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With the Measuring Rod against the Floods: Ferdinand Verbiest and His Forgotten Introduction of Benedetto Castelli's River Hydraulics to China

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Abstract: The monumental *Studies to Fathom Principles* (*Qiongli xue* 窮理學; 1683) by Ferdinand Verbiest S. J. (Nan Huairen 南懷仁, 1623–1688) was never printed, and its exact content is not known. A section of the only extant, though incomplete, manuscript deals with fluvial flood prevention and river control measures, a subject that until then had not cropped up in any Chinese-language work of the Jesuits. In this section, Verbiest not only described the already well-known Aristotelian theory of the origin of rivers, but also introduced to China new scientific propositions, concepts, and numerical examples originating from the seminal Renaissance work *Della misura dell'acque correnti* (*Of the Mensuration of Running Waters*; 1628) by Benedetto Castelli (1578–1643). In addition, Verbiest presented to his readers some noteworthy examples of pertinent Western achievements such as the pound-lock with miter gate, and he provided them with a simple economic analysis of flood control options. The significance and possible influence of Verbiest's text on further developments in Chinese approaches to water engineering are discussed, highlighting a hitherto largely

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In her PhD thesis, Sabine Kink investigates the transmission of Western hydrological knowledge and hydraulic technologies to late Ming China. Her dissertation is part of the research project "Translating Western Science, Technology and Medicine to Late Ming China: Convergences and Divergences in the Light of the *Kunyu gezhi* 坤輿格致 (Investigations of the Earth's Interior; 1640) and the *Taixi shuifa* 泰西水法 (Hydromethods of the Great West; 1612)." This project has been funded by the German Research Foundation (DFG) for the years 2018 to 2022 and is being carried out at the Department of Chinese Studies at Eberhard Karls University of Tübingen under the direction of Prof. Dr. Hans Ulrich Vogel.

disregarded facet of Western science and technology transfer in the field of river hydraulics and flood management.
Keywords: Ferdinand Verbiest, Benedetto Castelli, *Qiongli xue*, *Della misura dell'acque correnti*, hydrology, river hydraulics, flood prevention, Kangxi period

摘　要：耶稣会士南怀仁（Ferdinand Verbiest S. J.，1623～1688）的巨著《穷理学》从未印行过，其确切内容也不为人所知。目前仅存一份不完整的手稿，部分章节涉及河水防汛和治河措施，而此前耶稣会士所撰写的中文著作从未涉足这一主题。在《穷理学》的该章节中，南怀仁不仅介绍了众所周知的亚里士多德的河流发源理论，还向中国介绍了文艺复兴时期贝内代托·卡斯特利（Benedetto Castelli，1578～1643）的开创性著作《论流水的测量》（*Della misura dell'acque correnti*，1628）中全新的科学命题、概念和数值算例。此外，南怀仁还向读者介绍了一些与之相关的卓著西方成就，如带人字闸门的船闸，并对各种防洪方案进行了简要的经济分析。本文探讨了南怀仁的文本对当时中国水利工程方法进一步发展的重要意义和潜在影响，强调了一直以来为人忽视的在河流水力学和洪水管理等领域西方科学技术的转移。
关键词：南怀仁，贝内代托·卡斯特利，《穷理学》，《论流水的测量》，水文学，河流水力学，防洪，康熙朝

1 Introduction

A few years after Ferdinand Verbiest's arrival in China, where he stayed in the Jesuit Beijing residence from 1660,[1] a natural disaster of apocalyptic dimensions struck the area:

> But upon the Vigil of St. *Laurence*'s Day [August 10] in the Year 1668, after an Extraordinary Drought which had lasted all that Year, it began to Rain, and the Rain continu'd Day and Night till the sixteenth of *August*, with so much Violence, as if whole Rivers had pour'd down from Heaven. The Seventeenth of *August* about eight of the Clock in the Morning, of a sudden there came a Deluge that overflow'd the new City, the Suburbs and the Planes adjoining. Presently they shut up the Gates of the old City, and stopp'd up all the holes and clefts with Chalk and Bitumen mingled together, to prevent the entrance of the Water. But the third part of the Houses of the new City were overturn'd, and an infinite number of poor Creatures, especially Women and Children were either drown'd or buried in the Ruins. . . . Never was seen the like Consternation in that Court, where all Men were reduc'd to utmost despair, not being able to divine the Cause of so extraordinary a Deluge. At last, the King, having sent out certain People upon Rafts of Timber, for they have no Boats at *Pekim*, to examine the Reason, they found that the troubled River, of which we have already made mention, had broken down the Damms, and made it self a new Channel cross the Fields and Suburbs of the City which begat such an amazing Fear in the Minds of the People, that the King and the Grandees

1 Verbiest had reached Macao in June 1658. There, he studied Chinese and did his fourth vow, before he set off for mainland China in the spring of the next year. After a ten-month sojourn in Xi'an, he finally arrived in Beijing on June 6, 1660. See Golvers (2019).

> were just upon the point of removing to some other place. The same Fury of the Inundation carried away several Rocks, which knocking against the Piles of the famous Bridge, shook it in such a manner, that they broke down two of the Arches. (Magaillans 1688, 14–15)

Thus describes the early Portuguese Jesuit missionary Gabriel de Magalhães (An Wensi 安文思, 1610–1677) the disastrous flood of the Yongding River 永定河 (at that time called Wuding River 無定河 or Hunhe 渾河) in the summer months of 1668, which had caused the collapse of two arches of the famous Marco Polo Bridge (Lugou Bridge 盧溝橋) near Beijing in July. One month after this event, the remaining structure of the bridge collapsed (Intorcetta 1672, 65–66).[2] What is particularly striking in this impressive account is the bewildered and confused reaction of the court, which leaves the impression of a completely overwhelmed emperor who had to watch the disaster helplessly.

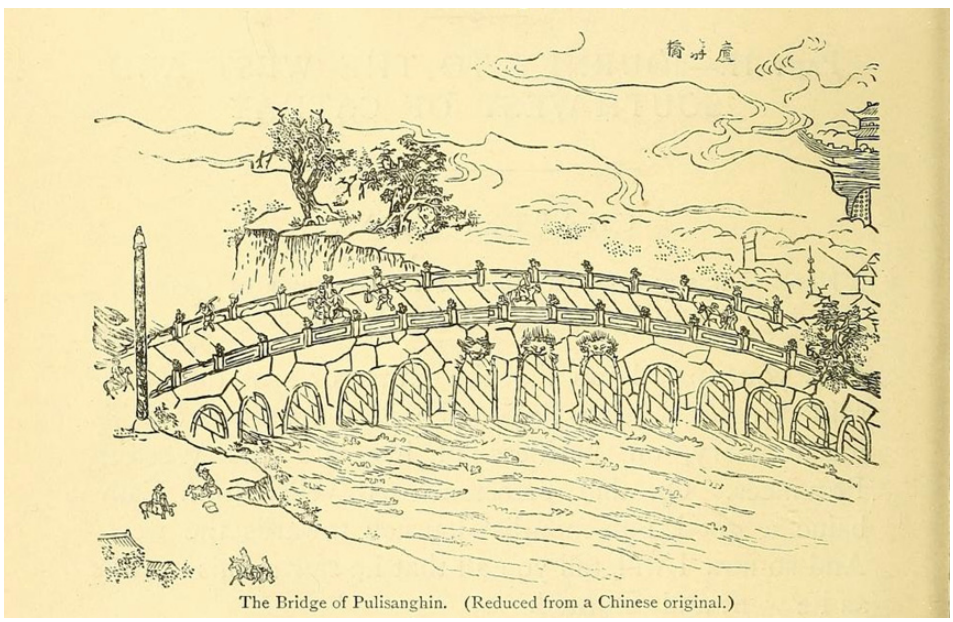


Figure 1: Marco Polo Bridge with thirteen arches, during the Ming dynasty (1368–1644) (Yule 1903, vol. 2, chap. xxxv:4, illustration reduced from a large Chinese engraving in *Gazetteer of the Capital* [*Jifu tongzhi* 畿輔通志; 1682]).

This is all the more remarkable as, since ancient times, flood control by means of large-scale water conservancy measures had been one of the main legitimizing tasks of the ruler in China. Thus, from the Song dynasty onwards the results of case-related practical experience in river regulation had been transmitted in the form of written

2 Intorcetta talks of July 25 and August 24, respectively.

treatises, which, however, lacked theoretical underpinnings and quantitative analysis. It is quite probable that, as personal adviser and teacher of Emperor Kangxi (r. 1661–1722), the Jesuit missionary Ferdinand Verbiest had recognized this weak point early on. But what is more, the indelible impression of the flood disaster in Beijing must have immediately aroused the technical interest of Verbiest, who had already demonstrated his up-to-date knowledge and practical expertise in mechanics and instrument making soon after his arrival in Beijing.[3] This is reflected in his description of this incident in the following passage of one of his writings:

> Outside the city walls of Peking, in western direction, there was a most famous bridge with many arches resting on very strong piers; for more than 400 years it had dominated the rushing river that runs by the Western Hills, while the swirling water, always seething and foaming, was raging around it, in vain. At long last, however, in the seventh year of the Kangxi Emperor [that is, 1668], this river was so swollen due to heavy rains which continued for many days, that it burst its banks far and wide, till it reached the city walls of Peking, thus threatening the whole city. It even swept away three enormous piers of the bridge, which was built over it, as if it was indignant at the yoke. (Golvers 1993, 112–113)[4]

Against this background it does not come as a surprise that Verbiest immediately realized the potential of this issue within the framework of his order's endeavor to transfer for proselytizing purposes Western scientific and technical knowledge to China,[5] and not least for this reason aspired to fill the methodological gap. To this end, in 1683, within the *Studies to Fathom Principles* (*Qiongli xue* 窮理學, hereafter *QLX*) as his most comprehensive oeuvre, he presented to the throne the Chinese rendering of parts of a recent European treatise on river hydraulics by Benedetto Castelli, which he had complemented with the description of supposedly innovative Western devices and developments in the field.

This article examines the origin, contents, and influence of the resultant and hitherto largely neglected section on floods in the *QLX*, thus presenting novel information about specific portions of this work which discuss Western natural philosophy together with instances of its practical application. Our study consists of six parts: 1. This introduction. 2. Background information about Verbiest and Castelli as well as a brief overview of the contemporary state of water conservancy knowledge in East and West. 3. A preliminary translation of large parts of Verbiest's forgotten text on flood prevention, with notes evidencing its close relation to Castelli's work and brief remarks on other possible European sources. 4. A discussion of the contents of Verbiest's text. 5. An

3 For Verbiest's self-perception as an *ingeniere*, see Golvers (2018).

4 This passage is part of Verbiest's description of one of his most famous technical accomplishments, the transport of extremely heavy stones for the mausoleum of the deceased Shunzhi Emperor across the provisionally repaired Lugou or Marco Polo Bridge in the summer of 1670.

5 For some facets of Matteo Ricci's underlying "accommodation strategy," see for example Elman (2005, 112–116).

evaluation of the reception and influence in China of the hydraulic knowledge transferred. 6. A summary of the findings of this examination, which is supplemented by two appendixes.

2 Background information

2.1 Verbiest, the hydraulic engineer

Verbiest, who lived in China from 1659 until he passed away in 1688, is well known as one of the giants in the history of the Jesuit mission during the Kangxi reign, partly through his own treatises such as the *Astronomia Europaea* and his written communication,[6] partly through specialized studies and various monographs on his life and work.[7]

Verbiest was born in the Flemish town of Pittem, Belgium, in 1623. He attended the Jesuit colleges in Bruges and Kortrijk until 1640, and then enrolled for the subjects of philosophy and mathematics in the Faculty of Arts of the secular University of Leuven. After the first year of study, Verbiest entered the Jesuit Order in Mechelen, where he completed the customary novitiate of two years. Thereafter, he resumed his studies at the Jesuit college in Leuven, taking part during the 1644–1645 term in a mandatory mathematics course held by the reputed scholar Andreas Tacquet S. J. (1612–1660). On the initiative of the eminent Jesuit Christopher Clavius, the inclusion of this course in the formation of its members had been stipulated in the official curriculum of the order, the 1599 *Ratio atque Institutio Studiorum Societatis Iesu*. This document, often abbreviated as *Ratio Studiorum*, standardized the global system of Jesuit teaching and at the same time modernized European university education.[8] Henceforth, at Jesuit colleges mathematics was taught independently of Aristotelian philosophy and included—besides astronomy, arithmetic, Euclidean geometry, and rudimentary mechanics—a selection of individual topics from the developing natural and engineering sciences.[9] Course contents differed widely, depending on the formation and interest of the teachers, the needs of students, as well as local requirements. Though rarely active in basic research, Jesuits also compiled

6 For the *Astronomia Europaea*, see Golvers (1993); for Verbiest's letters, Josson and Willaert (1938) and Golvers (2017).

7 For Verbiest's scientific activities, see, for example, Golvers (2003) or Witek (1994). A detailed overview of Verbiest's life and work is provided by Golvers (2019), on which the following statements about the China missionary are mainly based. Golvers also provides an extensive list of Western literature about Verbiest.

8 See Udías (2015), esp. chap. 1 "Clavius and mathematics in the Collegio Romano."

9 However, the regular and compulsory mathematics course was taught for forty-five minutes per day only during the second year of philosophy. The principal aim was to enhance the understanding of Aristotle's *Physica*. Usually, students had become familiar with Euclid's *Elements* after two months, so that the lessons could then switch to other topics. See Farrell (1970, 8).

comprehensive textbooks on mathematics and its applications for these classes, which were used all over Europe.[10]

From 1645 onwards Verbiest had to fulfill the compulsory teaching obligation which was part of the Jesuit formation, and thus was appointed professor of Latin and Greek at the Jesuit College of Brussels. In the academic year 1652–1653 he was then sent to the Collegio Romano in Rome to study theology. There he most likely met the famous scholar Athanasius Kircher S. J. (1602–1680), chair of mathematics, and his assistant Gaspar Schott S. J. (1608–1666), another mathematician and scientist. The remainder of Verbiest's theological studies was pursued at the Colégio San Hermenegildo in Seville because he had hoped to join the Jesuit mission in Chile, which belonged to the Spanish assistancy of the order. However, after obtaining a doctorate in 1655 and with no decision forthcoming from the Jesuit Father General in Rome about his missionary request, he left for Italy again. On his way to Rome, he surprisingly switched his application from the West to the East Indies (which included China), perhaps having become aware of the successful recruiting campaign of the China missionary Martino Martini (Wei Kuangguo 衛匡國, 1614–1660) back in Flanders.

Verbiest's petition was finally accepted on the condition that he successfully complete his mathematical education, which was of strategic importance for the proselytizing efforts in China at that time.[11] In the spring of 1656, he finally arrived—together with Martini—in Lisbon, only to find out that the fleet for the East Indies had just left. That Verbiest had already gone deeper into this increasingly important discipline is illustrated by the fact that he bridged the eight months of waiting time until the next departure of a carrier to the East Indies by holding a course of mathematics at the Jesuit College in Coimbra, as other Jesuits from various parts of Europe had done before him. Among them were Giovanni Paolo Lembo (ca. 1570–1618), Jan Ciermans (1602–1648) and his student Hendrick Uwens (1618–1667), and Valentin Stansel (1621–1705), some of whom had left behind their teaching manuscripts in Portugal, for example the *Tratado breve das Machinas Hydraulicas* by Lembo or the *Tratado da Estática* by Uwens with chapters on hydrostatics and hydraulic machines (Castel-Branco 2020, 351–368). Verbiest might well have studied these writings during his enforced interim stay.[12] In April 1657 he finally departed with the group of Martino

10 Examples of early Jesuit works dealing with hydrology, hydrostatics, hydrotechnics, or other water-related matters are given by Koenig (2018, 32–33).

11 To this end, Verbiest attended the course taught by the mathematician Giacomo Bonvicini S. J. (1619–1657) at the Jesuit College of Genoa.

12 Golvers (2019) points out that—in addition to the official courses—at the time of Verbiest's stay there existed among the Jesuits in the surroundings of the Coimbra College a rather vivid "mathematical culture" on an informal private basis and with personal holdings of novel publications (for example by Clavius or Cabeo) on the subject. Verbiest probably had access to such libraries.

Martini for Goa and Macau, where he arrived in June 1658. After studying the Chinese language for some months and a short sojourn in the Jesuit parish in Xi'an, he was called to Beijing by Emperor Shunzhi (r. 1644–1661) upon the suggestion of Adam Schall von Bell (Tang Ruowang 湯若望, 1591–1666) to become Schall's assistant in the Imperial Directorate of Astronomy (*Qintianjian* 欽天監).[13]

There, he not only demonstrated his talents in computational prediction, but also proved his proficiency in instrument building and "modern" European mechanics.[14] After some early successes in these fields, and despite the opposition of native astronomers, Verbiest was appointed acting director of the *Qintianjian* in 1669,[15] thus becoming an immediate employee of the Imperial Household. Besides working as mathematician and astronomer in charge of the calendar reform, he also served as "scientific adviser" and mathematics tutor to Emperor Kangxi. Thus, he taught Western algebra, geometry, and computational astronomy not only to the staff of the Calendar Bureau but starting in 1675 also to the emperor himself (Kurtz 2011, 66–67). What is more, due to his quite unconventional eagerness to acquire competence in useful Western technologies, Kangxi seems to have been especially interested in so-called "mixed mathematics" with its wide applicational scope. As described in his *Astronomia Europaea*, Verbiest was thus often asked by the emperor to solve (with and for him) concrete questions in other fields, among them hydragogics, hydrostatics, and hydraulics. While the Jesuit's related contributions to astronomy, calendar making, mathematics, geography, and mechanical knowledge have been extensively investigated, Verbiest's activities in water engineering have been reviewed only recently (Koenig 2018, 42–47). In addition to the mentioned teaching obligations, these hydrotechnical activities of Verbiest were both of a theoretical and practical nature and can be summarized as follows:

a) Compilation of written works with reference to water engineering

(i) *Illustrated Explanation of the Entire World* (*Kunyu tushuo* 坤輿圖說; 1672) reports on several outstanding hydraulic works and machines in Europe.

(ii) *Pictures of Newly-Made Instruments* (*Xinzhi yixiang tu* 新製儀象圖; 1674) contains two prints related to hydrotechnical work, one showing water being lifted from a well with the aid of pulleys, the other a leveling instrument and the procedure used to

13 See Hucker (1985, 169, no. 1185).

14 According to Golvers (2018, 10–13), Verbiest was well aware of the weak points and contestability of the Jesuits' astronomical expertise, which was one of the mainstays of their missionary strategy. He thus wanted to put his scientific activities and writings on a broader basis, including engineering skills (in mechanics, ballistics, and hydraulics) and medicine.

15 Actually, Verbiest was working in the Calendar Bureau under the unofficial designation of an "Administrator of the Calendar" (*Zhili lifa* 治理曆法). This means that he was lower in rank than Schall von Bell, who during the dynastic transition had been in charge of the affairs of the whole institution. See Jami and Han (2003, 90).

determine the gradient of land for irrigation purposes.

(iii) Its companion volume *Treatise of Newly Constructed Astronomical Instruments at the Imperial Observatory* (*Xinzhi lingtai yixiang zhi* 新制靈臺儀象志; 1674) describes some practical applications in water conservancy, such as digging and dredging rivers and canals for prevention of disasters, opening or closing waterways, and determining exact elevations for conveyance of water.

(iv) *Studies to Fathom Principles* (*Qiongli xue* 窮理學; 1683).

(v) In the *Astronomia Europaea* (1687) achievements are described in hydragogics (ch. 16), hydrostatics (ch. 22), and hydraulics (ch. 23), such as building a leveling instrument (*libella*) to determine the gradient of imperial lands and to evaluate the diversion of a river for irrigation, or the building of water pumps and other water-lifting machines.

b) Practical application of hydrotechnical knowledge

Verbiest was not only translating and writing books, but also applying his scientific and technical knowledge to designing and building all kinds of instruments, machines, and engineering works, such as pumps, waterwheels, fountains, and so on. Due to his growing reputation in the field, he was often requested to give advice on or supervise canal building works, but, as director of the astronomical bureau, he was not allowed by the emperor to carry out such fieldwork in the provinces.[16] Accordingly, no direct involvement on site of Verbiest in any Yellow River (Huanghe 黃河) Conservancy or Grand Canal (Da yunhe 大運河) projects is reported, and it seems that he was not personally in contact with the Director-General of the Grand Canal (*Hedao zongdu* 河道總督),[17] the government agency in charge of maintenance and operation of this vital waterway and additionally responsible for shipping on relevant sections of the Yellow River.

2.2 State of water conservancy and flood control in early Qing China

Although China had a long, distinguished history of water conservancy, similar to Europe, the basic approaches to solving flood control problems had not changed over a

16 For details on Verbiest's personal involvement in hydraulic activities at the Imperial Court in Beijing or for directing his Jesuit confreres in the provinces, see Koenig (2018, esp. 41–47).

17 In the Ming, the term *Hedao zongdu* was used as a variant designation for *Caoyun zongdu* 漕運總督 (Director-General of Grain Transport), who was in charge of tax grain transports along the Grand Canal from the Jiangnan area to Beijing. In the Qing, however, the responsibility of the *Hedao zongdu* was separated from that of the grain transport hierarchy. See Hucker (1985, 224–225, no. 2193, and 521, no. 6936). *Hedao zongdu* is sometimes also translated incorrectly as "Director-General of the Yellow River Conservancy" or "Viceroy of Rivers and Waterways."

long time.[18] In order to prevent rivers from overflowing their banks, dikes and dams were built and canals were dug to drain off part of the water and thus disburden the main riverbed. The biggest problem in China, however, was the pronounced silting of the Yellow River that made this waterway break its dikes and shift its course time and again,[19] and thus not only caused disastrous, harvest-destroying inundations but also threatened the vital grain transport to the capital on the Grand Canal. Earnest efforts were made to divert the flow of the Yellow River, namely, to split its course into smaller streams to prevent big floods, but this laborious traditional concept was increasingly abandoned from the latter half of the sixteenth century onwards.

At that time, a new hydraulic method was first brought up by Wan Gong 萬恭 (1515–1591). His idea had been to confine and narrow by means of additional "lining dikes" (*lüdi* 縷堤)[20] the watercourse of the Yellow River and thereby increase the velocity of its current, hoping that this would suffice to flush more silt towards the sea. Even though this technique had the negative side-effect of a dropping water table, in the following years it was officially adopted and fully implemented by Director-General of the Grand Canal Pan Jixun 潘季馴 (1521–1596). One of the main reasons for this change in strategy was that, due to the shift of the river course to the south, stretches of the Yellow River now formed part of the Grand Canal route, with the junction of the two waterways being a particularly critical issue. Here, it was imperative to avoid at any cost a pronounced rising of the Yellow River's bed, which would have disrupted the grain transport.[21] Therefore, together with the self-cleaning through an artificial increase of the current, it was planned to dam up Lake Hongze 洪澤湖 and other reservoirs of the region in order to discharge their waters in a controlled manner into the Yellow River for additional flushing. But despite such

18 First written accounts about technical details and organizational issues of river control in China began to crop up in the Northern Song dynasty (960–1127). While traditional techniques, for example, for building and repairing dikes and canals, continued to be applied until modern times, innovation during the Ming and Qing mainly consisted in the increasingly systematic cataloging of related projects and relevant measures. See Flessel (1974, 1 and 210–211).

19 In addition to the enormous amounts of alluvial particles from the loess uplands, it is mainly the low gradient at its lower reaches that retards the current of the Yellow River, elevates its riverbed, and makes it burst its banks frequently after heavy rainfalls. Under extreme conditions this led to altogether six major shifts of its course over time, and the relocation of its complete estuary to the south of the Shandong Peninsula in 1494 that persisted until the middle of the nineteenth century. See Amelung (2000, 3–6).

20 These dikes were erected close to the river, while the normal "outer dikes" for flood prevention were located up to three *li* away from the riverbed. A third measure was the building of stone-covered dams guiding the discharge of peak flow water at defined breakthroughs. See Vermeer (1987, 54–55).

21 See Amelung (2000, 59). Leonard (2019), though focused on later developments from the second half of the eighteenth century, provides excellent explanations about the issue of the critical junction of the Grand Canal with the Yellow River.

extensive efforts in river control, devastating floods like the one witnessed by Verbiest at the Yongding River in 1668 continued to occur and increasingly caused disputes among responsible officials.

In the early Qing, a novel approach to preventing such inundations was eventually presented under the tenure of Jin Fu 靳輔 (1633–1692), who served as Director-General of the Grand Canal starting in 1677 and thus during Verbiest's sojourn in China. A few years after the Jesuit had handed in his *QLX* in 1683, Jin Fu's close assistant Chen Huang 陳潢 (1637–1688) proposed to combine Pan Jixun's long-term measure of flushing the Yellow River's bed by means of the current with the ad hoc diversion through a number of floodgates of controlled amounts of peak flow waters which occurred after extensive rainfalls (Hummel 1943, 1:162). It is not clear whether this method was ever adopted, but the situation continued to remain extremely unstable. Altogether, despite its fundamental economic and political significance, river control work in China at that time appears to have been rather erratic, and after the Ming/Qing transition no pioneering ideas emerged either. In this context, Zhou Kuiyi (2015, 379–380) has identified the following weak points of traditional Chinese approaches to water conservancy that must have attracted the attention of Verbiest as well: (i) practical experience valued but theoretical generalization neglected; (ii) direct observation but little quantitative analysis and systematic measurement; and (iii) lack of scientific experiment.

In Europe, by contrast, novel approaches had led to a much better understanding of the causes and the control of floods starting in the early seventeenth century. Based on Galilean concepts of mechanics, mathematics, and geometry, the science of hydraulics[22] developed rapidly as a new discipline in Italy.[23] In 1628 the first work on such scientific river hydraulics, our *Della misura dell'acque correnti* (*Of the Mensuration of Running Waters*), was published there by the Benedictine monk Benedetto Castelli (1578–1643), a student and friend of Galileo Galilei (1564–1642). But whereas water control technologies continued to steadily progress in Europe and resulted in an "epistemic leap"[24] at that time, in China few attempts were made to apply to this field the corresponding mathematical knowledge transmitted from the West since the days of Matteo Ricci.

A notable exception in this regard was the distinguished astronomer Xue Fengzuo

22 At the time of Verbiest, hydraulics was understood as "the study and engineering of water (hydr-) moving through a (natural or artificial) riverbed (aul-), or in other words, a branch of mechanical engineering concerned with the use of fluids to perform mechanical tasks." See Golvers (2018, 16).

23 For this topic, see Maffioli (1994; 2010), who examines the newly emerging mathematization of nature in the field of water conservancy in northern Italy from the sixteenth century onwards.

24 See Davids (2005, 2). Davids compares the developments in river management in northern Italy and the Netherlands during the period 1400–1800 with those in China and explains the emerging epistemic gap mainly by differences in the underlying socio-political structures on both sides.

薛鳳祚 (1600–1680).[25] Since 1653 he had studied Western mathematics with the Polish Jesuit Nikolaus Smogulecki S. J. (Mu Nige 穆尼閣, 1610–1656), who is known for the introduction of logarithms to China.[26] In 1676, Xue was commissioned by the incumbent Director-General of the Grand Canal, Wang Guangyu 王光裕, to study the flood problems of both the Yellow River and the Grand Canal. Against this backdrop, he actively applied Western techniques of spherical trigonometry and logarithms to surveying for the first time. The resulting work, *Compendium on the Control of the Two Rivers* (*Lianghe qinghui* 兩河清彙; 1677?), was reprinted in the *Complete Collection in Four Treasuries* (*Siku quanshu* 四庫全書), with the editors noting that Xue Fengzuo's "mathematical expertise was an invaluable aid in analyzing problems related to flood control and canal upkeep. His use of European trigonometry was recognized as a clear improvement over the native forms of trigonometry" (Elman 2005, 195), which had dominated Chinese surveying techniques until that time.

This kind of Western precision surveying had already been practiced to great effect by Ferdinand Verbiest as an integral part of his river engineering works, either by using it directly (for example as later described in his *Astronomia Europaea*) or by having it applied on his behalf to canal building projects by confreres of his such as Giandomenico Gabiani (Bi Jia 畢嘉, 1623–1694) or Antoine Thomas (An Duo 安多,1644–1709).[27] Under the lasting impression of the devastating inundation in the Beijing area, however, he must have started early on to ponder over additional and more specific measures against such disasters that went beyond traditional Chinese approaches and were related to the hydraulics of the river flow instead. Verbiest's critical assessment of the whole situation—which of course was also a good opportunity to promote the Jesuits' missionary project and further distinguish himself in the field—becomes evident when, after another major flood in 1681 along the lower course of the Yellow River between the Grand Canal and the Pacific Ocean (Guy 2010, 244), he commented on the proposed but obviously inadequate Chinese countermeasures as follows: "Here incredible amounts are spent annually to prevent inundations of the Yellow River (the most celebrated river in the whole of China). This year more than four hundred thousand ounces of silver[28] (we call them Spanish

25 For Xue Fengzuo's life and work, see Chu (2015). On his collaboration with the Jesuits and his integration of Western and Chinese learning, see, for example, Han (2013) or Chu and Zhu (2019). His ideas, moreover, were analyzed by Xia and Yi (2010).

26 For the collaboration of Xue and Smogulecki, see Shi (2007).

27 See Koenig (2018, 42–50). For example, in the summer of 1689, Claudio Filippo Grimaldi S. J. talked about such Jesuit canal building activities in China with Gottfried Wilhelm Leibniz (1646–1716). He reported that currently up to 100 leagues of new canals were built under the direction of the Fathers ("*Fiat nunc canales novi in China, patrum ductu, ad 100 leucas*"). Depending on place and time, the length of one *leuca* measured about seven kilometers.

28 One Chinese ounce of silver (tael) equals 37.5 g silver.

patacas) have been thrown into its waters to no avail" (Koenig 2018, 47).[29]

This passage demonstrates Verbiest's keen interest in flood control and implies that he was convinced of being able to do much better at lower cost. On another occasion he made a similar remark: "*pecunias omnes saepe frustra in aquas suas proiciunt*" (they often throw all the money into their waters in vain) (Golvers 1993, 420 and 112). According to Jonathan Spence, during the Kangxi reign the annual expenditure on the entire river conservancy system was over 3,000,000 tael,[30] though in this field the officially listed amounts were usually much lower than the actual ones. In view of such high expenses for in a way insufficient river management and flood control measures, Verbiest had a strong motive to present to the emperor a new and more effective scientific approach to flood management, combining his own technical experience in water conservancy with the latest European theoretical developments. As we will see in the next subchapter, this kind of knowledge fit well into his *QLX*, which he was working on at that time.

2.3 The *Qiongli xue* and Verbiest's text on river engineering

Verbiest had intended the *QLX* as a comprehensive *Cursus Philosophicus*, namely a compendium of the manifold applications of Aristotelian syllogism (*litui* 理推, literally "inferences according to patterns" [Kurtz 2011, 72]) that formed the basis of Western science and technology, and as a textbook that could be used in preparation for the civil service exams in China.[31] This would have guaranteed an unprecedented

29 This remark was made by Verbiest in a letter of August 15, 1681, to the Jesuit Father General Charles Noyelle in Rome.

30 This huge amount was mentioned by Emperor Kangxi in his famous valedictory edict of 1717. See Spence (1974, 148). During his reign, annual government spending for river control had already increased more than tenfold, mainly due to socio-economic developments in the form of a gradual shift from levies and peasant corvée to hired labor. See Amelung (2022, 34–35). From the late eighteenth century on, costs burdening the state budget began to get out of hand and rose from one tenth of the total revenue in 1812 to more than 20 percent in the 1840s. See ibid., 40–41.

31 In China, the *QLX* has received comparatively little attention thus far. Among the studies on the work as a whole are Zhang Xiping (1999) and Shang (2003), which both investigate the background of the creation of the *QLX*, the basic structure and contents of the incomplete extant copy, and the main philosophical ideas presented by Verbiest in this work. Most recently, Yang and Shang (2021) have further analyzed the political reasons for the *QLX*'s non-acceptance on the part of Emperor Kangxi. On the Western side, the hitherto most detailed investigation of the overall structure and content of the *QLX* is Dudink and Standaert (1999). In his work on Chinese logic, Kurtz (2011) dedicates a subchapter to the *QLX* and the way in which it tries to convince its readers of the fundamental significance of syllogistic reasoning. Meynard (2017) focuses on the mainly Aristotelian sources of the *QLX* and traces Jesuit treatises in the Chinese language which were, as a whole or in part, incorporated into Verbiest's comprehensive work. The explanations provided in this subchapter of our article are mainly based both on Meynard (2017) and Dudink and Standaert (1999). In 2016, a punctuated modern edition of the *QLX* was published by Song and Gong.

dissemination of these teachings among literati.[32] However, after careful examination by the Ministry of Rites (*libu* 禮部) and members of the Hanlin Academy (*Hanlinyuan* 翰林院), Emperor Kangxi rejected the work in 1683, and allegedly even gave the order to burn the book immediately due to its heterodox contents and its "perverse, erroneous and illogical" style.[33] As the *QLX* was thus never printed, its exact content is not known. The only extant, though incomplete, manuscript is stored at the Peking University Library and consists of 14 out of a total of originally 60 *juan*. These preserved writings of the *QLX* belong to three main parts:

(i) *Five Universals in Logic* (*Libian zhi wu gongcheng* 理辨之五公稱) in 5 *juan* (numbered 1–5), which is a revised version of Francisco Furtado's (Fu Fanji 傅汎際, 1589–1653) *Investigation of Names and Principles* (*Mingli tan* 名理探; ca. 1636/1639).

(ii) *General Theory of Reasoning* (*Litui zhi zonglun* 理推之總論) in 5 *juan* (again numbered 1–5), which is based on the Coimbra commentary to Aristotle's *Analytica priora* and originally had been translated into Chinese by Furtado and Li Zhizao 李之藻 (1571–1630).

(iii) *Reasoning in Physics* (*Xingxing zhi litui* 形性之理推) in 4 *juan* (numbered 6–9), which is based on the Coimbra commentary to Aristotle's *De physica,* but also contains non-Aristotelian materials about mechanics and ballistics.

Dudink and Standaert (1999, 27), who have described the structure and content of this incomplete manuscript, mention a rather long section in the *Reasoning in Physics* part which deals with "floods, a subject not treated elsewhere in the works of Verbiest or other China Jesuits and, therefore, likely to be a newly written text."[34] This section on floods, which covers 14 out of 55 folios in *juan* 9 of this natural philosophy part of the *QLX*, is the object of investigation of our study. The whole chapter is dedicated to the Aristotelian elements of air and water and consists of altogether ten subdivisions. Thus, explanations are given here about topics like the thermo- and the hygrometer, the refraction both in water and air, the height of clouds and strange colors in the sky, water-leveling, and whirlwinds. The seventh or flood section is finally subdivided into, firstly, explanations on the water cycle and the origin of rivers, secondly, the causes of

32 Obviously, it had been Emperor Kangxi himself who had ordered Verbiest to translate books on the Western counterpart of the Neo-Confucian "investigation of things and fathoming of principles" (*gewu qiongli* 格物窮理) (Dudink and Standaert 1999, 16). This term had been used by the Jesuit Alfonso Vagnone (Gao Yizhi 高一志, 1566–1640) to designate the whole category of Western philosophy, comprising Aristotelian logic, ethics, and natural philosophy (Meynard 2017, 68).

33 It is more probable that the manuscript was not burned but just returned to Verbiest and the permission to have it printed was officially denied. See Dudink and Standaert (1999, 15–17). The main official stumbling block had been that in several places of the *QLX* human intelligence was said to be located in the brain and not in the heart mind (*xin* 心).

34 Being interested in the hydrotechnical activities of Jesuits in China, the above-mentioned statement by Dudink and Standaert aroused Albert Koenig's curiosity and induced him to have a closer look at this section, which eventually resulted in this co-authored article.

inundations of rivers and streams, thirdly, the relation between flow speed and water level, and fourthly, defense measures against inundations.

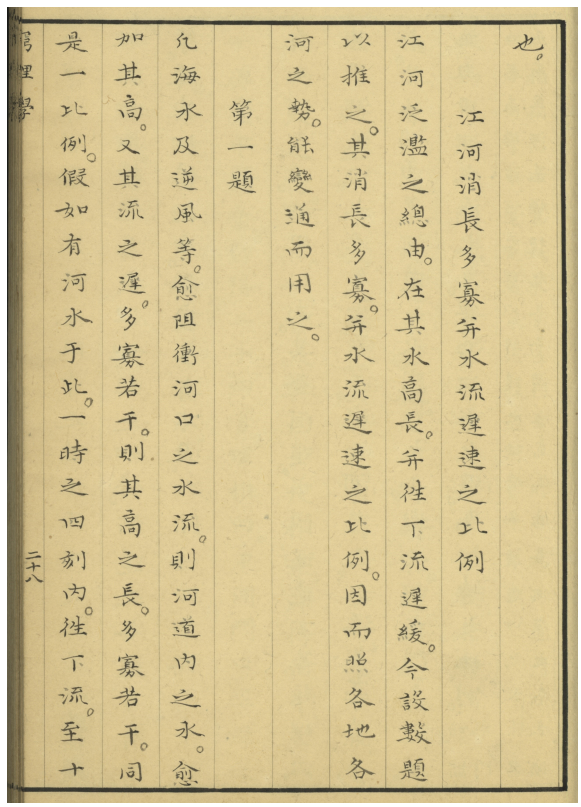


Figure 2: Excerpt from Verbiest's manuscript *Qiongli xue* 窮理學 (1683), *Reasoning in Physics* (*Xingxing zhi litui* 形性之理推), *juan* 9, fol. 28a, on the relation between flow velocity and water height (courtesy of the Peking University Library).

As the following analysis will show, Verbiest's text on river flooding, though wrapped in the tenets of natural philosophy and thus starting out with remarks obviously emanating from Aristotle's *Meteorologica*, also contains innovative ideas that clearly go beyond this traditional framework and probably were presented by him not least as a kind of lure in order to attract the attention of the emperor and his high-ranking scholar-officials to the Western art of syllogism, which was indispensable for understanding the principles or patterns of all sciences, and especially those of practical utility for the state.[35] These novel ideas are in large part taken from the already

35 In a memorial to the throne that Verbiest had submitted together with his *QLX* in 1683, he argued that "Without the syllogistic method, the military sciences, engineering, medicine, law, surveying, and measurement must remain superficial and can never become exact arts." For a translation of the complete memorial, see Kurtz (2011, 81–84, here 83).

mentioned pioneering work on river hydraulics by Castelli. Thus, before turning to Verbiest's rendering of the flood topic, a few remarks about this Italian work and its author are due.

2.4 Benedetto Castelli's *Della misura dell'acque correnti* and its influence in Europe

Benedetto Castelli O.S.B. was born as Antonio Castelli in Brescia, Italy, in 1578, and only upon entering the Benedictine order in 1595 took the name Benedetto. From 1604 to 1607 he lived in Padua and studied under Galileo Galilei, whom he also served as an assistant during a short stay in Florence. Galileo, who aspired to read the "book of nature" no longer with the tools of scholastic philosophy alone but to apply those of the mathematical discipline to it instead, is considered one of the pioneers of modern mechanics. The years under the guidance of this outstanding scholar set the course for Castelli's future career as a scientist. He was subsequently appointed professor of mathematics at the universities of Pisa (1613) and Rome (1626), where he also became a consultant to the Pope on the management of rivers in the Papal States.[36]

In 1628, Castelli's most important work on hydraulics was printed under the title *Della misura dell'acque correnti*. It consisted of two parts, the first of a more general nature and accessible without specialized prior knowledge, the second called "Demostrazioni geometriche della misura dell'acque correnti" and addressed to readers with a solid foundation in mathematics, who were thus able to follow the geometric-deductive procedure used by Castelli for the demonstration of his continuity equation.[37] A second edition of his work, enlarged by two new appendices and a letter to Galileo Galilei, was published in 1639, and a posthumous third one in 1660, namely seventeen years after Castelli had passed away in 1643. The third edition, to which the publisher had attached further reports and letters of Castelli on water-related themes, contained an additional treatise, the "Libro secondo" of 1642, with new propositions on the mensuration of waters.[38] This 1660 edition was extant in the Jesuit library in Beijing

36 More biographical information on Castelli can be found in De Ferrari (1978) and Blackman (2004).

37 According to this equation, the flow rate of incompressible liquids remains constant over any cross-section of a pipe or canal. First rudimentary ideas about this principle of fluid dynamics had already been formulated by Leonardo da Vinci (1452–1519), but Castelli was the first to express it mathematically.

38 The "Libro secondo" tries to relate the average speed of a fluid to the total height of that fluid. The resulting "second law" established by Castelli proved inaccurate, however. The correct square root law for the velocity of water flowing from a container was discovered by Evangelista Torricelli shortly after Castelli's death in 1643.

and presumably was Verbiest's source text.[39]

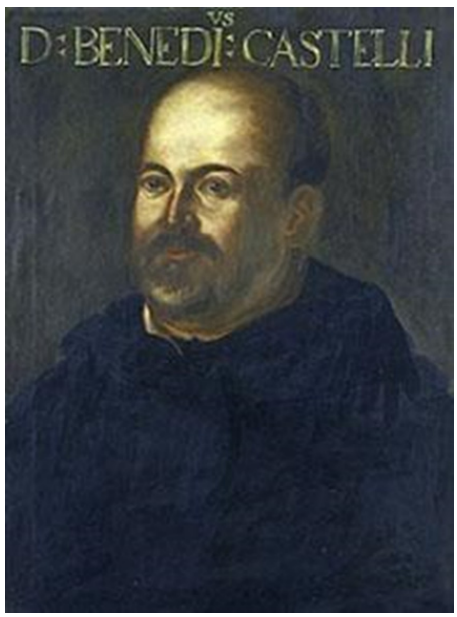


Figure 3: Benedetto Castelli, O.S.B. (1578–1643).

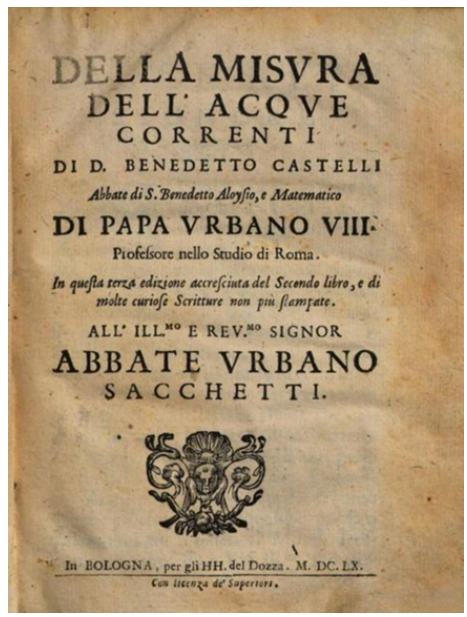

DELLA MISVRA
DELL' ACQVE
CORRENTI
DI D. BENEDETTO CASTELLI
Abbate di S. Benedetto Aloysio, e Matematico
DI PAPA VRBANO VIII.
Professore nello Studio di Roma.
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Figure 4: Title page of Benedetto Castelli's *Della misura dell'acque correnti*, 3rd edition (1660).

Castelli's aim had been to treat with unprecedented rigor and precision the subject of running waters and to establish thereby a kind of "*scienza matematica dei fiumi*" (Maffioli 2010, 34) that sufficiently took into account the complexity of their motion. His application of a mathematical-geometrical approach around the basic concept of velocity, which had resulted in the formulation of the continuity equation, made his work into the foundation of river hydraulics and modern hydrodynamics in the West. Its seminal influence on scientific developments in the field has been amply described in various works on the history of science and hydraulics.[40] Moreover, the immense importance of *Della misura dell'acque correnti* is demonstrated by early translations into English (Castelli 1661) and French (Castelli 1664).

Already one year after its first publication, Castelli wrote in a letter to Galileo in 1629 that, despite their general reservations about the Galilean school, the Jesuit fathers of the Collegio Romano had read and highly praised his work (Maffioli 2010, 234–235).

39 A clear hint supporting this assumption is the description of a calculation table in the section on the relationship between flow velocity and water height, proposition 8 of Verbiest's text in the *QLX*. This table is only found in the 1660 edition of Castelli's work (lib. II, proposizione IV, corollario II).

40 See, for example, Rouse and Ince (1957, 59–61), Shea (1998), Bertoloni Meli (2006), Ciriacono (2006), and Maffioli (1994; 2010).

Consequently, they aspired to make the innovative knowledge available to their classrooms by translating part of Castelli's book into Latin under the title *De mensuratione aquarum currentium*. This translation was internally circulated to students of the Collegio Romano by at least 1645, if not earlier, probably not for use in the common mathematical classes, but for the "special" courses to train expert mathematicians and candidates for the overseas mission. According to Maffioli (1994, 64–70), the Jesuit rendering left the introductory section out but comprised the entire second, mathematically demanding "Demostrazioni geometriche" of the edition of 1628. This Latin manuscript was never printed but could have been consulted by Verbiest, who attended the Collegio Romano in 1652–1653, and by Jesuit confreres and fellow students of his, such as Giandomenico Gabiani, who also arrived in China in 1659.

However, Castelli's work also attracted some criticisms, not the least from Jesuit mathematicians, among them Niccolò Cabeo (1586–1650), Giovanni Battista Riccioli (1598–1671), and the French Claude-François Milliet Dechales (1621–1678). In their own books they provided summaries of Castelli's most important propositions in Latin and consented to some of them. But there was disagreement, for example, about the question whether water does or does not accelerate in a river, which Castelli had answered in the affirmative.[41] In his *Architettura d'acque* (1656/1663) the Lombard engineer Giovanni Battista Barattieri (1601–1677) even went so far as to accuse Castelli of plagiarism when he claimed that the "Law of Continuity" had been discovered long before by Alessandro Betinzoli, who had worked as an engineer for the Republic of Venice and had already died in 1612 (Maffioli 1994, 64–70). However, even though the books of all these experts were most likely available in the Jesuit library in Beijing and probably were consulted by the Jesuit in the course of his composition of the *QLX*, Verbiest in his own writing did not take into account their criticisms but rather adopted the propositions of Castelli without any further modifications.

Below, we present a preliminary draft of the first English rendering of Verbiest's text on flood prevention in the *QLX*.[42] While its initial part about the origin of springs and rivers is not based on Castelli's work but on other sources and, therefore, is only

41 Cabeo was of the opinion that Castelli's measurement of the speed of water flow was not reliable because velocity changed according to width and depth within the cross-section of a canal. See Cabeo (1646, vol. 1, Ad Textus 60 "De mensura aquarum decurrentium," Quaestio 1–9). Cabeo's main point of critique was, however, that Castelli dared a mathematization of a phenomenon like the flow of water at all. Just like Cabeo, Riccioli (1661) rejected Castelli's assumption that water would accelerate in rivers, but he agreed with the rest of his approach. See Bertoloni Meli (2006, 166–173). In his 1674 work, Dechales finally disproved Castelli's proposition about the velocity increase, and showed that this acceleration took place not with the square but with the square *root* of the height of a river.

42 The presented translation was made by Sabine Kink.

paraphrased here, the rest of this text is translated literally.[43] With regard to its—certainly improvable—translation, our focus is not, however, on philological exactness but on the reproduction of novel scientific concepts and hydrotechnical methods. Enclosed as these innovative ideas are in the *QLX*'s traditional Aristotelian framework, their original presentation highlights a hitherto unknown facet of Western knowledge transfer in the field of river hydraulics. For comparative reasons, contemporary English translations taken from Salusbury's *Of the Mensuration of Running Waters* (Castelli 1661) are appended time and again in order to demonstrate Verbiest's direct reliance on Castelli in many parts of his work. Moreover, at the end of each translated subsection, Western books on hydraulics printed before 1680 and presumably available to Verbiest in the Jesuit Beitang Library are listed as possible or verified additional sources used by him.[44]

3 Verbiest's text on flood prevention in the *Qiongli xue*

In *juan* 9 of the *QLX*'s *Reasoning in Physics* (*Xingxing zhi litui* 形性之理推) part, Verbiest's Chinese text on rivers and flood control covers folios 23b–37a and constitutes the seventh out of altogether ten "topics" (*ti* 題).[45] A transcribed version of the original Chinese rendering is found in Appendix 1. Verbiest's text is divided into four subtopics (or subsections): The first, on the origin of springs and rivers, draws on the Western tradition going back to Aristotle's *Meteorologica*, whereas the other three are mainly based on Castelli. While subsections one and two are in large part of a general, introductory nature, the approach of subsections three and four (with an additional sample calculation) is much more mathematical and technical. This technical approach takes up about three quarters of the whole text and contains several numerical examples, for which the following units of length used in surveying apply:

43 The translation is based on a copy of the relevant section in the only extant but incomplete manuscript of the *QLX*, which is stored in the Peking University Library, and was kindly provided to us by Prof. Standaert, whom we thank for his support.

44 For these books, see also Koenig (2014). It should be underlined that the inclusion of monographs in catalogs like that by Verhaeren (1949) does not automatically mean that these books were actually extant in Beijing during Verbiest's sojourn, as in many cases the exact time of their arrival in the Beitang Library is not known.

45 At the beginning of *juan* 9 we find the entry that this chapter was written by Verbiest, Administrator of the Calendar (*Zhili lifa* 治理曆法) and, in addition, second-rank Right Vice-Minister in the Board of Works (*Gongbu youshilang* 工部右侍郎).

Chinese units	metric units (until 1702)[46]	metric units (from 1702)
1 *li* 里 = 180 *zhang*	444 m	556 m
1 *zhang* 丈 = 10 *chi*	2.47 m	3.09 m
1 *chi* 尺 = 10 *cun*	24.7 cm	30.9 cm
1 *cun* 寸 = 10 *fen* 分	2.47 cm	3.09 cm

3.1 Reasons for the perpetual flow of springs and sources

泉源在河海江湖諸水平上永流之所以然

The Hows and Whys of the Perpetual Flow of Springs and Sources Originating above the Water Level of Rivers, Oceans, Streams, and Lakes

fols. 23b–25b

This characteristically Aristotelian subsection of Verbiest's discourse that deals with the origin of springs feeding rivers and lakes is not found in Castelli's work. Therefore, its contents are only summarized here as follows:

While springs originate from high-lying places, rivers and streams come forth from the foot of the mountains. What is the reason for this phenomenon? To answer this question Verbiest first describes how the mountains are thoroughly pervaded by a system of various kinds of cavities. These caverns are joined to an extensive network of subterranean channels (*gou* 溝) filled with water and connected to the Great Sea (*dahai* 大海), which, by means of the periodically occurring tides, maintains a constant flowing back and forth of the water. When it happens that this water on its way to the mountains passes through ore veins containing heat-producing material like sulphur or lime, it becomes loaded with hot *qi* 氣, which often results in perpetually flowing hot springs (*tangquan* 湯泉) appearing on the surface above these places.

But extremely hot *qi* is also abundantly preserved in the deep-lying cavities below the roots of the mountains. Thus, once the water in the underground veins reaches these places, it becomes even hotter and is finally evaporated so that it now rises like smoke through the subterranean cavity network and up to the mountain peaks. These high-lying ridges usually reach the very cold middle region of the clouds (*yuncai zhi*

46 In 1702, the Jesuit Antoine Thomas was ordered by Emperor Kangxi to measure together with Chinese officials the length of an arc of one degree of the terrestrial meridian, which up to then had been defined as containing 250 *li*. Kangxi decided to change this ratio to 200 *li* per one degree, resulting now in a length of 556 m per *li*. See Jami (2012, 185, 244–245). This measurement was rather precise, as today we know that the perimeter of the equator divided by 360 [degrees] is equivalent to 111.319 m, which, again divided by 200 [*li*] is 556.6 m. For further conversion factors, see Qiu et al. (2001).

zhongyu 雲彩之中域),[47] for which reason they are permanently surrounded by fog and often covered with snow. When the hot vapor from the depths of the earth arrives inside the mountains in this freezing cold place, it is condensed (*ningjie* 凝结) and altered (*bian* 變) into water again. This, Verbiest says, is just like in the case of the generation of rain drops in the atmosphere outside the mountains, where water from the earth's surface gets evaporated and is made to rise by the heat of the sun. The whole process follows the same principle as the boiling of water on a stove fire: Once the vapor has reached the colder lid of the steamer (*zeng* 甑)—or, in the case of the mountains, their peak region—it is deprived (*tuo* 脫) of the heat that had caused it to rise, and therefore turns into heavier water again. In the steamer, this water first collects at the lid, but due to its hollowed shape it then runs down along the walls and back into the pot again. Similarly, inside the mountains the condensed water moves downward through empty spaces, and on its way it then searches for openings to leak out (*xiechu* 洩出), either to high ground outside the mountains or at their foot. As this process of evaporation and condensation repeats constantly, an eternal cycle feeding springs and rivers results and explains their perpetual and never exhausting flow.

Possible sources:
Aristotle, *Meteorologica,* I.13 and II.2; Albertus Magnus (1488/1890), liber secundus, tract. II (mainly cap. 1–12); *Commentarii Collegii Conimbricensis S. J.* (1593), tract. IX "De fontibus et fluminibus" (mainly cap. 1–5); Froidmont (1627), cap. II "De fontibus" and cap. III "De fluminibus"; Cabeo (1646), tom. I, add. 62. Q.1. "De origine fluviorum in universum"; Descartes (1650), *Meteora*, cap. III "De sale" and cap. VII "Cur aqua salis arena percolata dulcescat"; Kircher (1656), dialogus III, cap. III "Iter subterraneum"; Kircher (1665), tom. I, liber V "De lacuum, fontium, fluminumque origine"; Varenius (1672), liber I, cap. XVI "De fluviis in genere"; Dechales (1674), tom. II, tract. XV "De fontibus naturalibus, & fluminibus."

47 While in Aristotle's writings the idea of a three-layered atmosphere is not yet mentioned explicitly, it is already found in Seneca, *Naturales quaestiones*, book 2, and further elaborated in Albertus Magnus, *Libri quattuor meteororum*, I.1.7–9. Moreover, it is described in Clavius (1585); *Commentarii Collegii Conimbricensis S. J.* (Colegio de la Compañía de Jesús 1593, tract. I, cap. II); Geraldinus (1613); Della Porta (1614); Froidmont (1627); Eschinardi (1658); and Caramuel (1670). This theory had first been introduced to China by Matteo Ricci, who described and illustrated it in his world map *Kunyu wanguo quantu* 坤輿萬國全圖 of 1602, and then again in his *On the Structure of Heaven and Earth* (*Qiankun tiyi* 乾坤體義; ca. 1608). Soon after, it was discussed again in Sabatino de Ursis's and Xu Guangqi's *Hydromethods of the Great West* (*Taixi shuifa* 泰西水法; 1612). For more details on its introduction to China by the Jesuits, see Sun (2017) and Kink (2022).

3.2 Causes for fluvial flooding

江河泛濫之緣由
The Causes of Inundations by Rivers and Streams
fols. 25b–28a

Verbiest mentions seven distinct reasons why rivers might overflow and flood the adjacent areas. His text reads:

> In the case of so-called inundations (*fanlan* 泛濫) by rivers and streams, their two banks (*ya* 涯) to the left and right are not able to contain (*rong* 容) the running water. Such [inundations] being widespread, the reasons for them are numerous. Now, let us select (*tuiju* 推舉) some important points (*duan* 端) and talk about them.
>
> Firstly, whenever there is much water in the upper reaches of a river course (*hedao* 河道), but only little water enters the sea, this is the overall source of inundation.
>
> Secondly, the mouth where a river course enters the sea may meet with many obstructions. It might happen that the water of the upper reaches encounters an extraordinarily rising (*gaozhang* 高長) sea tide that rushes against the river course so that very little river water enters the sea, and, moreover, the downward current close to the sea gets strongly retarded, which results in obstruction. Since the water flowing [down] from above is fast, it rises through this and therefore the original river course is [no longer] able to hold it and [the water] must overflow from it. From an unusual increase in height of the sea tide that in comparison to its normal [level] might vary from one to two *zhang* 丈, one can infer the nature and scale of the impediment (*zudang* 阻當)[48] at the place where the river mouth enters the sea.
>
> Thirdly, it might happen that at the place where a river mouth enters the sea extraordinarily fierce winds act contrary to the force of the current for days and make the waves and bores (*botao* 波濤) of the sea and the twice daily sea tide rush against the flow of the water, which should stop its entering into the sea. Consequently, the distant water of the river course far away from the sea rises and overflows [its banks].[49]
>
> Fourthly, when the river water flows rapidly, it takes along a lot of sand, stones, and yellow earth, resulting in either a rise of the riverbed (*hedi* 河底) or a retention without motion in places close to the sea so that the mouth where the river enters the sea is

48 阻當 is the same as 阻擋.

49 This third issue is clearly taken from Castelli (1660), lib. I, corollario VII. Translation by Salusbury (1661): "Likewise, . . . the windes, which stop a River, and blowing against the Current, retard its course and ordinary velocity shall necessarily amplifie the measure of same River, and consequently shall be, in great part, causes; or we may say, potent con-causes of making the extraordinary inundations which Rivers use to make. And it is most certain, that as often as a strong and continual wind blows against the Current of a River, and shall reduce the water of the River to such tardity of motion, that in the time wherein before it run five miles, it now moveth but one, such a River will increase to five times the measure, though there should not be added any other quantity of water."

blocked, and the longer this lasts the higher [rises] the water that flows in from above. This is also a reason for inundations.[50]

Fifthly, when it rains heavily for days, the higher the water of rivers and lakes rises and thus enters the mouth of the river course, the more this course gets constricted and likewise surges up (*qian* 淺)[51] so that it is then [no longer] able to hold a strong current like this.

Sixthly, it often occurs that in the course of a single river there are either lock board (*zhaban* 閘板)[52] narrowings or many bridge arches (*qiao zhi kuang* 橋之鑛).[53] If the piers [supporting the] arches excel in width, they can impede a rapid current, and therefore the river water rises and smashes dikes and dams (*diba* 堤壩). In fact, if the lock gates (*zhakou* 閘口) and the arches [cause] a narrowing like this, the whole course of the river also results in such a narrowing, and consequently the rapid current of the whole water is impeded. Let us assume (*jiashu* 假如) that there are two such water tubes (*shuitong* 水筒), which are [thus] of unequal width. (See illustration 246.)[54] [In this illustration] *jia* 甲-*yi* 乙 and *bing* 丙-*ding* 丁 show [these two tubes]. The opening for the flowing water [marked with] *jia* of the *jia-yi* tube is equal [in size] to the *bing-ding* tube opening. The principle of the proposition (*tili* 題理)[55] says: Since the two tube openings are equal, even if the width [over the entire length] of the tubes is different, it is clear that the [amount of the] outflowing water must be equal.[56]

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50 This passage, which applies exactly to the situation of the Yellow River, might nevertheless have been inspired by Castelli (1660), lib. I, corollario V. Translation by Salusbury (1661): "if it shall happen that the Torrent grow torbid and muddy, and its streame be retarded in such a degree, that it is not able to carry away those minute grains of Earth, which compose the muddinesse; in this case the Torrent shall clear away the mud, and carry away the Sand at the bottome of its own Chanel, in the extreame parts of its mouth, which raised and voided Sand, shall again afterwards be carried away, when the River abating, the Torrent shall return to move with its primitive velocity."

51 淺：濺，水激射。

52 According to Needham et al. (1971, 349), the term *banzha* 板閘 stands for a lock gate made of boards, which is the same as a stop-log gate. Stop-logs are usually long rectangular wooden boards that are placed on top of each other and dropped into prefabricated slots inside a weir in order to adjust the water level in a river or canal.

53 鑛 here stands for 壙, which actually means "vault." In the above context, however, "arch" seems to be more adequate.

54 This illustration is missing in the manuscript.

55 For an overview of the usual Chinese renderings of Western syllogistic terms like "proposition" or "reasoning," see Kurtz (2011, mainly 71–84).

56 Again, the source is Castelli (1660), here lib. I, appendice VIII. Translation by Salusbury (1661): "The same contemplation discovereth the errour of those Architects, who being to erect a Bridge of sundry Arches over a River, consider the ordinary breadth of the River, which being v.g. fourty fathom, and the Bridge being to consist of four Arches, it suffices then, that the breadth of all the four Arches taken together be fourty fathom; not considering that in the ordinary Channel of a River, the Water hath onely two impediments which retard its velocity; namely, the touching and gliding along the two sides or shores of the River: but the same water in passing under the Bridge, in our case meeteth with eight of the same impediments, bearing, and thrusting upon two sides of each Arch (to omit the impediment of the bottom, for that is the same in the River, and under the Bridge) from which inadvertency sometimes follow great disorders, as quotidian practice shews us."

> Seventhly, as to a narrow river course, when it happens to rain exceptionally heavily, even if inside the river course the velocity of the flow does not decrease, once very much water enters this course from outside, it is not able to contain [all that water]. Let us assume that there is such a river course which is no more than 3 *zhang* wide and no more than 6 *chi* 尺 deep. If the flowing water just passes 1 *chi* of land within two seconds' time (*shike zhi er miao* 時刻之二秒), then every two seconds 180 cubic [*chi*] of flowing water must pass through. However, in view of the large amount of water entering the river course from the outside, the downstream (*xialiu* 下流) should be 300 cubic [*chi*] of flowing water, because only then could the disaster of an inundation be avoided. Now, since due to the narrowness of this river course that much [water] cannot flow down, the water level in the original course will at any rate increase by 4 *chi* in height, and this is the reason for its overflowing. If the course of the river had already been dredged (*tiaowa* 挑挖) to a width of 12 *zhang*, then, in accordance with what has been assumed before, despite an extraordinary augmentation of water from the outside, the water level inside the river course would increase in height no more than 1 *chi*. Consequently, an inundation could be avoided.

Possible sources:
Castelli (1660); Barattieri (1656/1663), lib. settimo "nel quale si tratta delle cause, delle quali derivono le innundationi de i fiumi"; Riccioli (1661), lib. sextus altimetricus, cap. XXIX "De incremento, & decremento altitudinis, ac velocitatis fluminum, fontium & canalium, earumque causis, ac mensuris"; Bonini (1663), del. lib. primo, cap. 6 "Cagioni naturali dell'inondationi de'fiumi"; Michelini (1664); Dechales (1674), tom. II, tract. XV "De fontibus naturalibus, & fluminibus, De aquis currentibus," propositio XLV "De causis intumescentiae fluminis." Verbiest seems to have relied primarily on Castelli here, many of whose propositions can be clearly identified. The other authors report mostly abridged versions of Castelli or deal descriptively with practical flood control works such as dike building.

3.3 The relationship between flow velocity and water height

江河消長多寡并水流遲速之比例
The Ratio of the Extent of Falling and Rising [of the Water Level] to the Velocity of the Water Flow of Rivers and Streams
fols. 28a–33a

This is the longest subsection of Verbiest's text. The altogether nine propositions presented by him all have a similar structure but are partly difficult to understand.

> The main reason for inundations by rivers and streams is the rising of their water [level] together with a slow (*chihuan* 遲緩) downward flow. Now, let us establish a number of propositions to infer from them the ratio (*bili* 比例) of the extent of their falling and rising

to the velocity of the water flow. Thereupon one can use this [ratio] to change the [river] courses in accordance with the circumstances of various places and rivers.

Proposition 1

The more seawater and contrary winds alike hinder the water flow at the mouth of a river, the more the water in the river course will increase in height. Moreover, the magnitude (*duogua ruogan* 多寡若干) of retardation of the flow and that of its rising are both of the same scale. Let us assume that there is such river water. After it has flowed downward 15 *li* 里 within a period of four quarters (*ke* 刻) of an hour (*shi* 時),[57] when seawater and contrary winds hinder it, its downflow is [then] no more than 3 *li*, and therefore its water level rises five times in height. But as the locations of whatever [river] course happen to be different, the reasons for this augmentation happen to be different in each place [too].[58]

Proposition 2

There are two such rivers [marked with] *jia*, *yi*, and *bing*. The velocity of the water flow of both courses is different. The principle of the proposition says: As to the magnitude of the water flow in river course *jia* and that in course *bing*, their ratio is compounded (*xiangjie* 相結) of the ratio between the sizes of the water doorway (*shuimenkou* 水門口) [that is, the cross-section] of the *jia* and the *bing* river courses,[59] and the ratio between the rapidity of the water flow of river *jia* and the sluggishness (*chi* 遲) of river *bing*.[60] (The so-called "water doorway" stands for a principle of river management. For better understanding, one crosscuts (*hengqie* 橫切) inside each river course the water from its surface to the river base. Moreover, the mutually corresponding measurements (*chicun* 尺寸) of width and depth of each river are taken to make the water outlet into a vertical door with a square opening shape. What is thus known as the "doorway of the river course" (*hedao menkou* 河道門口) serves to infer for each river course the arrangement and inner [design] (*shili* 勢里) of the passing water flow at a distinct place.) Let us assume that the cross-section of river *jia*, which is [marked with] *yi*, has 100 square *chi*, and that of the *bing* river at *ding* has 80 square *chi*. Moreover, the water of river *jia* flows down a distance of up to 40 *chi* within one minute's time. The water of river *bing*, equally within one minute, flows down a distance of up to 30 *chi*. The method [of calculation] says: 100 *chi* multiplied by 40 *chi* results in 4000. Moreover, 80 *chi* multiplied by 30 *chi* results in 2400. Reasoning (*litui* 理推) says: The ratio of the flowing water that has passed the cross-section of river *jia* to that

57 Even though the expression *shi* 時 (which actually stands for a double-hour) is used here, this passage obviously refers to the time system used in the Qing with 4 *ke* equaling 1 hour (*xiaoshi* 小時) or 60 minutes (*fen* 分), resulting in 96 *ke* per day. Further down in the text, however, Verbiest says that 100 *ke* are "one day and night," thus using the conventional pre-Qing units there. In this case, 4 *ke* would only be 57.6 minutes. This difference in absolute numbers is not relevant here, but it shows that Verbiest's approach contained quite a few quantitative inaccuracies.

58 The source for this passage is Castelli (1660), here lib. I, corollario VII.

59 Due to the parallelism in the passage "甲河道之**水門口大小**，與丙河**水門高大小**," the character 高 has been read as 口.

60 Compare Castelli (1660, 65): "Proposizione II. Se saranno due sezzioni di Fiumi: la quantità dell'acqua, che passa per la prima a quella, che passa per la seconda, ha la proporzione composta delle proporzioni della prima sezzione alla seconda, e della velocità per la prima, per la velocità per la seconda."

which has passed the cross-section of river *bing* is like 4000 to 2400.[61]

Proposition 3

If it has rained heavily for days, which has added much water to the river, the principle of the proposition says: The ratio between the amounts of water within the same river course after and before it has rained is compounded of the ratios between the water heights before and after the rain, and the rapidity of the perpetual flow. Let us assume the water height in the river is 4 *chi* and it flows down a distance of up to 40 *chi* within one minute. Afterwards, the rainwater increases its height by 2 *chi*, and consequently its rapidity is also increased, namely, it [now] flows down a distance of 50 *chi* within one minute. The method [of calculation] says: 4 *chi* multiplied by 40 [results in 160], and, moreover, 6 *chi* (to the former height of 4 *chi*, after [the rain] 2 *chi* are added, which together are 6 *chi*) multiplied by 5 *chi*[62] results in 300. The principle of the proposition says: The ratio of the magnitudes of the water downflow within the same time [that is, the ratio of the flow rates] inside the river course before it has rained and thereafter is 160 to 300.

Proposition 4

When a river increases in height, the sluggishness of its current before and its rapidity after this increase are in a ratio of the heights before and after [the increase], to which has been added the mean value of the first and the second term (*lü* 率)[63] of this ratio. Let us assume that there is a river like this. Before it has increased in height, its water is 8 *chi* deep, and thereafter it is 18 *chi* deep. Now one wants to infer the difference in flow velocity before and after [the increase in height]. The method [of calculation] says: In between 8 *chi* and 18 *chi* select another number like 12, which serves as the mean value of the continuous ratio (*lianbili zhi zhonglü* 連比例之中率). The principle of the proposition says: The ratio of the sluggishness of the current before to its rapidity after the increase in height is thus 8 to 18.[64]

Proposition 5

For one and the same course of a river or stream, the amount of flowing water that within the same period of time passes any place, whether it is wide or narrow, is always

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61 Again, the reference is Castelli (1660), here "Demostrazioni geometriche della misura dell'acque correnti," proposizione II. Translation by Salusbury (1661): "In two Sections of Rivers, the quantity of the Water which passeth by one Section, is to that which passeth by the second, in a Proportion compounded of the proportions of the first Section to the second, and of the velocitie through the first, to the velocitie of the second.

Let A, and B be two Sections of a River, I say, that the quantity of Water which passeth through A, is to that which passeth through B, in a proportion compounded of the proportions of the first Section A, to the Section B; and of the velocity through A, to the velocity through B."

62 Five *chi* is not correct and should read 50 *chi* instead.

63 Usually, *lü* 率 has the meaning of "rate" or "proportion." In the context of ratio calculation, however, this expression was applied by the Jesuits to express the mutually related terms or number values of a proportion (*numerus proportionalis* in Latin). See Jami (2012, 92 [footnote 44], 171–172).

64 It is not quite clear what the exact meaning of this passage is. According to the rule that has been established at the beginning of this proposition, 8 and 18 need to be increased by 12 each so that the actual ratio of the velocities would be 20:30.

uniform. Let us assume that *jia-yi-bing-ding* [mark] a river course. (See illustration 253.)[65] *Jia* to *yi* shows a [section of] wide, *bing* to *ding* a [section of] narrow places. The principle of the proposition says: The amount of downward flowing water in the four *zhang* [long] *jia-yi* [section of] wide places and in the four *zhang* [long] *bing-ding* [section of] narrow places is always uniform. In fact, if one thinks otherwise, and [therefore] assumes the flowing water passing the narrow places of *bing-ding* would be less and that passing the wide places of *jia-yi* would be more, then the places of the *jia-yi* [section of the] river course would have to increase in [water] height and the places of the *bing-ding* [section of the] river course would have to be lower [in water height]. However, such a theory does not conform with the evidence provided by common visual examination. Now, let us say [instead] that within a river course the amount of flowing water is equal in each place. For this reason, the current at narrow places in relation to wide ones is very fast (*xunji* 迅急), and as a consequence the amount of water that has flowed [through a narrow place] is equal to what has flowed through a wide place within the same time.[66]

Proposition 6

If there is a river *jia* that is entered by a river *bing*, and the amount of the incoming water remains uniform all the time, the proposition explains the measure of increase in height during an earlier against a later [period of water inflow], and the magnitude of increase in current velocity during a later against an earlier [period of water inflow], [both caused] by the water of river *bing* within this course [that is, the course of river *jia*]. Let us assume that the water level of river *jia*, after it has taken up the water of river *bing*, has risen by 12 *cun* 寸 within the earlier eight quarters [of an hour, that is, within two hours], and within one minute of this time it has flowed down 3 *zhang*. Within the later eight quarters' time, when river *jia* takes up the water of river *bing*, its water level rises [only] by 6 *cun*, but within one minute of this time it [now] flows down 6 *zhang*. The principle of the proposition says: As to the water of river *jia*, the ratio of its increase in height by 12 *cun* during one unit of the earlier time to its increase by 6 *cun* during one unit of the later time corresponds to (*xiangyu* 相與) the former and the latter number values (*lü* 率). This is because in river *jia*, whose water is shallow and which has a narrow riverbed, the power of the current is very sluggish during the earlier time, and consequently only little water flows down. Therefore, when the water of river *bing* enters, then the surface of river *jia* rises suddenly. Now that it has risen, the water also becomes more rapid and consequently more water flows down in river *jia*. For this reason, during the later period [of water inflow from *bing*] the increase in height [in river *jia*] is thereupon minor. From this one can infer for all rivers that when they receive water from the outside, during the first one or two days their water [level] all of a sudden rises a lot,

65 This illustration is also missing in the manuscript.

66 Source: Castelli (1660), here lib. I, "Demostrazioni geometriche della misura dell'acque correnti," proposizione I. Translation by Salusbury (1661): "The Sections of the same River discharge equal quantities of Water in equal times, although the Sections themselves be unequal."

while during the following one or two days it rises [only] a little.[67] Moreover, where the water is shallow as well as in deep[-lying] places, the river course should generally be narrowed, and where the water is deep as well as in high[-lying] places, it should be expanded instead.[68]

Proposition 7

For the case that river *jia* enters river *bing*, the proposition explains: The ratio between the water height of river *jia* within its own course and what it has added to the course of river *bing*, [that is] the water height at *yi* [of river *bing*], is compounded of the ratios between the widths of the two rivers *jia* and *bing*, and between the current velocities of rivers *jia* and *bing*. Let us assume that at *yi* the water is 100 *chi*, and at *ji* of [river] *jia* 60 *chi* wide. Moreover, from *yi* the water flows down a distance of 50 *chi* within one minute's time, and from *ji* of [river] *jia* 30 *chi* within the same time of one minute. The method [of calculation] says: 100 multiplied by 50 results in 5000. Moreover, 60 multiplied by 30 results in 1800. Now, supposing that the water height at *ji* of [river] *jia* is 5 *chi*, the principle of the proposition says: As to the water height at *ji* and the fraction it has added on top of the water height at *yi*, if [widths multiplied by downflows are] 4000[69] and 1800 [respectively], then, in the case that the original water height at *yi* had been 5 *chi*, the additional height is 1 *chi* and four fifths.[70]

Proposition 8

This proposition explains [how to] infer the amount of water that should be added if one wants to raise the water level of a river course to a fixed measure. One sets up a table of the additions and subtractions (*jiajian* 加減) of the river water in order to take advantage of its use. However, to apply this method one definitely needs the cross-section of the river course, because it is precisely the shape [that is a precondition] for this method. From the cases in which this method applies one can then infer those where it is not so. The method [of calculation] says: In the first row (*hang* 行) of the table one seeks out the measure of the original water height. Next, in the same row one seeks out the measure of the desired increased height. Then, in the opposite number position of the

67 This proposition clearly draws on Castelli (1660), lib. I, "Demostrazioni geometriche della misura dell'acque correnti," proposizione IV. Translation by Salusbury (1661): "If a River fall into another River, the height of the first in its own Chanel shall be to the height that it shall make in the second Chanel, in a proportion compounded of the proportions of the breadth of the Chanel of the second, to the breadth of the Chanel of the first, and of the velocitie acquired in the Chanel of the second, to that which it had in its proper and first Chanel.

Let the River AB, whose height is AC, and whose breadth CB, that is, whose section is ABC; let it enter, I say, into another River as broad as the line EF, and let it therein make the rise or height DE, that is to say, let it have its Section in the River whereinto it falls DEF; I say, that the height AC hath to the height DE the proportion compounded of the proportions of the breadth EF, to the breadth CB, and of the velocity through DF, to the velocity through AB. Let us suppose the Section C, equal in velocity through the Section AB, and in breadth equal to EF, which carrieth a quantity of Water equal to that which the Section AB carrieth, in equal times, and subsequently, equal to that which EF carrieth."

68 The meaning of this last sentences of proposition 6 is not quite clear.

69 This should read 5000.

70 River *jia* delivers 60 × 30 × 5 = 9000 cubic *chi*, which, given the measures in river *bing*, results in an additional height in *bing* of 9000 : (100 × 50) = 9 : 5 = 1 4/5 *chi*.

third row one obtains the respective square numbers (*fangshu* 方數).[71] The difference (*jiaoshu* 交數)[72] of these two square numbers represents the measure [that is, the amount of water] that needs to be added to the original height of the river water. Let us assume the original height is 5 *chi* and one wants to increase this height to 8 *chi*. The method [of calculation] says: The square numbers of 5 and 8 are 25 and 64. Their difference is 39. Only then does one obtain the number of cubic *chi* of water contained in an 8 *chi* high cross-section of the river.[73]

Proposition 9

There is a river course to which the method [described above] applies. For this [river course] one wants to know how much one needs to reduce the water height in order to obtain within this course a required distinct water height. Let us assume the original height is 30 *chi*. Now one wants to reduce [this height] and return to 24 *chi*. The method [of calculation] says: If 30 and 40[74] are subtracted from each other, the difference is 6. Reasoning says: This difference is consistent with the greater of the two water heights. . . .[75] Based on this one can infer that in order to obtain the required [height] one needs to reduce the original height of 30 *chi* by one fifth.

Possible sources:

Castelli (1660); Barattieri (1656/1663), lib. settimo "nel quale si tratta delle cause, delle quali derivono le innundationi de i fiumi"; Riccioli (1661), liber sextus altimetricus, cap. XXIX "De incremento, & decremento altitudinis, ac velocitatis fluminum, fontium & canalium, earumque causis, ac mensuris"; Dechales (1674), tom. II, tract. XV "De fontibus naturalibus, & fluminibus, De aquis currentibus," propositio XXXIX to XLV; Caramuel (1670), tom. II, XIX "Potamographia," articulus IV, "De fluxu fluminum," and articulus V "De directione fluviorum."

Most of these authors report abridged versions of Castelli's propositions. Only Dechales states some different views and presents some genuine improvement regarding the relationship between velocity and water height.

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71 Actually, square numbers are *pingfangshu* 平方數. The term *fangshu* 方數 instead usually stands for the homonymous 方术, which are medical prescriptions.

72 交 obviously stands for 較 here. (較數：数学上旧指用减法求得的馀数。)

73 This actually means that 39 units of water have to be added in order to reach a water height of 8 *chi*. Verbiest here clearly draws on the 1660 edition of Castelli (lib. II, proposizione IV, teorema IV, corollario II), because the table described by him is not found in the earlier editions. Translation by Salusbury (1661): "But if it should be desired, to know how much water it is requisite to add to make the River rise so, as that it may run in height 8. Of those parts of which before it ran but 5; one ought to take the sum of the number of Series of Additions standing under 8. 7. And 6, which are 15. 13. And 11. That is, 39. And this shall be the summe that must be added to 25: So that to make the River to run to 8. Of those parts in height, of which it before did run 5, it will be necessary to 39. Of those parts, of which the River before was 25." A similar example is found in Barattieri (1656/1663), lib. 6. The calculation example presented there is copied from Castelli.

74 This should read 24.

75 The correct meaning of the passage "與他數即六于與三十如一，于五。因此而推知原舊之高，三十尺者" is unclear.

3.4 Precautions against inundations

江河泛濫之防備
Precautions against Inundations by Rivers and Streams
fols. 33a–35b

In this fourth and last subsection, Verbiest discusses the pros and cons of both drainage canals and higher dikes for flood prevention as well as the appropriate design of the canals.

> From the several above established reasons for inundations one can infer several corresponding precautions [against them]. Now, a number of important points are briefly proposed in order to prepare for their required application.
>
> Firstly, whenever one opens up a new canal (*he* 河) in order to reduce the [excess] water after the swelling of either a great stream or another river and lead it through to enter the sea, the wider and the deeper this new canal route is, the faster it will prove effective. Therefore, whenever a [drainage] canal is opened at a stream or river alike, its cross-section must be proportional (*xiangchen* 相稱) in magnitude to the water level of the stream and, moreover, to the length of the river course. Reasoning says: Be it a stream or a river, a certain magnitude of the water level [of the river] and of the cross-section [of the canal] result in a certain velocity of the current inside the canal and a certain reduction of the water level [of the river]. Nevertheless, whenever one opens a new canal route in order to avoid the disaster of inundations, this sometimes is of use, sometimes not that much. In fact, if the swelling up of the water of rivers and streams persists only for a few quarters [of an hour], then it is of benefit to use [a canal], but if it persists for 20 or 30 days, its use does not have a big effect. The reasons for this have already been verified repeatedly. When it either rains constantly for days, or when small rivulets [constantly] enter big rivers and streams from the side, then this easily cancels out the [amount of] water swelling that the newly opened canal is able to reduce. Therefore, one should carefully consider whether to open a new canal or whether to make an extra effort in order to reinforce the dikes and dams of the original [river] course, and which of both [options] is more advantageous or rather spends more tax revenues (*qianliang* 錢粮), and then decide upon this. (For the supposition to prove the principle of this first paragraph, see the section below.)[76]

76 This refers to the appended calculation example, see 3.4.1. Source: Castelli (1660), lib. I, corollario XIII. Translation by Salusbury (1661): "And here I will pray those who rest not wholly satisfied with what hath been said, that for the love of truth, and the common good, they would please make diligent observation whether in the time of great Floods, the said Bank or Dam at Bondeno is cut, and that in few hours the main Po diminisheth, as has been said about a foot in its height, that they would observe I say, whether, a day or two being past, the Waters of the main Po return almost to their first height; for if this should follow, it would be very clear, that the benefit which resulteth from this diversion or Vent, is not so great as is universally presumed; because, though it may be granted for true, that the Waters of the main Po, abate at the beginning of the Vent, yet this benefit happens to be but temporary and for a few hours: if the rising of Po,

Secondly, when inundations by rivers and streams are generated either by the extraordinarily doubled water heights of sea tides, or by normal seawater tides together with contrary winds [blowing] for days, then the opening of new canals is inferior to the extra effort of raising the height and reinforcing the old dikes [along] the river course. However, [the effect of] heightening the dikes is lower in places close to the sea than in those distant from it. In fact, inundation disasters occur for the most part in places far away from the oceans, like [in the case of] the Baduo River 巴多河 [that is, the Po River] of the Far West. [There,] from places very close to the sea to those within 20 or 30 *li* 里, dikes usually do not exceed five or six *chi* in height, while at a distance of 120 or 150 *li* the dikes increase in height to more than 20 *chi*, and in other places it is even more difficult to avoid the disaster of inundation. To examine this, from ancient times until today the inspectors (*zhao* 照) of all regions have already carried out investigations over the years, and in correspondence with the varying maximum water levels of the great rivers, the height of their dikes has been increased.[77]

Thirdly, when one opens a new canal to release water from the original river and guide it into the sea, the straighter the course of the new canal, the faster the downflow of its water, and the more bends and windings (*wanqu* 灣曲) it has, the more sluggish is its downflow. At the same time, a rapid [current] damages the embankments on both sides of the canal course. Moreover, when alongside the canal there are many hollows and uneven [spots] or many roots of vegetation come forth, then this can strongly impede the current inside the water on the sides of the canal,

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and the dangers of breaking forth were of short duration, as it ordinarily befalleth in the overflowings of Torrents, in such a case the profit of the Vent would be of some esteem: But because the swellings of Po continue for thirty, or sometimes for fourty dayes, therefore the gain which results from the Vent proveth to be inconsiderable. It remaineth now to consider the notable harms which follow the said Sluice or Vent, that so reflection being made, and the profit and detriment being compared, one may rightly judge, and choose that which shall be most convenient."

77 Again, Verbiest's text is based on Castelli (1660), here lib. I, corollario XIV. Translation by Salusbury (1661): "In the Grand Rivers, which fall into the Sea, as here in Italy Po, Adige, and Arno, which are armed with Banks against their excrescencies, its observed that far from the Sea, they need Banks of notable height; which height goeth afterwards by degrees diminishing, the more it approacheth the Sea-coasts: in such sort, that the Po, distant from the Sea about fifty or sixty miles at Ferara, shall have Banks that be above twenty feet higher than the ordinary Water-marks; but ten or twelve miles from the Sea, the Banks are not twelve feet higher than the said ordinary Water-marks, though the breadth of the River be the same, so that the excrescence of the same Inundation happens to be far greater in measure remote from the Sea, then near; and yet it should seem, that the same quantity of Water passing by every place, the River should need to have the same altitude of Banks in all places: But we by our Principles and fundamentals may be able to render the reason of that effect, and say: That that excesse of quantity of Water, above the ordinary Water, goeth alwaies acquiring greater velocity; the nearer it approacheth the Sea, and therefore decreaseth in measure, and consequently in height. And this perhaps might have been the cause in great part, why the Tyber in the Innundation Anno 1578. issued not forth of its Channel below Rome towards the Sea."

and, as a consequence, [the water] gradually rises high and overflows.[78] Therefore, the canal sides should incline downwards like a slope. If it is not like this, the waterflow soaks the ground and inside the mire (*wa* 窪) many voids are formed so that on the lateral shores the banks (*tan* 灘) collapse. To discuss this more in general, the more the flowing water in this one part of the canal course rushes against either the canal sides or its bed, the more the favorable velocity downstream (*xiaxing* 下行) is reduced. Therefore, when one opens a new canal course, one must have proportional ratios (*li* 理) for its width and depth in order to avoid this rushing of the water and the collapse of the banks. Convention (*dingli* 定例) says: The depth of the canal should not surpass its width, and, moreover, half its width should not surpass its depth. If it is not like this, the flowing water rushes a lot against the sides and the canal bed, and consequently its velocity is strongly reduced. There is another illustrated explanation to demonstrate this theory. From this one can infer that when there are many lock gates in a river course, the wider their openings the faster the water flows down [through the drainage canals]. For a lock gate, on the left and right side one must prepare in advance doors (*men* 門) to release the water. Moreover, it is suitable to erect according to the method two[-winged] doorleaves (*men liang shan* 門兩扇)[79] which are connected like an edge (*leng* 稜) of triangular shape, in order to resist (*di* 敵) the power of the rushing water. The construction method of the two-winged doorleaves is of use for the "flushed river sand" (*shuichong hesha* 水沖河沙) method[80] as well as for the easy lock passage method of "barges in repose" (*hechuan anning* 河船安寧),[81] for which one allows all water in front of and behind the lock to return to one plane. As a result, several boats at a

78 This topic is also found in Castelli (1660), here lib. I, appendice IX. Translation by Salusbury (1661): "It is also worthy to consider the great and admirable benefit that those fields receive, which are wont to drink up the Rain-water with difficulty, through the height of the Water in the principal Ditches; in which case the careful Husbandman cutteth the reeds and rushes in the Ditches, through which the Water pass; whereupon may be presently seen, so soon as the reeds and rushes are cut, a notable Ebb in the level of the Water in the Ditches; insomuch that sometimes it is observed, that the water is abated after the said cutting by a third and more, of what it was before the cutting. The which effect might seemingly depend on this, That, before those weeds took up room in the Ditch, and for that cause the water kept a higher level, and the said Plants being afterwards cut and removed, the water came to abate, possessing the place that before was occupied by the weeds: Which opinion, though probable, and at first sight satisfactory, is nevertheless insufficient to give the total reason of that notable abatement which hath been spoken of: But it is necessary to have recourse to our consideration of the velocity in the course of the water, the chiefest and true cause of the variation of the measure of the Running-Water; for, that multitudes of reeds, weeds, and plants dispersed through the current of the Ditch, do chance notably to retard the course of the water, and therefore the measure of the water increaseth; and those impediments removed, the same water gaineth velocity, and therefore decreaseth in measure, and consequently in height."

79 Literally: "doors with two leaves."

80 This is probably a hint at Wan Gong's 萬恭 (1515–1591) idea to wash out the silt of the Yellow River by increasing its current velocity, which would be accomplished by means of dikes confining and narrowing its course. This new concept was then applied by Pan Jixun 潘季馴 (1521–1596), the most important hydraulic engineer of his time.

81 We have not yet been able to trace the origin and exact meaning of the term *hechuan anning* 河船安寧.

> time can pass straight through very easily and quickly, and there is no such thing as a dangerous and precipitous (*xianwei* 險危) [gradient between] the higher and the lower water [level]. There is another account that details this. When at times the farmland close to a river lies lower than the water level of the river course so that it is submerged (*yanjin* 淹浸) by rainwater running down from high-lying places, then one should dig out in the fields several crisscrossing (*zongheng* 縱橫) ditches (*gou* 溝) to take up and discharge the water and lead it close to the river side, where it is collected in one place. Thereafter, according to the method, with an implement like a windmill (*fenglun* 風輪) one raises the water across the embankment into the river. There is another discourse on hydromethods (*shuifa* 水法)[82] that gives an account of the so-called windmill that, by making use of wind power day and night without end, raises this water and causes it to pass the banks.

Possible sources:
Castelli (1660); Barattieri (1656/1663), lib. ottavo "nel quale si trattano diuerse maniere proposte per difendersi dall' innondationi de I fiumi, como anco la vera maniera di fabricare di argini"; Bonini (1663) Libro terzo to libro sesto; Michelini (1664); Zonca (1607), carte 10, "Porte per sostenner l'acqua d'alcun fiume per diversi bisogni." As in sections 3.2 and 3.3, Verbiest seems to have relied mostly on Castelli, many of whose propositions can be clearly identified.

3.4.1 Supplement to the first paragraph of the precautions section

以假如明証首端之論
Clear Proof of the Theory of the First Paragraph by Way of Supposition[83]
fols. 36a–37a

This is not a separate subchapter but just an appended calculation example to further explain the principle of the first countermeasure against fluvial flooding presented above. There, the question had been posed whether in the case of heavy rainfalls it makes more sense to open a drainage canal or to heighten dikes to prevent inundations. The impression of an appendix is reinforced by a change in layout of the Chinese text, with now ten instead of nine columns per folio.

> In the case of such a lake [to which the situation described in the first paragraph applies], which is 100 *li* wide and long, one wants to drain its water into the sea in order to avoid an inundation. For this reason, one opens a new canal which is 400 *li* long, 8 *zhang* wide, and 1

82 This might well be a hint at the *Taixi shuifa* 泰西水法, which, however, only talks in general of the use of wind power in the context of the Archimedean screw pump, without explicitly mentioning windmills. Instead, these devices are described in another joint Jesuit-Chinese treatise, the *Illustrations and Descriptions of Extraordinary Devices* (*Qiqi tushuo* 奇器圖說; 1628). See the discussion below and figures 10–12.
83 This appendix refers to the note at the very end of the first paragraph of the fourth section on fol. 35a, which says: "For the supposition to prove the principle of this first paragraph, see the section below."

zhang 2 *chi* deep, and takes it as a river course for draining the lake. However, when compared with the magnitude of the lake water, the inundation-avoiding effect of a new canal of these dimensions does not seem to correspond to that. This is because the ratio between the [volume of the] water filling the course of the new canal and [that of] the whole water surface of the lake at a depth of 1 *chi* is no more than one part out of 32. (If the width, length, and depth of the river course are like this and the calculation reverts to 1 cubic *chi* of water, then the cube number of the water of the full course compared to the cube number of the whole water surface of the lake at a depth of one *chi* amounts only to 1/32.) Examination [shows that], once 32 canal courses full of water have entered the sea, the water surface of the whole lake has been reduced in height by no more than 1 *chi*. Supposing within two days all the water filling the course of the new canal flows down the 400 *li* from the mouth of the lake into the sea once, then it must take 64 days until it flows down there 32 times. When this water has flowed down without interruption and entered the sea for 64 days, then the water level of the whole lake has only been reduced in height by 1 *chi*. Now, when it is raining constantly and one takes either a square or a high round device (*fangqi huo gaoyuan zhi qi* 方器或高圓之器) and gathers the rainwater inside, within one quarter [of an hour] it approximately increases 1 *fen* 分 in height. This has already been verified. Thus, when it rains for 100 quarters [of an hour] (that is, for one day and night)[84] on average without interruption, the water inside the device adds up to 100 *fen*, and it is clear that this is a height of 1 *chi*. But as they both follow the same principle, the water level of the big lake increases in height by just the same ratio as that inside the device.[85] If one takes this into account, then when it rains constantly for days without interruption, the rainwater of one day and night is offset within 64 days by the new canal's reduction of the height of the whole lake's water surface by 1 *chi*.[86] Moreover, it is very

84 In the Qing, 100 *ke* actually were 1500 minutes and therefore 1 hour more than Verbiest's "one day and night."

85 Source for this description of the use of a rain gauge is Castelli (1660), lib. I, "Copia di Lettera Al Sig. Galileo Galilei Primo Filosofo del Serenissimo Gran Duca di Toscana." This is the first Western report on the use of this measuring device. Translation by Salusbury (1661): "Being returned to Perugia, there followed a Rain, not very great, but constant, and even, which lasted for the space of eight hours, or thereabouts; and it came into my thoughts to examine, being in Perugia, how much the Lake was increased and raised by this Rain, supposing (as it was probable enough) that the Rain had been universal over all the Lake; and like to that which fell in Perugia, and to this purpose I took a Glasse formed like a Cylinder, about a palme high, and half a palme broad; and having put in water sufficient to cover the bottome of the Glasse, I noted diligently the mark of the height of the Water in the Glasse, and afterwards exposed it to the open weather, to receive the Raine-water, which fell into it; and I let it stand for the space of one hour; and having observed that in that time the Water was risen in the Vessel the height of the following line —, I considered that if I had exposed to the same rain such other vessels equal to that, the Water would have risen in them all according to that measure: And thereupon concluded, that also in all the whole extent of the Lake, it was necessary, the Water should be raised in the space of an hour the same measure."

86 A similar passage is found in Castelli (1660), lib. I, appendice XII (and likewise in appendice XIII). Translation by Salusbury (1661): "That which has been demonstrated in the Vessel, falls out exactly also in our Lake of *Perugia*, and its Emissary; and because the immensity of the superficies of the Lake is in proportion to the superficies of the Emissary or Sluice, as many millions to one, as may be easily calculated; it is manifest, that such abatement shall be imperceptible, and almost nothing, in two dayes space, nay in four or six: and all this will be true, when we suppose that for that time there entreth no other Water into the Lake from Ditches or Rivolets, which falling into the Lake would render such abatement yet less."

> clear that when mountain water flowing down from the surroundings together with torrential water from small rivulets enters the lake, this further increases its height. Therefore, if one does not make an extra effort to heighten and reinforce the dikes of the lake and the riversides, then it is to be feared that—since after days of rain lake and canal are already filled with water—there still might happen an inundation disaster. Now, [digging] the 400 *li* long new canal with the given width and depth amounts to 611,200,000 cubic *chi* of soil.[87] If the dikes of the lake on all sides are heightened by 5 *chi*, then this amounts to only 72,000,000 cubic *chi* of soil.[88] Therefore, the expenses for opening up a new canal are nine times higher than those for heightening the lake dikes by 5 *chi*. But as the heightening of the lake dikes is one thing and the dredging of a new canal another, the manifold increase in benefit (*jia wu bei zhi li* 加五倍之利)[89] becomes all the more clear.

Possible sources:
Castelli (1660); Riccioli (1661), lib. sextus altimetricus, cap. XXIX "De incremento, & decremento altitudinis, ac velocitatis fluminum, fontium & canalium, earumque causis, ac mensuris," propositio XXX; Dechales (1674), tom. II, tract. XV, "De fontibus naturalibus, & fluminibus, De aquis currentibus," propositio LVI, theorema "Ut se habet superficies vasis, aut lacus ad sectionem alvei per quem exoneratur; ita velocitas aquae in praedicto alveo, ad decrementum aquae in lacu, aut vase." Riccioli and Dechales are only citing from Castelli's letter to Galileo Galilei.

4 Discussion of Verbiest's text

At first sight, Verbiest's text on fluvial flooding does not seem to pose major problems of understandability for its Chinese readers. The terminology chosen by the author both for the Aristotelian part on the origin of running waters and for the technical account appears familiar and consistent. Metaphors or allusions to the ancient classics, moreover, are strikingly absent, so that in the eyes of Verbiest's learned addressees the writing was certainly not elegant in style. However, closer scrutiny reveals that quite a few passages of Verbiest's account are difficult to comprehend both from a linguistic

87 Above, the canal is said to be 400 *li* long, 8 *zhang* wide, and 1 *zhang* 2 *chi* deep. This would result in (400 × 1800) × 80 × 12 = 6**9**1,200,000 instead of 6**1**1,200,000 cubic *chi*. This discrepancy might be due to a copying error, but its actual reason cannot be fully verified, because Verbiest's working assumptions are not known.

88 Roughly speaking, this would mean that the lake dikes are heightened by 5 *chi* or about one meter, and that their crest has a thickness of 20 *chi* or almost seven meters. This is in accordance with Flessel (1974, 29), who for the region of the lower reaches of the Yellow River gives a thickness at the top of the dikes of more than 5 and sometimes up to 30 meters.

89 五倍：五次，表示再三、多次。 This expression goes probably back to a passage in the book *Mengzi* 孟子, "Gaozi xia" 告子下, where different ways to become a perfectly virtuous gentleman (*junzi* 君子) are described, among them the example of the humble and wise farmer Yi Ying, who despite his hard work had spared no effort to visit and consult those in power time and again (五就湯，五就桀者，伊尹也).

and a specialist point of view, with the terms used to express scholastic concepts of logic making things even less digestible in these cases. Thus, the text remains quite theoretical and abstract in these sections, which contain quite a few inaccuracies and rather appear as a demonstration of computational skills imbedded in the system of Western syllogism than as an instruction usable for practical application.

In the following, we will leave such philological considerations aside and focus on some content-related issues of Verbiest's rendering instead.

4.1 Origin of springs and rivers

The first section of Verbiest's text in the *QLX* deals with the origin of springs and rivers, a common topic in Renaissance literature but not part of Castelli's work. The above-listed possible Western sources for this topic that were held in the Beitang Library[90] are mostly based on Aristotle's *Meteorologica*, parts of which had already been discussed in Sabatino de Ursis's S. J. (Xiong Sanba 熊三拔, 1575–1620) and Xu Guangqi's 徐光啟 (1562–1633) *Hydromethods of the Great West* (*Taixi shuifa* 泰西水法; 1612). The subject matter was then translated into Chinese in a more systematic way by Alfonso Vagnone S. J. (Gao Yizhi 高一志, 1566–1640) in his *Investigation into the Phenomena in the Atmosphere* (*Kongji gezhi* 空際格致; ca. 1633), which kept rather close to the structure of the Jesuit Coimbra commentary (1593) on Aristotle's work.[91] Unlike Verbiest in the *QLX*, in the context of the origin of rivers and springs both the *Taixi shuifa* and the *Kongji gezhi* had focused less on the condensation of *qi* inside the mountains, but mainly described the feeding of springs by water from the sea through a network of underground channels. The comparison of the cycle of evaporation and condensation of water with that in a steamer, however, is also found in the *Taixi shuifa*, though only with regard to the formation of rain in the atmosphere.

For the corresponding chapter on rivers and streams (*jianghe* 江河; fols. 19b–21a) of his *Illustrated Explanation of the Entire World* (*Kunyu tushuo* 坤輿圖說; 1672), Verbiest had still copied verbatim some passages from Vagnone's work. The contents

90 The Beitang holdings included the main corpus of pertinent literature about the origin of rivers and springs up to 1680. Missing in this collection were only the books of Palissy (1563; 1580), Besson (1569), and Perrault (1674). These works, however, were all written in vernacular French. Despite their in parts significant advancement over Aristotle's theories, they were therefore less widespread among contemporary scholars in Europe. Similarly, the 1653 treatise by Jean François, a French confrere of Verbiest, is usually omitted from the list of important hydrologists and was absent from the Beijing library as well. Contrary to Aristotle, these French authors counted rain and snowfall among the origins of rivers. An excellent summary of related theories is found in Biswas (1970).

91 The in-depth examination of the Aristotelian explanations for natural phenomena provided in the *Taixi shuifa* is an integral part of the PhD thesis written by Sabine Kink MA within the framework of the above-mentioned Tübingen project, which also encompasses an analysis of the *Kongji gezhi* by Anna Strob MA.

of the respective section in the *QLX*, however, are much more detailed. For example, Verbiest now focuses on the mechanism that allows the water that had been delivered from the sea to rise to higher ground. The reason for this phenomenon, he says, is that inside the mountains the water gets hot enough to be evaporated, and he illustrates this by making reference to an experience had by miners:

> In extremely deep places inside the earth there are often cavities with very hot *qi* 氣 inside.[92] Whenever those who had opened mines [to look] for the Five Metals (*wujin* 五金) and precious stones (*shi* 石) had regularly tried to verify [this by going down to these places], they were not able to endure (*dangshou* 當受) this heat. This kind of hot *qi* is often preserved and retained (*cunliu* 存留) in the low-lying cavities below the roots of the mountains. Moreover, when water veins spreading everywhere in deep places inside the earth have passed through materials producing heat and fire, and they then penetrate to the hot places of the cavities beneath the roots of the mountains, their heat must be doubled.[93]

In Verbiest's depiction, it is this extremely hot *qi* which is able to make water boil and thus rise as steam inside mountains like in a pot on a stove fire. By running from the cold peak region back down along crevices as tiny droplets and leaking out through small apertures in the rock, it can then assemble into perpetually flowing fountainheads emerging on high ground. This explanation presupposes the existence of a large, ocean-fed reservoir of underground water, which had been rejected by Aristotle. Consequently, in the Coimbra commentary as well as in de Ursis's and Vagnone's renderings, such a hidden water pool is also not seen as causative for the formation of springs and rivers. This means that for the concept presented in the *QLX* Verbiest must have drawn instead on another source, namely on Athanasius Kircher's *Mundus subterraneus*, which had been published in 1665 and was probably available to him in Beijing.[94] In this work, Kircher had been the first to connect a vast subterranean network of water-carrying channels to an equally extensive system of fiery caverns down in the earth. In the *QLX*, Verbiest thus combines traditional Aristotelian thought already familiar to his Chinese readers with more recent ideas from the field, and puts this nevertheless conventional prelude in front of the innovative technical sections that follow.

92 This is clearly an allusion to the idea of an *ignis centralis* that was first formulated by Plato.
93 See fols. 24a–24b of Verbiest's text on river flooding in the *QLX*. We thank Prof. Hans Ulrich Vogel and Dr. Cao Jin for valuable hints concerning this passage.
94 Verhaeren's Beitang catalog (1949) lists both the 1665 and a 1678 edition of Kircher's *Mundus subterraneus* (no. 1919 and 1920). Books III and V of this work deal with water circulation and the origin of springs, rivers, and lakes, respectively.

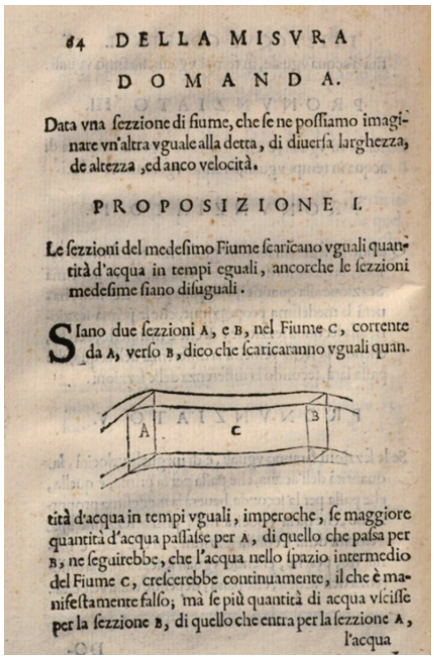


Figure 5: Kircher's view of mountain caverns being supplied with seawater (Kircher 1665, *Mundus subterraneus*, tom. I, liber V:233).

4.2 A mathematical approach to flood management

This technical part, however, first starts out with a rather conventional account of possible reasons for riverine floods as well. Thus, in paragraphs one to five of this section Verbiest speaks not only of natural causes mentioned by Castelli but also of others well known to his Chinese audience, such as a retardation of the water flow through contrary tidal waves and fierce winds at the mouth of rivers, the silting of riverbeds, or the rising of adjacent lakes and tributaries after heavy rainfalls. Only then does he turn to less established ideas by blaming man-made impediments to the current like lock boards or bridge piers, which, according to Castelli, have the same effect as if the whole river course would be narrower by nature. Moreover, Verbiest here already offers a first, strictly quantitative assessment of the dredging of riverbeds as a possible countermeasure to flooding.

This increasingly innovative approach is continued in the subsequent, highly theoretical third section that introduces important propositions connecting flow velocity to water height. It could be clearly established that Verbiest took many of the passages of this section directly from Castelli's work *Della misura dell'acque correnti*. In doing so, the Jesuit more or less follows the scholastic pattern of reasoning of his template. Thus, he usually begins his explanations with a general statement (*fan* 凡, *you* 有) or by posing a particular problem (*sheling* 設令), then he introduces the related principle of the proposition (*tili* 題理), establishes exemplary assumptions with distinct numbers (*jiaru* 假如), and subsequently applies the method [of computation] (*fa* 法) to



84 DELLA MISVRA

DOMANDA.

Data vna ſezzione di fiume, che ſe ne poſſiamo imaginare vn'altra vguale alla detta, di diuerſa larghezza, de altezza, ed anco velocità.

PROPOSIZIONE I.

Le ſezzioni del medeſimo Fiume ſcaricano vguali quantità d'acqua in tempi eguali, ancorche le ſezzioni medeſime ſiano diſuguali.

Siano due ſezzioni A, e B, nel Fiume C, corrente da A, verſo B, dico che ſcaricaranno vguali quan.

tità d'acqua in tempi vguali, imperoche, ſe maggiore quantità d'acqua paſſaſſe per A, di quello che paſſa per B, ne ſeguirebbe, che l'acqua nello ſpazio intermedio del Fiume C, creſcerebbe continuamente, il che è manifeſtamente falſo; mà ſe più quantità di acqua vciſſe per la ſezzione B, di quello che entra per la ſezzione A, l'acqua

Figure 6: Propos. I of "Demostrazioni geometriche": Sections of a river that carry equal quantities of water in equal times although these sections are unequal (Castelli 1660, *Della misura dell'acque correnti*, 64).

these examples, in some cases even using numbers exactly identical with those found in Castelli (for example, in propositions 1 and 8). Moreover, in the context of heightening dikes to prevent fluvial inundations (section four), Verbiest borrows Castelli's example of the Italian river Po by referring to it in Chinese as "Baduo River" 巴多河, derived from the Po's Latin name Padus.[95] Other passages show the influence of Castelli in a more indirect manner, either through modifications of the source text or by the inclusion of additional numerical examples for a better understanding of the propositions.

The continuity equation, Castelli's most important contribution to river hydraulics, which states that the flow of a river is equal to the area of the cross-section multiplied by the velocity, is incorporated in Verbiest's propositions 2, 3, and 7 of the section on the relationship between flow velocity and water height, without, however, dwelling on the critical question of how to assess velocity at all. We will come back to this issue below. Apart from that, Verbiest here employs the same main assumptions as Castelli throughout his text, namely, rectangular cross-section of the river, uniform flow velocity over the entire cross-section, and velocity increases with the square of the height.[96] The latter assumption is actually false because the velocity increases

95 Surprisingly, however, Verbiest makes no reference at all to the Roman Tiber River, which is mentioned in Castelli's work repeatedly. Moreover, its great flood of 1660 had inspired the books on modern quantitative flood control by Bonini (1663) and Michelini (1664). The remedies against such inundations proposed by the abbot Bonini (1612–1680), who had himself witnessed this deluge, were partly based on Castelli. For example, Bonini adopted Castelli's theory that flow velocity increases with the square of the water height while taking a more critical stance in other questions. The priest and mathematician Famiano Michelini (1604–1665) was, like Castelli, a follower and admirer of Galileo Galilei (1564–1642). He took a special interest in hydraulics and the active defense against erosion of riverbanks. While the book by Bonini might have already been available to Verbiest, that by Michelini was obviously donated to the China mission only in 1689. Both titles are listed in Verhaeren (1949).

96 The assumption that velocity increases with the square of the height is stated in Castelli (1660, lib. II, proposizione IV, theor. II, corollario II), as well as independently in Barattieri (1656/1663, lib. 6).

proportionally with the square *root* of the height, as was stated for the first time by Dechales (1674, tom. II, tract. XV, propositio L). Since Verbiest does not report the square root method, it appears that he had not seen Dechales's *Cursus seu mundus mathematicus* at the time of his writing. On the other hand, just like Verbiest in his appendix to the first paragraph, Dechales states that a round vessel *or* a square vessel could be used to collect rain, a fact reported neither by Castelli nor Riccioli (1661). Hence, Verbiest might nevertheless have consulted the book by Dechales, but with regard to the author's mathematical expertise relied on Castelli.

Finally, it should be noted that trapezoidal cross-sections of rivers or canals with dikes were frequently employed in early Chinese books on mathematics or river conservancy, so Verbiest's Castelli-based approach was not that unfamiliar to Chinese scholars at first sight. Moreover, even though in China volume calculations were not applied to water flows, they were most important there for determining labor requirements in the case of canal excavation or dike construction. Wagner (2012) gives the example of a canal with varying trapezoidal cross-sections taken from *Comprehensive Discussion of River Flood Prevention* (*Hefang tongyi* 河防通議), a lost eleventh-century treatise on river conservancy reconstructed and edited by the Yuan scholar Shakeshi 沙克什 (1278–1351) in 1321. Other examples in this context refer to dike construction, one with a simple trapezoidal section taken from *The Nine Chapters on the Mathematical Art* (*Jiuzhang suanshu* 九章算術; first century CE), another with a varying trapezoidal section taken from *Continuation of Ancient Mathematics* (*Jigu suanjing* 緝古算經; after 626 CE) by the astronomer Wang Xiaotong 王孝通 (sixth–seventh century CE) (Wagner 2013). Finally, Qin Jiushao 秦九韶 (ca. 1202–1261) determines in *Mathematical Treatise in Nine Sections* (*Shushu jiuzhang* 數書九章; 1247), problem VII.4, the volume of a wedge with a trapezoidal base and one sloping side for a canal excavation (Libbrecht 1973, 113–119). The calculations for more complicated volumes with varying trapezoidal sections were in some cases incorrect, however, as noted by recent authors.

After the very abstract and mathematical explanations of the third section, which probably were difficult to comprehend for most Chinese readers, Verbiest returns to more practical considerations in the last subchapter of his text, which deals with suitable precautions against fluvial inundations. At the center of these considerations is the question whether it would make more sense to heighten existing dikes or to dig new drainage canals in order to prevent fluvial inundations. While with the canals Verbiest's main focus is on their appropriate design, he makes quite some effort to convince his audience that in certain situations the dike option is to be preferred. As proof of this assessment, he even appends an extra paragraph with the calculation of a hypothetical example. But what is more, in this fourth and last part of his text he takes the opportunity to present to his Chinese readers some noteworthy examples of known Western science and technology, highlighted below.

4.3 Pound-lock with miter gate

Thus, in the third paragraph of section four Verbiest mentions briefly the safety problems posed for boats by traditional sluices—actually roof-like double slipways—in rivers and canals, which can be found in China to this day. Louis Le Comte S. J. (Li Ming 李明, 1655–1728) described such contemporary locks in *Nouveaux mémoires sur l'état present de la Chine* (1697),[97] explaining their danger to boats as shown in Figure 7. In his text in the *QLX*, Verbiest proposes instead a new type of lock or sluice (*zhakou* 閘口) with a laterally moved two-winged miter gate (*men liang shan* 門兩扇), which is similar to the one illustrated in Zonca (1607, carte 10). Verbiest may have seen similar sluices in his home province of Flanders, which were developed independently from those in northern Italy,[98] and obviously he took them to be an advanced technology from the West. However, Needham (1963) and Needham et al. (1971)[99] presume that pound-locks (without miter gates) had actually been invented in China already during the Song dynasty, namely around 984 by the official Qiao Weiyue 喬維嶽 (926–1001). This would be much earlier than in Europe, where the first pound-lock of this kind was reportedly

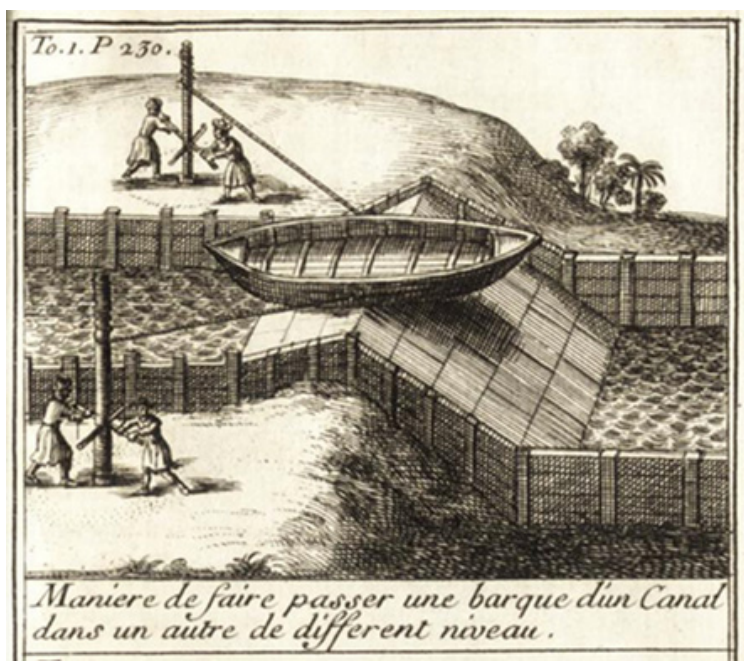


Figure 7: Example of Chinese ship-lock (Le Comte 1697, tome I, illustration to p. 230).

97 This description is found in Le Comte (1697, 159–202, "Lettre IV. A Monsieur Le Comte de Creci. Du climat, des terres, des canaux, des rivieres & des fruits de la Chine," esp. 180–188).
98 See Stevin (1617, 4). See also chap. 4 "Hydraulic Engineering" of Forbes et al. (1966).
99 Chap. (v) "Sluice-gates, locks and double slipways" is found in Needham et al. (1971, Section 28 (f) (9) "Techniques of Hydraulic Engineering," 344–365).

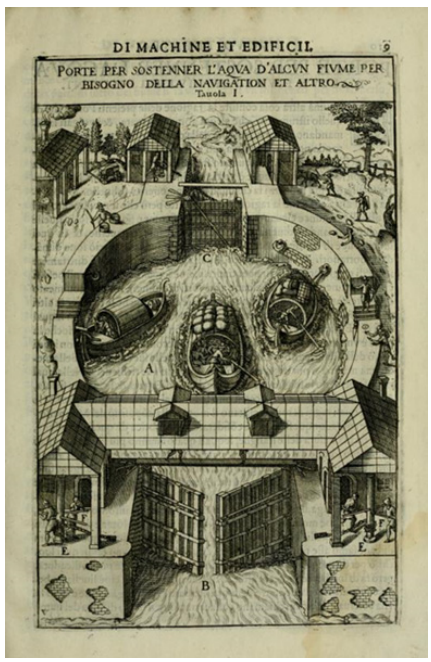


Figure 8: Gates to contain water of a river for the purpose of navigation. Letter B shows the triangular upper gate in two parts (Zonca 1607, 9, carte 10).

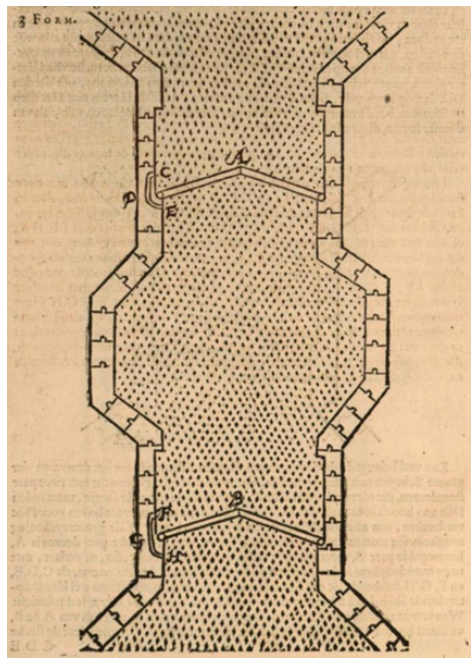


Figure 9: Plan view of Stevin's (1617, 4) pivotted sluice lock.

built in 1373 at Vreeswijk in Holland. The invention of the European miter gate is attributed by Needham et al. (1971, 358) to Leonardo da Vinci in 1497 at the latest. However, by the time of Verbiest the use of the original pound-locks had apparently fallen into oblivion in China, and officials may not have been aware of them anymore. Interestingly, the first modern pound-lock was built in China only in 1933 in the Xiaoqing River 小清河 near Jinan, Shandong, for ships up to 200 tons (Haasler 1939).

4.4 Windmills for drainage (drainage mills)

At the end of the same paragraph, Verbiest also refers to the use of windmills (*fenglun* 風輪, literally "wind wheels") to drain flooded fields, without, however, providing any information about the type of water-lifting device to be employed to this end, whether a simple scoop-wheel, a Chinese square-pallet chain-pump, or perhaps an Archimedean screw, which was first introduced to China in the *Taixi shuifa* in 1612 and to which Verbiest obviously refers here when he mentions another "discourse on hydromethods" (*shuifa zhi lun* 水法之論). This treatise indeed had recommended the use of wind power as a driving force for the screw pump, without, however,

mentioning windmills explicitly. Similarly, it is not clear which particular type of windmill Verbiest had in mind when he wrote his text on floods for the *QLX*. In Europe, horizontal-axle windmills for drainage, which usually employed scoop-wheels, had been built in the Low Countries since about 1408 (Davids 2008, 72–78), and Verbiest, a native of Flanders, might have seen this standard type in operation.

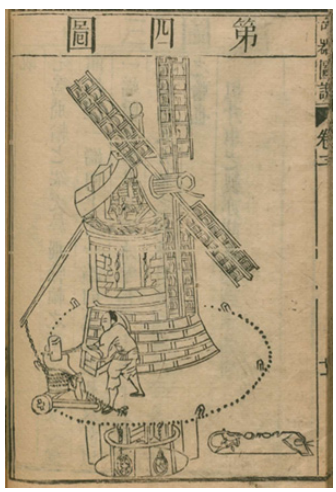


Figure 10: Horizontal-axle windmill from Ramelli (1588) in *QQTS* (1628, *juan* 3:fol. 11b).

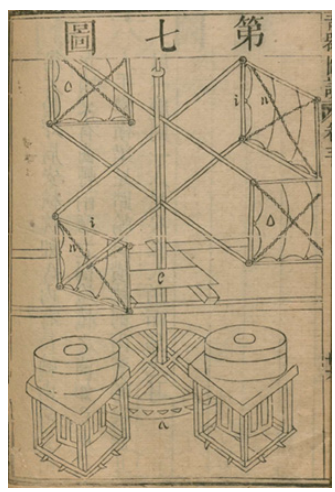


Figure 11: Vertical-axle windmill from Veranzio (1615) in *QQTS* (1628, *juan* 3:fol. 36b).

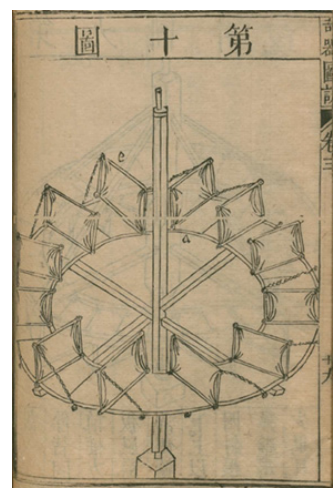


Figure 12: Vertical-axle windmill from Veranzio (1615) in *QQTS* (1628, *juan* 3:fol. 39b).

First depictions of European windmills had been presented to the Chinese in the *Record of the Best Illustrations and Descriptions of Extraordinary Devices of the Far West* (*Yuanxi qiqi tushuo luzui* 遠西奇器圖說錄最 [short *QQTS*]; 1628) composed by Wang Zheng 王徵 (1571–1644) and the German Jesuit Johannes Schreck alias Terrentius (Deng Yuhan 鄧玉函, 1575–1630). This book contains in its third *juan* an illustration of a horizontal-axle windmill for water lifting. This illustration had been copied by Schreck from Ramelli's (1588, plate 73) *Le diverse et artificiose machine*, which was available in the Beitang Library. In addition, in the *QQTS* two Western vertical-axle windmills are shown, this time reproduced from the 1615 machinery book *Machinae novae* by Fausto Veranzio, which Schreck had brought with him from Europe. As this type, however, is not explained in the *QQTS* with regard to its application, it rather appears to be an ornamental technical gadget.

In China, however, such windmills had already been known at least since the Southern Song dynasty (1127–1279), with the vertical-axle version enjoying the widest application for irrigation and drainage purposes in agriculture.[100] These mills were

100 For a brief overview of distinct types of Chinese windmills, see Zhang (2009).

noticed with interest by a Dutch Embassy to the Qing court, which took place in 1655–1657. About his observations while traveling on the Grand Canal in central Jiangsu, Joan Nieuhof, steward to the ambassadors, reports:

> They boast likewise of store of Wind-mills, whose Sails are made of Mats. The great product of the Countrey consists of Rice, which the Peasant stands obliged to look after very narrowly, lest it perish upon the ground by too much moisture or too much heat and drought; so that their Eyes are continually upon the Crop, otherwise it suddenly withers to nothing, or a small encrease. The Wind-mills therefore are to draw out the Water in a moist season, or to let it in as they think fit, to keep their hopes from burning up in a dry and hot season; so that by this means the *Chineses* enjoy twice a year a rich Harvest of Fruits. . . . The Royal Navigation [that is, the Grand Canal] runs quite through the Countrey up to the very Walls [of Paoing][101] . . . by which means they water their grounds in a dry season. This part of the Countrey is also full of drainage Mills, to drain upon occasion. (Nieuhof 1669, 91)[102]

It is rather unlikely that Verbiest had knowledge of this account. But since he had traveled along the same route to the north in 1659, he might have seen with his own eyes these Chinese drain mills, which were much lighter as well as easier and cheaper to build, move, and operate than the heavy, massive Dutch drainage mills.[103] It is also possible that he became aware of them through a description found in an account about China by the Spanish Dominican missionary Domingo Fernández Navarrete (Min Mingwo 閔明我, ca. 1610–1689), who says: "In our way to the Imperial City . . . we saw another odd Invention for drawing of Water, which we



Figure 13: Vertical-axle windmills in Baoying 寶應 (Nieuhof 1665, image from p. 119 of the 1693 edition).

101 That here Paoying or Baoying 寶應, located north of Lake Gaoyou 高邮 in Jiangsu, is meant becomes clear from the illustration on the next page that shows the city walls of Baoying and several windmills along the canal, without, however, depicting any drainage or irrigation devices.
102 The German sinophile and polymath Gottfried Wilhelm Leibniz (1646–1716) was so impressed by the illustration in Nieuhof that he proposed a Chinese vertical-axle windmill in combination with an Archimedean screw pump to dewater the German Harz mines. However, his prototype mill used wood instead of the Chinese sail cloth for the vanes. Therefore, it was too heavy and inefficient at low windspeeds, causing the failure of his water management scheme. See Gottschalk (2007, 109–124).
103 Interestingly, reports of 1687 imply that Verbiest himself had erected a windmill for water lifting at the imperial court in Beijing, but probably only after he had completed the *QLX*. See Koenig (2018, 45–46).

could not but admire and laugh at. These sort of Mills stood in a Plain upon the flat Ground, and were full of Sails made of Mat, as is usual in that Country; and the Wind twirling them about, they flew like Lightning, and drew abundance of Water without being attended by any body" (Navarrete [1676] 1732, 33).[104] In any case, in the context of the *QLX* this driving mechanism for drainage devices seems to have appeared technically meaningful and thus remarkable to Verbiest.[105]

4.5 Rain gauge

In his sample calculation about the advantages of heightened dikes over drainage canals in the case of an abrupt rising of the water level (appendix to section 4 of his text) Verbiest, among other things, describes the use of a rain gauge to determine the amount of precipitation within a fixed period of time—an important method to adjust river control measures. The information about this instrument, which is simply described here as a square or a high and round device (*fang qi huo gaoyuan zhi qi* 方器或高圓之器), is obviously taken from Castelli's letter of 1639 to Galileo Galilei, which was included in the second (1639) and third (1660) editions of *Della misura dell'acque correnti*. As reported extensively in the hydrological literature, Castelli's letter to Galileo is the first mention of the application of a rain gauge in the West.[106] Comparison with the figures of recent floodings [107] shows that



Figure 14: Rain gauge pedestal with a restored rain gauge of the Gyeongsang-do Provincial Office in Daegu (Korean National Treasure No. 330), crafted in the Gengyin year of Qianlong's reign (1770) (picture by Koh Gyeongmin, https://fr.wikipedia.org/wiki/Cheugugi).

104 This book was available in the Jesuit library in Beijing (Verhaeren 1949, no. 3819). Like Nieuhof, Navarrete hints at the vanes of the mills, which were often made of thin bamboo strips ("mats") and shaped like the sails of traditional Chinese boats.

105 Although windmills were already well known in China, Verbiest's description of their usage in combination with novel Western hydraulic devices might have attracted the attention of Emperor Kangxi. A brief note of April 30, 1687, in the diary of Frans Flettinger, member of the Fourth Dutch VOC Embassy to Beijing (1685–1687), says that heavy rain and fierce winds had destroyed a windmill constructed by the Jesuits in the royal court. See Wills (1985, 289).

106 See, for example, Biswas (1970) or Strangeways (2007, 143).

107 During the inundations in Germany in July 2021, 150 mm of precipitation were measured

Verbiest's assumption that after 24 hours of heavy rain the water inside the device would add up to 100 *fen* (about 310 mm) was quite realistic and thus probably based on actual measurement.

Similar to the pound-lock, Needham and Wang (1959) report that rain gauges were invented and applied in China much earlier than in Europe.[108] Thus, the first account of this instrument is to be found in section 2, "Heavenly Phenomena" (*tianshi lei* 天時類), of Qin Jiushao's *Mathematical Treatise in Nine Sections* (*Shushu jiuzhang* 數書九章; 1247) (Libbrecht 1973, 474). This "crater lake basin" (*tianchipen* 天池盆) gauge had the form of a conical jar and is reported to have been available in all prefectures and departments at that time. Moreover, Qin Jiushao says that in order to deliver comparable results this jar needed to be of a distinct shape, and he gives a short example of how to calculate the volume of fallen rain from the level of water collected in the jar. Beyond that, however, he provides no further information about the concrete application of the device. After the Song dynasty was overthrown by the Mongols in 1268, knowledge about such rain gauges was apparently forgotten in China.[109]

In Korea, a cylindrical rain gauge was introduced by the engineer Jang Yeong-sil 蔣英實 (1390–after 1442) only two centuries later, namely in 1441 during the reign of King Sejong the Great 世宗大王 (r. 1418–1450) of the Joseon dynasty 大朝鮮國 (1392–1897) (Kwun 2006). But what is decisive is that, already a year later, this "rain measuring device" (*cheugugi* 測雨器) was standardized and distributed to all provinces together with binding instructions for use. The measurements were to be reported from the periphery to the main meteorological office at the end of every heavy rain event. That way, an early warning system in the form of a downright network for the observation of rainfall throughout the country was created for the first time. After the interruption of measurements during the turmoil of the sixteenth and seventeenth centuries, this rain gauge network was reactivated in 1770 by King Yeongjo 英祖 (r. 1724–1776) and continued right up to the early twentieth century. Thus, Korea has probably the world's longest history of systematic rainfall recordkeeping, whereas in China reports to the emperor and responsible officials about critical amounts of precipitation did exist but continued to lack quantitative preciseness during the whole Qing period.[110]

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within 15 hours, while for a flooding in Zhengzhou, China, at about the same time, 450 mm within 24 hours were reported.

108 See Needham and Wang (1959), Section 21 "Meteorology," (d) "Precipitation," 471–472 on rain gauge.

109 According to Udías (2003, 277–281), in China the first regular measurements with modern rain gauges or pluviometers were commenced by French Jesuits at their newly founded meteorological observatory in Zikawei (now Xujiahui 徐家匯) near Shanghai only in 1873. This observatory revived somewhat the ancient Jesuit tradition of the Imperial Observatory in Beijing.

110 Already since the Ming, water levels along the upper reaches of the Yellow River had been regularly gauged, and a rise had to be reported by way of express couriers to officials in charge of the lower reaches. However, information about rainfalls, which is no less important for timely

4.6 Economic analysis of alternative flood control options

In corollary 13 of the first book of his *Della misura dell'acque correnti*, Castelli had raised the question whether—at a time of great flooding—the opening in the Po River of a sluice to divert the water into another channel would have a long-term beneficial effect. In his appended calculation to assess the advantages of heightened dikes over drainage canals, Verbiest addresses this question as well, and advises to additionally consider the expenditures for the two proposed options, namely, to conduct a crude cost-benefit analysis.

Verbiest's approach can be traced back to Benedetto Castelli's 1639 letter to Galileo Galilei about his studies at Lake Trasimeno near Perugia in Italy.[111] This lake is of volcanic origin, with a surface area of about 120 km^2 and no natural outlets. Since Etruscan and Roman times, depending on meteorological and climatic conditions, the high variability of its water level had repeatedly caused floods and droughts, and therefore provoked human intervention to regulate and control the situation through hydraulic works (Burzigotti et al. 2003). Based on the amount of rain found in a vessel, that is inside the first Western rain gauge, Castelli had determined how much the lake would increase in height after a given amount of rainfall. He then proceeded to calculate the rate at which the water level would fall again after a drainage canal was built. To this end he compared the lake's total surface area with the cross-sectional area of the artificial outlet, just as Verbiest does in the *QLX*.

But what is more, the practice-oriented engineer Verbiest extends Castelli's analysis through a concrete numerical example. Given the rise of the water level of a lake due to extraordinary precipitation, he asks whether in this case it would in principle be more effective for flood control to dig a new drainage canal or to raise the height of the lake dikes for intermediate storage of the excessive water. Moreover, by using the volumetric amount of earth moved in both cases as an indicator, Verbiest's calculation also shows that in his example the cost for constructing the new canal would be nine times higher than with the dike heightening option.[112] Taking additionally into account the obviously more complicated and challenging task of the realization of a new canal, he comes to the final conclusion that raising the lake's dikes would be much more beneficial than the drainage option. This leaves the impression that similar cost-benefit analyses for flood control, if any, in Verbiest's eyes were not conducted sufficiently in China at the time. Thus, as already mentioned, he had complained in his internal letter

countermeasures against inundations, was usually limited to general remarks about their intensity and duration. See Amelung (2000, 245–247).

111 See Castelli (1660, lib. I., appendice XII and XIII, and "Copia di lettera al Sig. Galileo Galilei Primo Filosofo dell Serenissimo Gran Duca di Toscana," June 18, 1639).

112 Instead of the Chinese earthwork unit *tufang* 土方 (see below), Verbiest more precisely uses here "cubic [meters] of soil in *chi*" (*lifangtu zhi chi* 立方土之尺).

of 1681 about the "incredible amounts . . . spent annually" and in vain for inadequate large-scale river conservancy projects in China, and in his *Astronomia Europea* (1687) he continued to denounce the related waste of money when he wrote, in reference to his own hydraulic work, that in 1672/1673 he had first built a smaller model of a water wheel as a try-out "so as not to throw the Emperor's money needed for these expenses into the water" (Golvers 1993, 118–119) in case it malfunctioned. His text on the prevention of fluvial flooding in the *QLX* was now a good opportunity to once again point out the importance of this issue and promote the budget-conscious Jesuit approach of combining the application of scientific river hydraulics with economic analysis, rendering potentially huge savings.[113]

5 Reception and possible influence of Verbiest's text

In the 1680s and 1690s, following another devastating inundation by the Yellow River in 1676, Emperor Kangxi had initiated the most extensive and expensive flood control project ever undertaken in the history of China, especially targeted at the sensitive stretches of the Grand Canal between the Yangtze and Huai Rivers (Gandar 1894). However, since the available river conservancy knowledge at the time was still mostly empirical and had little rational foundation, many project proposals remained controversial and were subject to intense debates at the court. The lake example described by Verbiest in the *QLX* may allude to such a controversy. In fact, the dimensions selected in his sample calculation strongly suggest that he might have had in mind the Hongze Lake 洪澤湖 west of Huai'an 淮安 (or perhaps the Gaoyou Lake 高郵湖 north of Yangzhou 揚州), which was about to be harnessed for the flushing of the Yellow River at the nearby junction with the Grand Canal.

Thus, it seems that in the *QLX* Verbiest deliberately took the opportunity to introduce to China the most recent Western theories about effective flood control measures at a time when he could expect to knock on an open door with his expertise. To this end, Castelli's work *Della misura dell'acque correnti* was indeed the most suitable and most advanced text available to him, unique even in Europe because of its innovative scientific approach. Verbiest with his longstanding interest and experience

113 Amelung (2022) examines the role of fiscal considerations behind river control measures in the course of the Qing. He comes to the conclusion that it was less ecological constraints or inefficiency together with corruption that would make costs related to hydraulic engineering along the Yellow River and the Grand Canal explode during the late imperial period. Rather, the Kangxi and then also the Qianlong emperors' strong commitment to the Confucian task of "taking care of the 'economic needs of the state and the livelihood of the people' (*guoji minsheng* 國計民生)" (ibid., 34) precluded an overburdening of the farmers and resulted in an abandonment of corvée labor for large-scale river control measures. However, both rulers failed to raise additional sources of money for the wageworkers hired instead, which led to a steadily increasing fiscal deficit at that time.

in hydrotechnical questions had quickly recognized the importance of this treatise (Koenig 2018). Moreover, he had grown up in the Low Countries, which, together with Italy, had pioneered the development of modern hydraulics in Europe. The construction of dikes along rivers and the seacoast or the operation of dams and sluices for flood control and navigation were thus familiar topics to Verbiest, as was the use of windmills for water lifting and drainage purposes.

Since the *QLX* was never printed, there is no written information available on the degree of reception of Verbiest's text on flood control among experts in China. Therefore, only educated guesses can be made here about its probable further impact. In any case, successful river management was vital for both the well-being of the people and the revenues of the agrarian Chinese state, and thus of utmost importance for the destiny of the dynasty. That explains why in this field the demonstration of imperial patronage counted among the main legitimizing tasks of any ruler, and Kangxi was no exception in this regard. But in his case it went much further, as Verbiest had taught the emperor Western mathematics and—due to Kangxi's passion for Western sciences in general and water conservancy in particular—probably also subdisciplines such as hydragogics, hydrostatics, and hydraulics for many years. Thus, the emperor is indeed reported to have visited river works time and again and to have given instructions directly on site on these occasions. Therefore, it is quite possible that despite his eventual rejection of the *QLX* he might have read Verbiest's short treatise on flood management out of personal interest. However, the selective and appropriative handling of the information obtained from this and other sources, while on the whole keeping to traditional practical approaches of water conservancy, is characteristic of the way Western knowledge began to be increasingly dealt with in China at that time.

An example highlighting this eclectic appropriation are reports that credit Kangxi with having personally specified a method for measuring water flow rates (Li 1932, 69; Zhou 2015, 355). At first sight such remarks appear questionable and part of the usual glorifying rhetoric, but without doubt he was much more directly involved in astronomical, mathematical, and also hydraulic affairs than other Chinese emperors.[114] This commitment went so far that he repeatedly summoned to the court selected high-ranking and renowned scholar-officials to discuss related topics with them and thereby display his skills in the subject matter. Thus, on one of these occasions in 1692, he demonstrated to his audience gathered at Qianqingmen 乾清門 (Gate of Heavenly Purity) in the Forbidden City his expertise in questions of what we today would call basic fluid dynamics. Here, the emperor stressed the importance of precise calculations

114 For Kangxi's lifelong efforts to integrate Western mathematical methods with the Chinese tradition and the resulting "imperial mathematics" as a body of universal knowledge, see the comprehensive survey by Jami (2012).

and took as an example the crucial determination of the discharge rate, that is the quantity of water running through the sluice gate of a river within one day and night.[115] The method put forward by him was to first measure the width of the lock opening and to then assess how much water ran through it within one second. From this the amount flowing through it within twenty-four hours could be inferred.[116] In essence, this is correct and strongly reminds us of Verbiest's account in the *QLX*, even though Kangxi's specifications are somewhat imprecise, as instead of the cross-section of the river only its width is considered here.

At the same time, this incomplete measuring instruction probably indicates the importance Kangxi attached to Chen Huang's above-mentioned idea to relieve the lower reaches of the Yellow River through the drainage of peak flow waters. Apparently, Chen himself had already realized that in order to apply this method in a controlled manner it was necessary to somehow quantify his approach. Thus, in his *Record of a Dialogue on River Flood Prevention* (*Hefang shuyan* 河防述言; 1689),[117] he expressed as follows what he had intuitively understood through observation about the relationship between flow velocity and water drainage:[118] "When the water [of a river] flows quickly, it is like a hurriedly walking man who can manage 200 *li* a day. When it flows slowly, it is like a leisurely walking man who can manage 70 or 80 *li* a day. Now, with the soil-cube method (*tufang zhi fa* 土方之法) that takes water of one *zhang* in length and breadth and one *zhang* in height as one [water-]cube, one calculates how many cubes this river manages [in a day]. From what remains after it has taken up this [amount] one calculates [how much needs] to be drained

115 The official depiction of these events, which rather than discussions were "Sagely Instructions," is documented in the *Imperial Diaries* (*Qiju zhu* 起居注) and then the *Veritable Records of the Qing Dynasty* (*Qing shilu* 清實錄). See Jami (2012, 229 and 233). The above example had already been recorded in Li Di's 李迪 (1986) seminal article about early efforts to calculate water discharge rates during the Ming and Qing periods.

116 See Li (1986, 371), where he quotes the description of this event given in Yu Jin's 余金 *New Words in a Prosperous and Peaceful Reign* (*Xichao xinyu* 熙朝新語; 1818), which also contains the citation of Kangxi's instruction: "又曰：算數精密，即河道閘口流水亦可算晝夜所流分數。其法先量閘口闊狹，計一秒所流幾何，積至一晝夜，則所流多寡可以計矣" (edition *Xuxiu Siku quanshu* 續修四庫全書, *juan* 5, fol. 2b).

117 This work had first been published in 1689—and thus three years before Kangxi's instructions—as a part of Jin Fu's *Strategies for River Control* (*Zhihe fanglüe* 治河方略).

118 The volumetric flow rate is the volume of fluid which passes a cross-section per unit of time and today is usually given in cubic meters per second (m3/s). It can also be defined as the product of flow velocity and cross-section area, but either way time measuring is indispensable.

off."[119] In other words, Chen correlated his vague idea of velocity as distance covered per day with that of volume managed per unit of time, which in essence is nothing else than the flow rate.

When one compares this explanation with that of Emperor Kangxi, the latter appears to be an attempt to systematize and render Chen's genuinely Chinese approach more precisely according to Western standards of calculation as transmitted by the Jesuits. What is strikingly missing, however, not only in Kangxi's instruction but also in Verbiest's text on flood prevention in the *QLX*, is any indication of *how* to measure the crucial time intervals in a reasonably accurate manner, without which the whole calculation of flow rates remained a very rough estimate if not entirely impracticable, as the procedure used by Chen Huang suggests. That in view of the increasingly precarious situation of centrally coordinated river engineering measures quite some thought must have been given by the emperor and his advisors to this unsolved issue becomes discernible from a Jesuit textbook manuscript written probably by Verbiest's assistant Antoine Thomas[120] for the emperor's lectures in 1690.[121] This manuscript is entitled *Explanation of the Proportional Compass* (*Biligui jie* 比例規解) and, among other things, uses the calculation of the discharge-related cross-section of a river as an

119 *Strategies for River Control, juan* 9, fol. 40a: "水流速，則乃急行人，日可行二百里，水流徐，則乃緩行人，日可行七八十里。即用土方之法，以水縱橫一丈、高一丈為一方，計此河行幾方。然後受之，其餘者皆以計宣泄之。" *Tufang* 土方 is an old Chinese unit used in earthwork to calculate the workload. One *tufang* equals one cubic meter of soil that is dug out or piled up. (挖土、填土、运土的工作量常都用立方米计算，一立方米称为一个土方。这一类的工作叫土方工程。) This method was described by Zhang Guowei 張國維, who in his *Complete Treatise on Water Conservancy of Wu* (*Wuzhong shuili quanshu* 吳中水利全書; 1636), *juan* 16, fol. 98a, cites Nie Shaochang's 聂绍昌 *Tiaoshang junzhu shiyi shen* 条上浚筑事宜申 of 1609: "汛工當論土，方不論丈尺。**土方之法，每去土，四旁上下各一丈為一方**。每方計土一千尺，該一千六百挑。每方用起工，一人名，曰一置擔土，二人名，曰二挑。" For Chen Huang's basic idea about how to calculate the flow rate, see also Amelung (2000, 248–249), who points out that Chen's method—if at all—seems to have been little in use, as it is not mentioned in any of the handbooks on river engineering scoured by him. Chen's approach is, moreover, mentioned in the subchapter on flow survey (with examples of flow measurement) in Zhou (2021, 168–171).

120 For Thomas's life and work, see Golvers (2020). Thomas had reached Beijing in 1686, where he supported Verbiest in his astronomical and engineering activities. After Verbiest's death in 1688, he was appointed his interim successor as head of the *Qintianjian* and henceforth had access to all remaining papers and documents written by Verbiest as well as to his personal specialized library, which also stored Western astronomical and measuring instruments. The application of some of these instruments is described by Thomas in different treatises.

121 A "libellum de usu circini proportioniis ac de logarithmorum usu" (booklet about the use of the proportional compass as well as the use of logarithms), written by Antoine Thomas, is mentioned in the Jesuit "Annotationes annuae intermissae et postmodum continuatae" for the years 1686–1694 (ARSI, JS 149, f. 532r.). See Golvers (2020). On the upper end of this folio is written "**Anno 1690**, Januari 15," which supports the assumed authorship and time of the creation of the *Biligui jie*.

example for the use of this mathematical instrument (Jami 2012, 361–362).[122] The accompanying illustration suggests that Thomas might have been inspired for his sample by Verbiest's partial translation of *Della misura dell'acque correnti*.



Figure 15: Measurement of River Flow in *Biligui jie* 比例規解 (1690) (Bibliothèque municipale de Lyon, Collections anciennes et spécialisées, MS 75-80 D, fol. 4b).

122 On the cover of this manuscript textbook, which is stored in the Bibliothèque Municipal de Lyon, the title in French is given as *Usage de instrument par rapport a la geometrie*. The Chinese title *Explanation of the Proportional Compass* had been borrowed by the author of this textbook from a work written by Giacomo Rho S. J. (Luo Yagu 羅雅谷, 1592–1638) in 1630, itself adapted from Galileo Galilei's *Le operazioni del compasso geometrico e militare* (1606). This original *Explanation of the Proportional Compass* as *juan* 21 was part of the *Books on Calendrical Astronomy according to the New Western Method* (*Xiyang xinfa lishu* 西洋新法曆書; 1645), which was available to the composers of Kangxi's *Essence of Numbers and Their Principles*, see below. Interestingly, chapters 39 and 40 of this latter compendium are also entitled *Explanation of the Proportional Compass* and, even more comprehensively than their namesake, deal with the proportional compass called *bilichi* 比例尺 (literally "proportional ruler") in the text. For Rho's 1630 *Explanation of the Proportional Compass*, see CCT database at http://heron-net.be/pa_cct/index.php/Detail/objects/237 and Jami (2012, 193, 321, 357).

In this illustration, the critical measures of the river's cross-section, namely its width (*kuan* 寬) and depth (*shen* 深), are precisely marked. Again, the method of timekeeping is not discussed in the textbook example (Jami 2012, 362), but the drawing shows a wooden stick flowing on the river somewhat downstream, which was perhaps required to determine the flow velocity, at first sight seemingly in the rudimentary way described by Chen Huang, namely through comparison with the speed of a man walking more or less hurriedly along the river bank in parallel.[123] In this context it is quite telling that the need for precise time measurement *is* actually mentioned in the 1660 edition of Castelli's *Della misura dell'acque correnti*, where in the "Libro secondo" he proposed the use of a pendulum device for this purpose.[124] Verbiest, however, had not included this essential component, which is indispensable for the practical application of the continuity equation, in his Chinese rendering of Castelli's ideas. Instead, he had discussed the second-pendulum in connection with a quite different topic, the ballistics of cannon balls, and well ahead of his text on flood prevention.[125] While in another Jesuit textbook for the emperor's lectures this device again crops up in a similar context in the early 1690s,[126] it took about three more decades until it finally found its way into officially endorsed Chinese hydrodynamics, namely in the monumental and richly illustrated work *Essence of Numbers and Their Principles Imperially Composed* (*Yuzhi shuli jingyun* 御製數理精蘊), which was composed on the orders of Kangxi and first printed

123 Actually, Antoine Thomas had quite some knowledge about the intricacies of measuring the flow velocity, which actually decreases from top to bottom. In his *Synopsis mathematica*, a textbook he had written during his sojourn in Coimbra (1678–1680) before his departure for the East Indies, he explains the use of the hydrometric rod, a wooden stick held in an upright position by a weight when floating in water, and assuming an inclined position when exposed to the current of a river.

124 See Castelli (1660, lib. II, supposizione), where the author deals with the comparison of the flow velocity of two rivers. The pendulum method for timekeeping had been taught to Castelli by his teacher Galileo Galilei.

125 See Dudink and Standaert (1999, 25–27). The sixth subsection of *juan* 8 of the *QLX*'s *Reasoning in Physics* takes up corresponding parts of an earlier work of Verbiest, the above-mentioned *Treatise of Newly Constructed Astronomical Instruments at the Imperial Observatory* of 1674. There, the pendulum timepiece is described by the Jesuit more pictorially as a "ball hanging on a string" (*chuixian qiu* 垂線球) and swinging back and forth (see Verbiest 1674, *Xinzhi lingtai yixiang zhi*, *juan* 4, fols. 36a–36b of the *Xuxiu SKQS* 續修四庫全書 edition). Nicole Halsberghe has pointed out the inclusion of this timekeeping instrument in the *Treatise of Newly Constructed Astronomical Instruments at the Imperial Observatory* already in 1994. It is also mentioned in Jami (2007, 157–158). Most probably, Verbiest's source had been Giovanni Battista Riccioli's S. J. *Almagestum novum* (1651), which explains the second-pendulum and was extant in the Beitang Library.

126 This textbook, the *Outline of the Essentials of Calculation* (*Suanfa zuanyao zonggang* 算法纂要總綱), was composed by Antoine Thomas between 1689 and 1695. See Han and Jami (2003, 153). In this work a second-pendulum is used for measuring distances by means of the velocity of the sound of cannons. See Jami (2012, 246).

in 1723, shortly after his death.[127] This work can be considered a compilation of the Jesuits'—and for the most part of Antoine Thomas's—lecture notes for Emperor Kangxi (Jami and Han 2003, 91) and eventually describes exactly the hitherto missing procedure to determine, for the calculation of the volumetric flow rate, the velocity of the river current by means of the enigmatic wooden stick and an "instrument for verifying the time [with a] plummet" (*yan shi yi zhuizi* 驗時儀墜子) (see Appendix 2).

The eye-catching correlation between the *QLX* (1683), the Jesuit textbooks written for the emperor's study in the 1690s, and finally the imperial *Shuli jingyun* (1723) suggests that Castelli's ideas had found their way from Verbiest's text via Thomas's tutorship of the emperor into Kangxi's compendium. This would mean that, even though the *QLX* had fallen out of favor with the imperial court and was never officially published, its knowledge about the measuring of the water flow was not lost but integrated with other approaches, and in this form remained in circulation among scholar-officials. At the same time, it must not be forgotten that Emperor Kangxi had claimed exclusively for himself the Jesuits' scientific writings—including the lecture notes they had composed for him—in order to assert his authority against Chinese experts in such fields (Jami 2012, 7–8).[128] This personal appropriation, also and especially of the written word, makes one wonder whether the emperor could indeed be seen as the originator of the mentioned mathematical principles of fluid dynamics, which seem to have drawn on Castelli's ideas instead.[129]

Another hint of the possible influence of Verbiest's treatise on river floods and their prevention is perhaps the fact that the new Marco Polo Bridge, reconstructed by Emperor Kangxi in 1698, was supported by only eleven arches instead of thirteen for the old bridge (Yule 1903, II.35:4–8). Verbiest's text had mentioned bridge piers among the causes of inundations because they narrow the river course and impede the flow, but Chinese hydraulic engineers had already been aware of this obstructing effect since the Song dynasty. In this regard, the late fifteenth-century controversy about the famous multi-arched Chuihong Bridge (Chuihong qiao 垂虹橋), located in the present-day Wujiang District 吳江区 of Suzhou 苏州, can serve as a pertinent example (Li 2010). The main difference in the approach to this issue is, however, that while the Chinese dispute had

127 Jami (2012, 362) assumes that the use of the pendulum device was not yet widespread among Chinese officials at that time. This might in part explain its retarded incorporation into the assessment of the flow (or discharge) rate in China.

128 While during the 1690s Kangxi had been instructed by French Jesuits, with Antoine Thomas as their most prominent representative, he later began to distance himself from them and increasingly counted on the expertise of Chinese scholars. Thus, he established an office of mathematics (*suanxue guan* 算學館) in 1713, which conducted an editorial project concerned with Western teachings, including the compilation of imperially commissioned mathematical treatises such as the *Essence of Numbers and Their Principles*.

129 For the sweeping appropriation of the Jesuit lecture notes in the *Essence of Numbers and Their Principles*, see Jami and Han (2003, esp. 106–108).

been based on mere observation and experience, Verbiest tried to additionally establish a general theoretical quantitative principle underlying this man-made hindrance of the flow, which would then allow for more specific and measured counteraction.

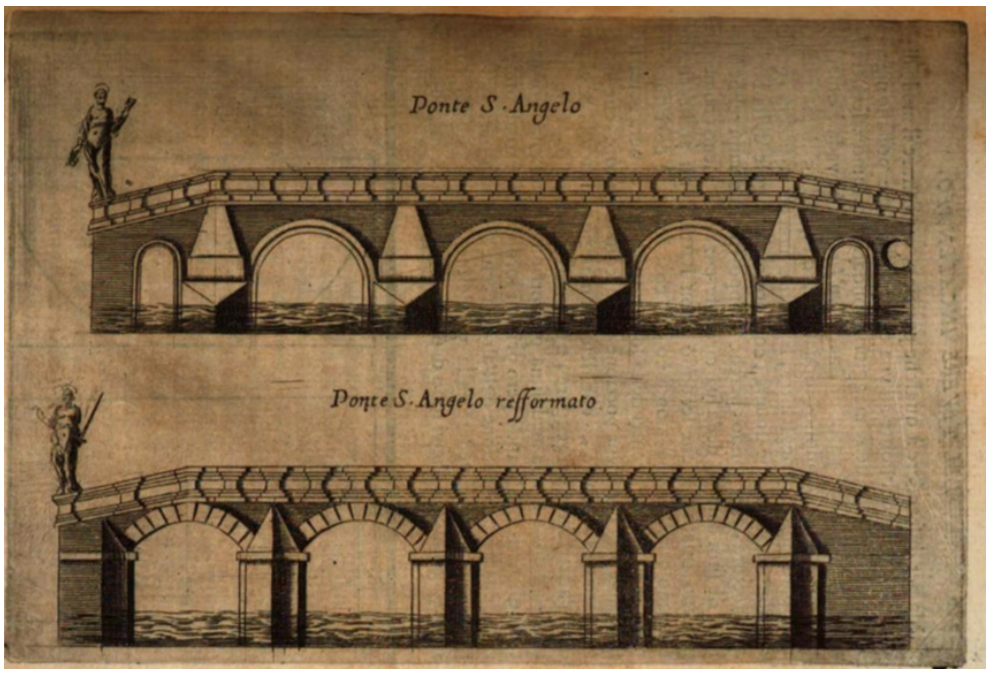


Figure 16: Influence of piers and cross-sections on water flow under the Ponte Sant'Angelo in Rome, original plan (above) and improved plan (below) (Bonini 1663, 315).

Beyond these educated guesses, however, no concrete indications at possible practical applications of Castelli's propositions were documented in Chinese hydraulic treatises at least until the middle of the nineteenth century. In this context, Davids (2006)—probably without knowing about Verbiest's translation of Castelli's treatise—has suggested that the monopoly of knowledge by the imperial bureaucracy precluded, if not internal debates and occasional shifts in procedure, any effort to develop new mathematical approaches to river hydraulics, let alone (we might now add) the adoption of related foreign ideas transferred by Jesuit missionaries. This contrasted strongly with the situation in Europe with its diversity of institutions, scholars, and administrations, which gave rise to new kinds of propositional knowledge much more easily.[130]

130 The distinction between propositional "what" (observation, classification, measuring, and/or the establishment of principles and laws of natural phenomena from them) and prescriptive "how" (instructions, techniques, etc.) knowledge has been coined by Mokyr (2002). In China, and particularly in the field of hydraulics, the former was approached in a mere "ad hoc" fashion on a case-by-case basis, with little reflections of an abstract nature until the nineteenth century. See Davids (2006, 67–68).

6 Summary of results

The results of this study can be summarized as follows: Induced not least by the lasting impression of a flood disaster in Beijing, Ferdinand Verbiest had recognized the importance of effective fluvial flood prevention for the legitimization of the Qing dynasty already in the 1670s. As a hydraulic engineer, he was not only well acquainted with the latest specialist developments in the field in Europe, but had also become aware of the methodological stagnation and the ongoing controversies about adequate approaches to river management in China. He thus decided to fill this gap by introducing in the *QLX* a Chinese rendering of large parts of Castelli's treatise on running waters. This transfer of innovative Western hydraulic knowledge, moreover, fit perfectly into the Jesuit agenda of an indirect mission by means of the so-called "apostolate of the book."[131]

As the preliminary translation and the discussion of its contents have shown, Verbiest's text was still preluded by a traditional Aristotelian account of the origin of rivers and springs, but for the rest is clearly marked by Castelli's pioneering quantitative assessment of the connection between canal design and flow velocities. However, on the more practical side and perhaps with the exclusion of his efforts at a simple cost-benefit analysis for dikes and canals, the supposedly advanced Western gadgets like windmills or sluices presented by Verbiest in the subchapter on precautions against floodings turned out to be more of a revival of devices pre-existing in China but partly forgotten there. Nevertheless, it was the specific way that Verbiest proposed to apply them in river control that stands out and deserves our attention. All these findings shed new light on Verbiest's known hydrotechnical activities and confirm his role as the paramount Jesuit hydraulic engineer in imperial China.

As the *QLX* was never printed, only assumptions could be made about the reception in China of Castelli's ideas via Verbiest's text on river floods and their prevention. Here, Emperor Kangxi's fascination with Western science and technology and his accompanying appropriation of relevant Jesuit writings might help to trace further signs in this direction, especially when taking into account imperially endorsed mathematical compendia of the time. An example here are the computational exercises pertaining to the field of what Catherine Jami (2012, 362) has called "outdoor mathematics," practiced by Kangxi often under the guidance of his Jesuit tutors. These textbook calculations might be related to the basic mensuration approaches that had once been presented by Verbiest in the *QLX*. This would mean that the emperor's aim to monopolize for his own purposes the Western expertise and to strengthen thereby his political authority vis-à-vis leading scholar-officials led to an obliteration of the true

131 In Verbiest's text on flood prevention, however, no hints at his religious missionary ambitions are detectable.

authorship of such methods and, at least in part, ended up in their slumbering in thick tomes. If more actively applied to the planning and the practical execution of hydraulic projects instead, the innovative approaches presented by Verbiest would certainly have had the potential to further the still inadequate river conservancy measures in China—regardless of the shortcomings of the Jesuit's text. But obviously the overall circumstances of the Manchu rule and the increasingly felt need to demonstrate its deep bonds with the Chinese past and culture precluded the open adoption of Western hydrotechnical methods introduced by Christian missionaries for the time being. Nevertheless, even though Verbiest's efforts to introduce Castelli's ideas to China in his ill-fated *QLX* were soon forgotten in China, the scientific knowledge he had transferred was not lost altogether but survived in imperial garb.

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Appendixes

Appendix 1

Transcription of Verbiest's text on river flooding in *QLX*, section *Reasoning in Physics* (*Xingxing zhi litui* 形性之理推), *juan* 9, topic 7 (*di qi ti* 第七題), fols. 23b–37a:[132]

132 For better readability, the text here has been given a modern, sentence style punctuation that therefore differs from that of the original manuscript version.

[23b]**泉源在河海江湖諸水平上永流之所以然**

夫江河湖等，大概從高山之根而發，又其山之高處，多有泉源，永遠不竭。今推其緣由之前，須備設数端之論，以明之。

一曰：夫地深處各方多有藏永水之穴，并多有水遍流通貫之溝，如人渾身之血，遍流于脉絡然。亦有多水從大海，[24a]通至地内極遠處，而海潮永補永存其徃来之流也。

二曰：如此之水脉，多通高山之根。

三曰：高山從下根最深之地，至山頂之處多有空缺，如石洞窪穴，及湾曲之礦等項。

四曰：前所說地深處，多有水流。其水常有經過地脉内，生熱生火之土料，如硫磺石灰等類之鑛脉。因而經過之水，亦帶其熱氣，而其地面各方，多見湯泉永流者，皆由之故耳。

五曰：地内極深之處，多有空缺，其内之氣甚熱。凡開五金[24b]及諸石之鑛者，常經試驗，不能當受其熱。此等熱氣諸山根下空缺深處，極多存留。又地内深處，遍流之水脉，祝經過生熱生火之諸料，而通達山根下空缺之熱處，則其熱必加倍。而其在深窪之水，如地面上，在鍋内之水。由下爐火之滾沸然，自水面徃上，恒有稀開水氣，如烟徃上升。凡過山之空缺，如石洞窪穴。及灣曲之鑛等項，無所不通入之，從大地中極深之處，至地面相近。各國土内，多有養火之空缺道路，通達遍流者，另有本論詳載。

六曰：大山之頂，并與相連之脊嶺。其高常有相近雲彩之[25a]中域，即最冷之所，恒為或冷雲所圍，或霧雪等所遮蓋。故其頂高嶺，恒屬大冷。（山處加冷之緣由，另有他所以然以證之。見光響異驗之理推第三篇。）今查光响異驗之理推，第五篇内，其地面上，所驗下雨之緣由，而以為山空内，不斷下水如雨之比論。盖山根下深處之熱，恒吸起水氣而升，至山顶。正如太陽之熱，從地面上，吸起大海江河之水氣，升至雲彩中域然。又山頂相近之處，既屬多冷，則將其水氣凝结，而變之如雨點然。猶如雲彩之中域，既屬冷，將太陽吸起之水氣，凝结，而變為雨水無異也。其山頂亦可比蒸露水之甑盖。山項空窪，既已[25b]凝结其水氣，則水氣脫所帶之熱，歸于元冷加重。因而随山内空窪之各處，遍流徃下，尋其洩出之門，或山外面之高地，或山根下。順其所得之路，而洩出，正如其蒸露之水氣，既已凝結，而粘在于甑盖。因随甑内窪面形，從上徃下流，于管筒而出，同是一理也。今山根下，既恒有如此之多水。又山頂相近之處，既永存加冷之緣由，故其水氣永遠輪轉，從下徃上，從上徃下流。因而或山外面之高地，或山根下，其泉源永遠不竭，而為江河永流之緣由矣。

江河泛濫之緣由

[26a]所謂江河泛濫者，即其左右兩涯，不能容流水。如此之廣大也，其緣由甚多。今推舉其數端言之耳。

一曰：凡河道上流之水多，入海之水少，此其泛濫之總根也。

二曰：河道入海之口，遇有多阻障。其上流之水，如或有際遇異常高長之海潮， 衝入河道，則河水入海最少，而其近海流下又甚遲，以致阻障。從上流水速，故自高長，而河之原道，不能容收，必出而泛濫也。夫海潮比其常時，異常之加高，或有至一丈至二丈不等。因而推知河口入海之處，[26b]有何等多大之阻當也。

三曰：凡或有異常大風，於河口入海之處，連日逆其流水之力，將海之波濤，并每日兩次之海潮，以衝河水之流，當住其入海。則離海之河道，遠處之水，因而高長亦從茲泛濫也。

四曰：凡河水湧流，隨帶多沙石黃土等，以致或河底高起，或于近海之處，存留不動，因而堵河入海之口，以致從上流之水，愈長愈高。此亦為泛濫之緣由也。

五曰：凡連日下大雨，則江湖之水愈加高長，而入河道之[27a]口，其河道愈狹窄，并高淺，則不能容受如此之多流也。

六曰：凡一河之道中，多有或閘板窄小，或有多橋之鑲。其鑲柱愈寬大者，則能阻當河道之急流，故河水高長，而破堤壩矣。盖閘口　鑲如此之窄小，則河之全道，亦歸於如是之窄小，因而阻當全水之急流也。假如有兩水筩於此，其寬窄不等。（見二百四十六圖。）如甲乙丙丁是也。其甲乙筩流水之口，甲者，與丙丁筩口為相等。題理曰：其兩筩口既相等，其筩寬窄雖不等，則其出流之水，必為相等明矣。

七曰：河道之窄小，凡遇非常之雨，雖河道内不減其流之[27b]速，但從外來入河道之水，既甚多，則河道不能容之。假如有河道于此。其寬不過三丈，其深不過六尺。設令其流之水，時刻之二秒，内，惟經過一尺長之地，則每二秒内，必經過一百八十流水之立方體。然照從外来，入河道水之多，每二秒内，該下流三百流水之立方体，纔可免泛濫之災。今因本河道之窄小，既不能下流如此之多，則河原道之水平，必加四尺之高，此其泛濫之緣由也。若本河之道，原已經挑挖，至十二丈之寬，則照前所設，從外来非常加倍之水，但其河道内之水平加高，不過一尺。因而可免泛濫[28a]也。

江河消長多寡并水流遲速之比例

江河泛濫之總由，在其水高長，并徃下流遲緩。今設數題以推之，其消長多寡，并水流遲速之比例。因而照各地各河之勢，能變道而用之。

第一題

凡海水及逆風等，愈阻衝河口之水流，則河道内之水，愈加其高。又其流之遲，多寡若干，則其高之長，多寡若干，同是一比例。假如有河水于此。一時之四刻内，徃下流，至十[28b]五里後，因海水及逆風阻當之，其下流不過三里，則其水平，比前加五倍之高矣。但如此之加倍，何道之各處，多有不等。緣其加倍之由，各處多有不同。

第二題

有兩河于此，如甲乙丙。其各河道，水流遲速不等。題理曰：其甲河道内，水流多寡若干，與丙河道内，水流多寡若干，者，其比例由甲河道之水門口大小，與丙河水門高大小，並由甲河水流之急，與丙河水之遲，為相結之比例。（所謂水門口者，盖講治河之理。以明悟之功，將各河道内之水，從其水面至河基横切之。又各河之寬，並其各河深之尺寸，彼[29a]此相稱，而以其放水為立門方口形。名為‘河道門口’以便推各河于本道定處水流經過之勢里耳。）假如甲河之門口，乙者，有一百尺之平方數，其两河之門口，丁者，有八十尺之平方數。又甲河之水，時刻之一分，内，徃下流，至四十尺之遠。其丙河之水同一分内，徃下流，至三十尺之遠。法曰：一百尺與四十尺相乘。即得四千。又八十與三十相乘。即得二千四百。理推曰：甲河門口經過之流水。與丙河門口經過之流水，其比例如四千與二千四百之比例。

第三題

[29b]設令連日下大雨，多加河内之水，題理曰：下雨之後，河道之水，多寡與未下雨之前，本道之水，多寡之比例，由其下雨前後水高，并由其永流之急，相結之比例。假如河水高四尺，其徃下流時刻之分内，至四十尺之遠。後雨水長加其高，至二尺。因而亦加其急，即時刻之一分，徃下流至五十尺之遠。法曰：將四尺與四十相乘，又將六尺（前高四尺，後加二尺，共為六尺），與五尺相乘，即得三百。題理曰：未下雨之前，河道内，與下雨之後，河道水，同一時徃下流，多寡若干之比例，如一百六十與三百之比例。

[30a]第四題

凡河水加高，其加高之前，流水之遲，與加高之後，流水之急，為前後之高，與加之比例中，第一率與第二率之比例。假如有河於此。其水未加高之前，深八尺，既加高之後，深十八尺。今欲推其先後急流之分別。法曰：八尺及十八尺中，另推一數，如十二，以爲連比例之中率。題理曰：加高前，流水之遲，與加高後，流水之急，如八與十八之比例焉。

第五題

江河之一道，不拘寬窄之各處，其同一時候，經過之流水[30b]多寡，皆如一。假如甲乙丙丁為河道。（見二百五十三圖。）甲乙為其寬處，丙丁為其窄處。題理曰：甲乙四丈寬處，徃下流之水，與丙丁四丈窄，徃下流之水，多寡皆如一。盖若以為不然，而設若丙丁窄處，經過流水少，其甲乙寬處，經過流水多，則甲乙河道之處，必加高，而丙丁河道之處，必為低。然如此之說，不合於人目常試驗之據。今謂河道内，各處水流多寡相等者，緣窄處交寬處水流之迅急，因而同一時所流之水，多寡，與寬處同一時，所流之水相等。

第六題

[31a]設令有甲河，入丙河，而前後所進入之水，多寡如一。題解曰：丙河水本道内。前後加高之尺寸。與後前之水流。加急多寡若干矣。假如甲河於前時八刻内，既已受丙河之水，則甲河水面，加高十二寸，而其刻之一分内，下流至三丈。後時八刻内，甲河既受丙河之水，則其水面加高六寸，而本刻之一分内，下流至六丈。題理曰：前一時，甲河水加高至十二寸，與後一時，加高至六寸之比例，此為前後率相與之比例。

盖甲河水，於前一時，既為淺而底窄，其水流之勢甚遲，因而下流之水少。故丙河進入其水時，則甲河水[31b]面，忽然加高。今既加高，則亦加水之急，因而甲河水，下流之則多。故於後一時，加高之遂少，為此之故耳。由此而推諸河，凡受從外来之水，前一二日之内，忽然水多加高，至後一二日之内，水少加高。况河道於水淺及底之處，大槩窄小，於水深，及高之處，反寬大。

第七題

設令甲河入丙河。題解曰：甲河在本道内之水高，與其所加丙河道内，於乙處水高之比例，由甲丙两河之寬，并由甲河水流之急，與丙河水流之急，相结之比例。假如乙處水[32a]寬一百尺，甲己處寬六十尺。又其乙處於時刻之一分内，徃下流五十尺之遠，其甲己處之水，於同時刻一分内，徃下流三十尺之遠。法曰：一百與五十相乘，即得五

千。又六十與三十相乘，即得一千八百。今設甲己之水高五尺。題理曰：甲己處水高，與其乙處水高，上所加之分，如四千與一千八百，即其乙處於本水之高，五尺者，另加高一尺零五分之四。

第八題

若欲河道內，水面長高，至一定尺寸，則推知應加水多寡[32b]若干，解此題。惟設河水加減之表，以便其用。但其用法必須河道之水門，就其有法之形。由其有法者可推知，無法者。法曰：於表第一行內。查其一行之水。原高之尺寸。次查本行。其欲加高之尺寸。則第三行內。於正對數目之位。得其各方數。其兩方數之交數。即為應加於原河水高之尺寸也。假如原高為五尺，其欲加高為八尺。法曰：五與八两數之方數，為二十五，與六十四。其交數為三十九。纔得河道內，八尺高之水門，所容水立方尺之數。

第九題

[33a]有法之河道。於此欲知其水高，須減若干，以得本道內一定所求之水高。假如原高為三十尺。今欲減而歸於二十四尺。法曰：三十與四十相減，其較數為六。理推曰：其較數與兩水高之大者如一，與他數即六于與三十如一，于五。因此而推知原舊之高，三十尺者，須減其五分，以得其所求也。

江河泛濫之防備

從前所設泛濫之緣由若干，可推知其相應之防備若干。今畧舉數端，以預備其便用。

[33b]一曰：凡開新河，已減其或大江，或他河之水漲，而引通入海，其新河道愈寬愈深，則其效驗愈速。故凡於江及河等所開之河門，其門口與江之水面，又於河道之長短，必須相稱之寬大。理推曰：不拘或江或河，其各水面之寬大，與各水門之寬大若干，則各水門內，流水之急，與各水面減其高若干。雖然，如此凡開新河之道，以免泛濫之災，或有時得其用，亦或有時無大用也。盖江河等，其水之漲高，若留存不過數刻之久，則用之有其利，若留存之二十三十日之久，其用無大效也。緣屢次已經試驗。凡連日或照常[34a]下雨，或從旁邊小河之水，入大江大河，則易補其新開之河，所能減去之水漲。故或開新河，或加工以堅固舊河之堤壩，兩者何一更有利用，何一更費錢粮，宜细心酌量而定焉。（看後篇之假如以証此一端之理。）

二曰：凡江河之泛濫，或從海潮加倍異常之水高，或從連日之逆風，並海水之常潮而生，則開新河，不如加工，起高堅固，其河道之舊堤。但若高其堤，則於海相近處，不如離海之遠處。盖泛濫之災，大槩在於離海之遠處，如遠西巴多河。從海密近之處，至二十三十里內，其堤常常不過五[34b]六尺之高，至一百二十里，或一百五十里遠，則其堤加高至二十尺有餘，而彼處尚難免泛濫之災。審此，各地照從古至今，歷年已經驗，大河水最高處，多寡不等，相應其堤高，加倍若干。

三曰：凡開新河以放舊河之水，引導入海，其新河道愈直，其水下流愈速，其河道灣曲愈多，則其水下流愈遲。並急壞河道兩邊堤岸。又凡河邊多空缺不平，或多生草木等根，則在河邊水內者，多能阻當水流之速，因而漸次高起泛濫，其河邊宜偏下来如斜坡然。不然，水流亦浸其土，而[35a]窪內多空以致邊岸灘塌。總而論之，此一端河道流水，愈冲磨，或河邊，或河底，愈減其下行之順速。故開河道之時，其寬與深，必須有

相稱之理，以免水之冲磨灘塌。定例曰：河道之深，不宜勝過河道之寬，又其寬之半，不宜勝過河道之深也。不然，流水多冲磨河邊河底，因而多減其流水之速。另有圖說以證此論。由此而推知，凡河道中多有閘口，其口愈寬大其水徃下流愈速。夫閘口左右兩邊必須預備放水之門。又於其閘口宜然，法立門兩扇，並合於三角形之稜，以敵水冲之力。其門兩扇之造法，用“水沖河沙”[35b]之法，及“河船安寧”易過閘之法，俾閘之前後之水，皆歸於一平面。因而甚易甚速，諸船一齊可以徑過，並無上水下水之險危者，另有本論詳之。若或近河之田地，比河道之水面更低，因而從各處高地有雨水下来，淹浸之，則田地之中，宜縱横挑挖受水之數溝，引到其水至河旁相近之處，總歸一所。然後照法以風輪等器，將其水堤起過岸入於河内。所謂風輪者，即借風力而晝夜不斷，提起此水，使之過岸，別有水法之論載之也。

[36a]**以假如明証首端之論**

設令有湖於此，其寬長各一百里，欲洩本湖之水入海，以免泛濫。故開新河，長四百里，寬八丈，深一丈二尺，以為洩湖之河道。然新河之寬大如此，相較湖水之寬大如彼，則其免泛濫之效，似不相當焉。蓋新河满道之水，比全湖水面一尺之深，不過三十二分之一分耳。（河道水寬、長、深如此，若以筭歸於一尺立方之水，其滿道水之立方數相比於全湖水面一尺深之立方數，則僅為三十二分之一分耳。）審此，則三十二條河道滿水既已入海，其全湖之水面，減其高，不過一尺而已。今設令新河滿道之水，兩日内，從湖口起，一次，全下流四[36b]百里入海，則三十二次全下流，必須六十四日，方可入海也。其六十四日，河水不斷下流，既已經入海，則全湖之水面，減其高，僅一尺耳。今照常下雨之時，若或將方器或高圓之器，内收雨水，其器内之雨水，每一刻，約加一分之高者。係已經試驗也。則一百刻（即一晝夜），平常下雨不斷之時，其器内之水，加一百分，即一尺之高者明矣。然大湖水面加其高，與器内水面加其高，無異，同是一理。揆此，則照常連日不斷下雨，其雨水一晝夜内，即還補其新河六十四日内，所減全湖水面一尺之高矣。况從周圍下流山水，並小河之澗水入湖，愈增湖水之高者，甚明矣。故若不加工高[37a]起堅固湖堤，並河邊，則連日下雨，其湖與河，既已滿水，恐仍有泛濫之災矣。今四百里之新河，寬、深如此，共計有六億零一千一百二十萬立方土之尺。若湖提四面高起五尺者，則共計不過七千二百萬立方土之尺耳。故挑新河之費用，較湖堤高起五尺之費用，加九倍。而湖堤之高起如此，新河之挑挖如彼，則其加五倍之利益明矣。

Appendix 2

Example of measuring the flow of a river in *Essence of Numbers and Their Principles Imperially Composed* (*Yuzhi shuli jingyun* 御製數理精蘊; 1723), second part (*xiabian* 下編), *juan* 37, final section 7 (*mobu qi* 末部七), “Difficult Problems” (*nanti* 難題), fols. 14a–15a.[133]

133 Edition *Qinding siku quanshu* 欽定四庫全書, *Masters and Philosophers 6* (*Zibu liu* 子部六), *Astronomy and Mathematics* (*Tianwen suanfa lei* 天文算法類). Punctuation of the Chinese text and translation by Sabine Kink.

設如河口上寬十尺下寬六尺深五尺求每日流水幾何

Supposing a river opening [that is, its cross-section] has a width of 10 *chi* at the top, of 6 *chi* at the bottom, and a depth of 5 *chi*, one seeks how much water flows [through it] every day.

法：以木板一塊，置於水面，用驗時儀墜子候之。看六十秒內，木板流遠幾丈。如流遠十丈，即以十丈變為一百尺。乃以河上寬十尺與下寬六尺，相加折半得八尺，與河深五尺相乘，得四十尺。又與木板流遠一百尺相乘，得四千尺，即六十秒內所流之數。又以六十秒收為一分，為一率，水流四千尺為二率。以每日二十四小時，化為一千四百四十分一小時為四刻，一刻為十五分為三率。求得四率五千七百六十萬尺，即一日內所流之數也。此法先用木板，以驗所流之緩急。水急則木隨水流亦急，水緩則木隨水流亦緩。看木之緩急，即知水流之多少。故先求得河口面積，再以遠乘之，即得水流之積數也。

As to the method [for figuring this out]: Place a wooden stick in one piece on the surface of the water and watch out for it by using a chronometer (*yan shi yi* 驗時儀)[134] [with a] plummet (*zhuizi* 墜子). Examine [thereby] how many *zhang* in distance the stick has flowed within 60 seconds. If the flow distance is 10 *zhang*, take these 10 *zhang* and change them into 100 *chi*. Then take the river width of 10 *chi* at the top and of 6 *chi* at the bottom, add them, and divide [the result] in half to obtain 8 *chi*. Multiplied by the river depth of 5 *chi*, one obtains 40 *chi*. Again, multiplied by the wooden stick's flow distance of 100 *chi*, one obtains 4000 *chi*, which is the flow [distance] within 60 seconds. Further, gather (*shou* 收) 60 seconds into one minute as the first term (*lü* 率), and take the water flow of 4000 *chi* as the second term [of the proportion]. Transform the 24 hours (*xiaoshi* 小時) of every day into 1440 minutes—1 hour has 4 quarters and 1 quarter 15 minutes—as the third term. [These terms] are required to obtain the fourth term of 57,600,000 *chi*, which is the amount of flow within one day. This method first uses a wooden stick to verify the velocity of the flow. When the water is fast, then by following the water flow the wooden [stick] is also fast, and when the water is slow, then by following the water flow the wooden [stick] is also slow. By examining the velocity of the wooden [stick], one then knows how much water has flowed. Therefore, it is first required to obtain the surface area of the river opening [that is, of its cross-section], next [this area] is multiplied by the distance [the stick has flowed], and then one obtains the [amount of volumetric] water

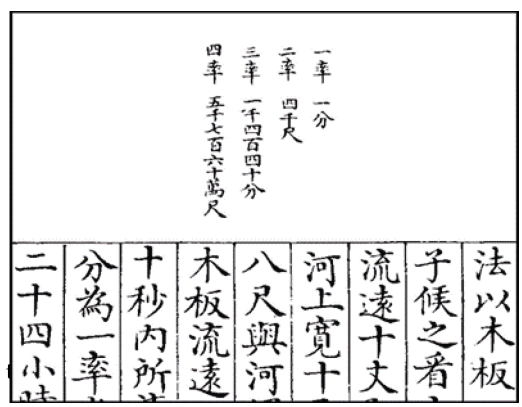


Figure 17: Textbook style presentation of number values (*lü* 率). *Essence of Numbers and Their Principles Imperially Composed*, *juan* II.37, fol. 14b (detail).

134 Literally: "An instrument for verifying the time."

flow as the product [of these terms].

This (from today's point of view) simple arithmetical task contains an easily recognizable calculation error, as with 57,600,000 the result of the multiplication of the established three terms (1 × 4000 × 1440) is one order of magnitude higher than the correct value, which is 5,760,000. One can only speculate about the reason why this remarkable mistake was overseen by the compilers of the *Shuli jingyun*, a committee of appointed editors working at the imperially supervised office of mathematics (*suanxue guan* 算學館) since 1713.[135] But in light of the extraordinary length of this compendium (the whole text covers almost 4900 pages), it would not be surprising to find other similar mistakes.

In addition to the obvious relationship with Castelli's ideas and Verbiest's text on floods in the *QLX*, closer scrutiny reveals that this sample computation contains several hints at their connection with the above-mentioned Jesuit lecture notes written for Emperor Kangxi. Thus, form and style of this short example clearly show that it was meant as a calculation instruction for learners. This includes a detailed step-by-step procedure, the repeated insertion of further explanations, and even a visual aid on top of the pages. Moreover, the timepiece with a plummet (*zhuizi* 墜子) mentioned in the text makes use of the fact that for a distinct period of time the length of the pendulum is proportional to the square of the number of its swings. This time-measuring instrument had been introduced by Verbiest already in the 1670s, but interestingly it is not mentioned in the *Explanation of the Proportional Compass* (*Biligui jie* 比例規解), on which the above calculation example is based (Jami 2012, 362). And finally, the step-by-step development of the calculation process by using the expression *lü* 率 is used here in a way strongly resembling similar approaches found in Antoine Thomas's *Outline of the Essentials of Calculation* (*Suanfa zuanyao zonggang* 算法纂要總綱).

135 For the members of this committee, see Jami (2012, 373–378).