *A New Treatise on the Elements of the Differential and Integral Calculus*. Sekiguchi Hiraki was a renowned mathematics educator of the Meiji period. He began by studying Wasan, then taught himself English and Western mathematics. He worked as a mathematics instructor in the Kanazawa Domain 金泽藩, teaching in normal and common schools and running a private school that produced many disciples, several of whom later became distinguished mathematicians. Sekiguchi was also among the first members of the Tokyo Mathematical Society. He translated many Western mathematical works, though they mostly circulated as manuscripts among his students. Owing to his efforts, Kanazawa became one of the centers of Western mathematics in the early Meiji era (Editorial Committee 1983, 54).

(2) *Appendix to Differential Calculus* 微分術附录, also by Sekiguchi Hiraki, compiled in the third month of the seventh year of the Meiji era (1874). This treatise is essentially a solution collection for extremum problems. At its outset, Sekiguchi presents detailed solution methods for six representative problems, employing Western notation and calculation techniques throughout. Following that, he lists one hundred extremum problems along with their answers, providing only hints for the methods of solution. It is noteworthy that many of the collected problems originated in the Wasan tradition.

(3) *Drafts on Differential Calculus* 微分草稿, an extant manuscript translated by Yokota Tatsuro 横田龙郎; the base text and the date of the translation are not recorded. Yokota Tatsuro (1849–1875) was a retainer of the Nanbu Domain 南部藩, so the work can be dated to before 1875 (Yanamoto 1991). The book is divided into five parts, treating respectively the basic concepts of calculus, the differential of inverse functions, methods for finding higher-order derivatives, Taylor's theorem, and Maclaurin's theorem. In this translation, many newly coined mathematical terms are rendered directly into katakana 片假名. The notation and calculational procedures are notably advanced. Several examples are given below:

$$y', f'(x), \frac{\partial y}{\partial x}, \lim \frac{\Delta y}{\Delta x} = \lim \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

In the early 1880s, several Japanese printed works on calculus began to appear. In 1880, Fukuda Han's *Introduction for Algebra and Calculus* 笔算微积分入门 was published, marking the first calculus book both authored and formally published by a Japanese writer. The work is divided into two parts. The first part (280 pages) treats differential calculus. When compiling this treatise, Fukuda chiefly consulted the English works of E. Loomis, H. N. Robinson, Ch. Davies, I. Todhunter, and T. G. Hall. The second part deals mainly with integral calculus, and the final ten or so pages provide an introduction to differential equations. Fukuda's translation also drew on material from the Chinese work *Daiweiji sheji*. A comparison of the mathematical terminology in *Introduction for Algebra and Calculus* with the terminology used in *Daiweiji sheji* shows that most of the terms for analytic geometry, differential calculus, and integral calculus were adopted from the latter. A few terms were refined: for example, the word "limit," which had previously been translated by Alexander Wylie and Li Shanlan as "限," was rendered by Fukuda as "极限," matching the modern Japanese and Chinese usage. The modern Japanese and Chinese mathematical term "极限" (limit) may have entered common usage via Fukuda's work.

A number of new Japanese translations of calculus texts were published thereafter. The major ones include:

(1) *Differential Calculus* 微分学, translated by Nagasawa Kamenosuke 长泽龟之助, published in the fourteenth year of the Meiji era (1881). This was the first printed Japanese translation of a treatise on differential calculus, its source text being I. Todhunter's *A Treatise on the Differential Calculus* (1871).

(2) *Integral Calculus* 积分学, also translated by Nagasawa Kamenosuke, published in the fifteenth year of the Meiji era (1882). This was the first printed Japanese translation devoted to integral calculus, its source text being I. Todhunter's *A Treatise on the Integral Calculus*.

(3) *Differential and Integral Calculus* 微分积分学, vol. 1, enlarged and translated by

Okamoto Norifumi 岡本则录, published in the sixteenth year of the Meiji era (1883). The base text was Albert E. Church's *Elements of the Differential and Integral Calculus* (revised edition, 1874).

(4) *Differential Equations* 微分方程式, translated by Nagasawa Kamenosuke, published in the eighteenth year of the Meiji era (1885). This was the first Japanese monograph on differential equations to appear in print. It runs 756 pages and covers foundational knowledge in both ordinary and partial differential equations. The original was G. Boole's *A Treatise on the Differential Equations* (1859); the translation relied on I. Todhunter's revised second edition (1865).

The *Journal of the Tokyo Mathematical Society*, launched in the eleventh month of the tenth year of the Meiji era (1877), included from its first issue a column entitled "Miscellaneous Questions in Algebra and Calculus" 代微积杂问, which published articles introducing and discussing topics in analytic geometry and calculus. The inaugural issue contained an article by one of the society's cofounders, Yanagi Narayoshi 柳 檣悦 (1832–1891), entitled "A Quick Western Method for Finding Maxima and Minima" 極大極小ヲ求ムル捷法 (西式). In this article, Yanagi (1877, 14) explicitly observed that "differential and integral calculus has already begun to become an important subject of study and research for Japanese mathematicians." He compared the extremum methods of Wasan with those of differential calculus, highlighting the superiority of the Western techniques. Yanagi had studied the Seki School of Wasan 关流和算 in his youth and later attended the Nagasaki Naval Training Center in the Bakumatsu period where he received instruction in Western mathematics, and was thus conversant with both Wasan and Western mathematics. In the Meiji era, he served as a rear admiral in the Japanese Navy and, like Kanda Takahira, was one of the representatives and founding directors of the Tokyo Mathematical Society. Beginning with issue 7 (the sixth month of the eleventh year of the Meiji era), the journal split the original "Miscellaneous Questions in Algebra and Calculus" column into two separate sections, "Miscellaneous Questions in Algebra and Geometry" 代数几何杂问 and "Miscellaneous Questions in Differential and Integral Calculus" 微分积分杂问, and further added a new column, "Miscellaneous Questions in Differential Equations" 微分方程式杂问. In the journal's early years, the contributors who frequently published on calculus topics included Yanagi Narayoshi, Kikuchi Dairoku, Sekiguchi Hiraki, Okamoto Norifumi, Omura Isshu, Yamamoto Nobuzane, Fukuda Riken, Fukuda Han, Arakawa Juhei 荒川重平, Akamatsu Noriyoshi 赤松则良, and Nakagawa Masayuki 中川将行.

Japanese university-level education in mathematics majors began in the tenth year of the Meiji era (1877). In that year, the Tokyo Kaisei School 东京开成学校 and the Tokyo Medical School 东京医学校, both under the Ministry of Education, were merged

and renamed the University of Tokyo. When the university was founded, the Department of Mathematics, Physics, and Astronomy was established under the Faculty of Science, so that mathematics was taught together with physics and astronomy; courses in pure and applied mathematics were offered, although the exact syllabi are not recorded. In May 1877, Kikuchi Dairoku (1855–1917), who had spent eight years studying in Britain, graduated from the University of Cambridge (with a Bachelor's degree in mathematics) and returned to Japan. In June, he was appointed professor of mathematics at the University of Tokyo. Fujisawa Rikitaro 藤泽利喜太郎, who later became a prominent mathematician, was among the first graduates of the Department of Mathematics, Physics, and Astronomy. Following Kikuchi's advice, Fujisawa specialized in mathematics upon graduation and was subsequently sent to study in Europe. It was not until 1881 that the University of Tokyo established an independent Department of Mathematics. The graduating class of 1884 numbered only one student, and the second class in 1885 consisted of only two. According to Takahashi Toyoo 高桥丰夫, a member of the inaugural class, Kikuchi lectured from the original English editions of the prominent British mathematician I. Todhunter's (1820–1884) *A Treatise on the Differential Calculus* and *A Treatise on the Integral Calculus*, together with G. Boole's (1815–1864) *A Treatise on the Differential Equations* (Editorial Committee 1983, 104). Boole's work was the leading British work on differential equations at the time and enjoyed considerable scholarly esteem. In 1887, Kikuchi's pupil Fujisawa Rikitaro (1861–1933) returned to Japan from Germany. To catch up to the rapid development of mathematics in Europe, he reformed the mathematics curriculum at the University of Tokyo. In the early 1890s, he introduced a variety of new courses for second- and third-year students, such as the elliptic function theory and general function theory. First-year university calculus had already adopted Karl Weierstrass's rigorous formulation of ε - δ . The first-year program also included an introduction to differential geometry and both ordinary and partial differential equations (Editorial Committee 1983, 164–167). By the end of the nineteenth century, Japanese analysis had caught up with that of the leading European countries, and Japanese mathematics thus entered the modern era. It is worth emphasizing that Fujisawa imported the German seminar model. Starting from 1896, he organized an ongoing sequence of seminars in which students participated, thereby nurturing a new generation of research mathematicians such as Teiji Takagi 高木贞治 and Yoshie Takuji 吉江琢儿. After graduating from the Department of Mathematics in 1897, Teiji entered graduate school and, a year later, was sent by the government to study in Germany. There, he studied the then-nascent algebraic number theory under the great mathematician D. Hilbert and returned to Japan in 1901. Working thereafter on his own, Teiji published seven papers in Japanese journals between 1914 and 1920, founded the class field theory, and produced work of first-rate international significance. The emergence of Teiji Takagi attests to the

remarkable achievements of Japan's foundational professional education in mathematics.

3 Comparison of the two countries' situations and undertakings

In the 1850s, mathematical study in both China and Japan remained within the domain of traditional mathematics. In the late 1850s and early 1860s, the introduction of calculus inaugurated the modernization of mathematics in both countries. As indicated, the two nations set out from nearly identical starting points—China, in fact, enjoying a headstart—yet by the end of the century their outcomes diverged dramatically. Japan rapidly assimilated and absorbed Western calculus, continuously imported new mathematical knowledge, and thereby narrowed the gap with the West. Within a short period, Japan built a modern system of mathematical education and completed the transformation from traditional Wasan to modern mathematics. China, by contrast, lingered in the initial stage for a long time, remaining at an early, introductory phase of modern mathematics and failing to quickly complete the transition from traditional to modern mathematics. The mathematical relationship between China and Japan was rapidly reversed: China, which had initially been a source of knowledge, became a recipient. After the defeat in the First Sino-Japanese War, China began to send students to Japan and was compelled to learn and import the advanced knowledge and experience that Japan had acquired. What, then, accounted for so striking a divergence in outcomes? This is a question that requires careful consideration and must be answered through a comparative analysis.

It should be said that the transmission of calculus played a catalytic role in the transition of mathematics in both China and Japan. Before calculus was introduced, both countries had already incorporated Western elementary mathematics. However, those imports generally did not surpass the sophistication of the indigenous traditional mathematics. Consequently, mathematicians in China and Japan had a rather constrained understanding of Western mathematics and often remained skeptical of its superiority. Prior to the arrival of calculus, mathematical development in both countries had already reached a considerable level and had begun to tackle problems within the domain of higher mathematics. Each tradition had by then devised distinctive methods that successfully resolved concrete problems—for example, in Wasan the Yenri tetsu-jutsu (圓理綴術, Method of Linkage in Studies on the Circle) and the Method for Extrema (極数術); in traditional Chinese mathematics, the Continued Proportion Method for Cutting a Circle (割圓連比例) and the Pointed-Cone Method (尖錐術)—all of which reached a high level of refinement. The introduction of calculus made it possible to compare these advanced elements of traditional mathematics with calculus-based methods. Through such comparison, Chinese and Japanese

mathematicians gradually recognized the advantages of modern mathematics, thus propelling the transformation of mathematics in both countries. Both China and Japan had, before the formal transmission of calculus, produced certain nascent ideas and methods anticipating it. Objectively, these intellectual beginnings laid the groundwork for the reception of Western calculus and represented a similarity between the two countries at the time. Nevertheless, the trajectories of mathematical modernization diverged markedly because of substantial differences in their political, economic, educational, and socio-cultural contexts.

First, differences in the political, economic, and social-institutional transformations of the two countries produced divergent effects on the process of mathematical modernization.

During the Bakumatsu period, Japan had already embarked on the importation of Western machinery, industry, and technology and had implemented institutional reforms. If one compares the Tokugawa shogunate 徳川幕府 and the various domains' 藩 efforts to introduce Western military industry and technology with China's Self-Strengthening Movement, one finds that, although the measures were broadly similar in kind, China's initiatives proved less effective than Japan's. As early as 1854, the shogunate established works in Edo (Tokyo) for the manufacture of Western-style firearms and artillery and even began constructing Western-style warships, preceding Zeng Guofan's 曾国藩 establishment of the Anqing Arsenal 安庆军械所 in Anhui in 1861. Motivated by diplomatic imperatives and the need to introduce Western science and technology, the shogunate also founded the Bansho Shirabesho (蕃书调所, Institute for the Study of Barbarian Books) in Edo in 1855 to translate works on general Western knowledge, natural sciences, and mathematics. In 1857, the Bansho Shirabesho started to evolve into an educational institution, engaging outstanding individuals from shogunal retainers 幕臣 and those of the domains to teach Dutch, English, and Western studies. Only shogunal retainers were permitted to attend that year, while the following year domain retainers 藩士 were allowed to enroll as well. Ichikawa Kaneyasu 市川齐宫, a Fukui Domain retainer, served as professor of mathematics at the Bansho Shirabesho. In 1862, the institution's name was changed to the Institute for the Study of Western Books 洋书调所, and it later became Kaiseijo 开成所. Kanda Takahira was appointed professor of mathematics at the Institute for the Study of Western Books in 1862. It was there that Kikuchi Dairoku received his initial exposure to Western mathematics. In contrast, the Qing government did not establish the School of Combined Learning, an institution for the study of foreign languages, until 1862, and the Department of Astronomy and Mathematics—created “for machinery manufacture” (因制造机器)—was not founded until 1867. The shogunate's establishment of the Nagasaki Naval Training Center in 1855, staffed by Dutch instructors who provided systematic instruction in Western military techniques and in

the sciences, including mathematics, was far more consequential. The center trained a corps of military and technical personnel and played a significant role in Japan's technological modernization. Its first cohort comprised thirty shogunal retainers and 129 domain retainers, and many prominent Meiji-era mathematicians were among its alumni. While the shogunate was providing instruction in military and technical knowledge, it also began constructing a large ironworks in Nagasaki under the supervision of Dutch engineers in 1859 to support shipbuilding, and the works were completed in 1862. This enterprise was the precursor to the renowned Japanese Mitsubishi Nagasaki Shipyard and preceded the establishment of Shanghai's Jiangnan Arsenal 江南制造局 in 1867. The shogunate also began formally sending students to the West in 1862, selecting promising candidates from the Nagasaki Naval Training Center, the Institute for the Study of Western Books, and Kaiseijo and dispatching them to Europe to study Western technology, natural sciences, medicine, and the humanities. Kikuchi Dairoku, for example, was sent to Britain in 1866 to study mathematics. The Qing government, on the other hand, did not partake in official student dispatches until 1872—ten years later—and the early cohorts did not include students specializing in mathematics; only in the twentieth century did this begin to change.

The reforms of Bakumatsu Japan and China's Self-Strengthening Movement were comparable in nature and thus invite direct comparison. By contrast, the Meiji Restoration, which began in 1868, was a far more sweeping social transformation that reshaped political institutions, economic structures, scholarship, culture, technology, and education. It was fundamentally different in nature from China's Self-Strengthening Movement: the Meiji government enacted a range of measures to import Western science and culture that cannot be equated with the more limited initiatives in China's Self-Strengthening era. Japan rapidly built a modern education system modeled on Western examples. The government pursued an aggressive program of wholesale Westernization in science and education—for example, eliminating traditional Wasan at the primary education level. On the third day of the eighth month of the fifth year of the Meiji era (1872), the Ministry of Education promulgated a new Fundamental Code of Education 学制令. Chapter 27, which dealt with elementary-school textbooks, explicitly stipulated that “only Western methods are to be used” (唯用洋法) under the “Arithmetic” (算术) section. The Elementary School Rules of Teaching 小学教则 announced on the eighth day of the ninth month that same year likewise mandated teaching of “Western-style arithmetic” (洋法算术). Because of a shortage of Western-arithmetic teachers and textbooks, however, the Ministry's Notice No. 37 (April 1873) relaxed the rule: “while the Elementary School Rules of Teaching prescribed the use of Western arithmetic for arithmetic, it allowed for the concurrent

study of traditional arithmetic.”⁹ Consequently, a mathematics text compiled by the Wasan mathematician Takaku Morishizu 高久守静 was adopted by Tokyo elementary schools. In October 1874, the Ministry of Education issued Notice No. 10, which declared “the repeal of Notice No. 37 issued by the ministry in the sixth year of the Meiji era”¹⁰ and ordered the nationwide abolition of Wasan. In 1876, the authorities required Takaku to switch to teaching Western arithmetic, and he was compelled to tender his resignation in the following year (the first month of the tenth year of the Meiji era). Japan created a nationwide system of compulsory primary education and established diverse secondary and university professional education systems. The institutionalization of university-level professional mathematics education played a decisive role in transforming and advancing mathematics. All of the Meiji government’s policies and measures were aimed at absorbing Western science and industrial technologies with high efficiency. Through the sequence of reforms launched by the Meiji Restoration, Japan stepped into the capitalist era and developed a virtuous, self-reinforcing mechanism for importing Western technology and disseminating Western scientific knowledge—a catalytic dynamic that was crucial to the rapid modernization of mathematics in Japan. The Qing dynasty, in contrast, having been reduced to a semi-feudal, semi-colonial society since the Opium War, displayed no pressing desire to import modern Western science and technology. A modern educational system did not take root until well into the twentieth century. Those delays severely impeded the dissemination and development of modern mathematics in China, resulting in a protracted transition from traditional to modern mathematics.

Second, differences in intellectual and cultural outlooks also exerted divergent influences on the respective trajectories of mathematical modernization in the two countries.

China was a long-established feudal civilization with several millennia of continuous cultural development, and well into the mid-nineteenth century many Chinese still regarded Chinese civilization as the true civilization and disregarded foreign cultures as “barbarian” (夷). A defensive negative attitude toward Western civilization was widespread among officials, scholars, and ordinary people, and there was substantial resistance to the introduction of Western science and technology. In 1866, when Yixin 奕訢, one of the leading figures of the Qing Self-Strengthening faction, proposed establishing the Department of Astronomy and Mathematics within the School of Combined Learning, his proposal elicited vigorous opposition from numerous high officials. Only after a lengthy and ferocious dispute lasting six or seven months did the Self-Strengthening faction prevail. The conservatives’ chief objection was to “let those who follow the orthodox civil-examination path engage in technical

9 “小学教则中算术规定使用洋算，但可兼学从来之算术。”

10 “废止明治六年当省第三十七号公告。”

pursuits,”¹¹ arguing that “astronomy and mathematics confer negligible benefit; to have Western instructors teach the orthodox literati will do great harm,”¹² and that the result would be “to turn the talents whom the state cultivates and keeps in reserve for service into followers of the barbarians” (Zhu 1883, 551–575).¹³ Such entrenched isolationism impeded Chinese mathematicians’ acceptance of any foreign ideas that conflicted with long-standing Chinese cultural traditions. Even Li Shanlan, who had mastered calculus and recognized its superiority, did not fully embrace the Western system of notation. He adopted new ideas and methods, yet retained traditional forms to a considerable degree. Building on the mathematical notation and formulas of the English original of *Daiweiiji sheji*, which he did not directly adopt, he developed a Sinicized system of notation and formulas and translated the original expressions accordingly, hampering the subsequent transmission of Western mathematical knowledge. Japan, on the other hand, had a longstanding predisposition to learning and absorbing foreign cultural elements—first from China and India, later from the West—without the same doctrinal scruples or cultural hostility. In sharp contrast to the contentious establishment of the Department of Astronomy and Mathematics in China, the establishment of institutions such as the Institute for the Study of Western Books and the Nagasaki Naval Training Center proceeded with comparatively little friction. Japanese authorities had no qualms about sending shogunal retainers to study Western science and technology at these institutions, and the Institute for the Study of Western Books even initially restricted admission to shogunal retainers. In short, from the Bakumatsu period onward, Japan encountered minimal internal resistance to absorbing Western civilization. Upon recognizing the superiority of Western science and technology, the Japanese promptly adopted a proactive approach to assimilating Western knowledge and applied it effectively to the advancement of their national science and technology. Consequently, Japan’s mathematical modernization accelerated: Japanese mathematicians adopted advanced Western notation and became directly connected to international mathematical discourse by the early 1870s.

Third, differences in scholarly traditions and the modes by which Western mathematics was introduced also produced divergent outcomes in the two countries’ transmission and development of modern mathematics.

Mathematicians in Qing China, shaped by long-established scholarly traditions, tended—consciously or not—to approach Western mathematics, including calculus, through traditional methods and perspectives. The late-Qing Chinese treatises on calculus, cited above, largely explicated the two Chinese translations produced by Li Shanlan and by Hua Hengfang in collaboration with Westerners, supplemented them

11 “令正途科甲人员习为技巧之事。”

12 “天文算学，为益甚微；西人教习正途，所损甚大。”

13 “举聪明俊秀，国家所培养而储以用者，变而从夷。”

with calculation examples, and employed the traditional Chinese mathematical commentary and textual research. In fact, these works sought to subsume calculus within the trajectory of traditional Chinese mathematics, so understandings of calculus were frequently framed within traditional mathematical thought. For example, Zhang Bingshu's teacher, in a preface to "*Daiweiji sheji*" with *Supplemented Solutions* 代微积拾级补草, observed that "in Chinese methods, there is the Method of Interpolation (zhuishu 缀术): applied to circle division, it yields deduced formulas which, when numerical values are substituted, correspond to the various kinds of calculus formulas" (Liu 1918).¹⁴ Xia Luanxiang, though he possessed a profound grasp of calculus methods and used them as tools in his research, employed exceedingly complicated formulas in his writings and eschewed even Li Shanlan's symbols in favor of textual descriptions that made his work difficult to comprehend. In pursuing the integration of Chinese and Western mathematics, Li Shanlan inadvertently impeded late-Qing mathematicians from capturing the essential spirit of modern Western mathematics.

Japan's situation differed: Even when the earliest learners of calculus were Wasan mathematicians, there was no inclination to incorporate calculus into the Wasan framework. Rather, some subsumed the Wasan Method for Extrema 极数术 into the realm of differential calculus.

From the Ming dynasty onward, translations of Western scientific works in China were carried out almost entirely through collaborations between Chinese scholars and Western missionaries. Chinese scholars typically lacked proficiency in Western languages and rarely learned those languages proactively. Therefore, missionaries provided oral interpretation while Chinese scholars produced the written Chinese renderings. This collaborative mode persisted well into the late Qing and, consequently, few Chinese scholars could read Western originals directly. It was not until the early twentieth century that this situation began to change. Because Chinese scholars had long relied principally on translations, their research was severely constrained.

In contrast, Edo-period Japan had cultivated a tradition of Dutch learning and a corps of *rangaku* scholars competent in Dutch. This tradition was inherited in the Meiji era, yielding a cohort of scholars versed in Western languages. Freed from dependence on Western intermediaries for translations of scientific works, Japan enjoyed a comparative advantage and was able to proactively import and absorb modern Western mathematics, whereas China remained a passive recipient.

After the Meiji Restoration in particular, Japan continuously introduced Western mathematical knowledge by dispatching students to European centers of mathematics to study the most advanced developments, thereby ensuring the timely renewal, effective absorption, and application of new mathematical knowledge. China did not

14 “中法有缀术，用之割圆者，皆推定算式，临用以数入之，与微积各类之公式同。”

implement similar measures until the twentieth century.

Furthermore, differences in institutionalization of mathematics shaped divergent developmental pathways of the modernization of mathematics in China and Japan.

In Japan, various Wasan schools had crystallized during the Edo period, and Wasan education flourished. Many Wasan mathematicians made a profession of teaching, transmitting mathematical knowledge through a distinctive “way of art” 艺道. By the Bakumatsu and early Meiji periods, alongside governmental institutions for Western learning, numerous domains had appointed full-time mathematics instructors, and many private Wasan schools and Western-style mathematics academies were in operation throughout the country. The popularization of mathematics in Japan was therefore far more extensive and profound than the contemporaneous diffusion of traditional Chinese mathematics in China. Hence, Japan was able not only to introduce mathematics into various types of schools at an early date but also to found national mathematical associations sooner. In 1877, the Tokyo Mathematical Society was founded with an inaugural membership of 117—a figure comparable to Western societies of the period. For reference, the London Mathematical Society had 27 members when founded in 1865; the Société mathématique de France reached 147 members by 1875 after its founding in 1872; the American Mathematical Society had 16 members at its founding in 1888; and the German Mathematical Society numbered 205 members at its founding in 1890 (Editorial Committee 1983, 83). The Tokyo Mathematical Society also launched its own journal, the *Journal of the Tokyo Mathematical Society*, which played an important role in disseminating calculus and facilitating scholarly mathematical exchange. In contrast, China’s institutionalization and social popularization of mathematics proceeded far more slowly. It was not until 1868, when the School of Combined Learning established mathematics instructors within its Department of Astronomy and Mathematics, that China had professional mathematics teachers. A national mathematics society, the Chinese Mathematical Society, did not emerge until 1935, and Chinese mathematical periodicals only began to appear at the end of the nineteenth century. In these respects, China lagged significantly behind Japan.

The foregoing analysis indicates that differences in political, economic, and cultural conditions, together with factors intrinsic to the discipline of mathematics, produced markedly divergent outcomes in the two countries’ introduction and dissemination of calculus and other Western mathematical knowledge. Japan accomplished the modernization of its mathematics within a relatively short period, whereas China required a considerably longer and more protracted transition.

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